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Mathematics for Common Schools

A MANUAL FOR TEACHERS

INCLUDING

DEFINITIONS, PRINCIPLES, AND RULES
AND SOLUTIONS OF THE MORE
DIFFICULT PROBLEMS

BY

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SUPPLEMENT.

MANUAL FOR TEACHERS

I

INTRODUCTORY

Plan and Scope of the Work. — In addition to the subjects generally included in the ordinary text-books in arithmetic, *Mathematics for Common Schools* contains such simple work in algebraic equations and constructive geometry as can be studied to advantage by pupils of the elementary schools.

The arithmetical portion is divided into thirteen chapters, each of which, except the first, contains the work of a term of five months. The following extracts from the table of contents will show the arrangement of topics:

First and Second Years

Chapter I. — Numbers of Three Figures. Addition and Subtraction.

Third Year

Chapters II. and III. — Numbers of Five Figures. Multipliers and Divisors of One Figure. Addition and Subtraction of Halves, of Fourths, of Thirds. Multiplication by Mixed Numbers. Pint, Quart, and Gallon; Ounce and Pound. Roman Notation.

Fourth Year

Chapters IV. and V. — Numbers of Six Figures. Multipliers and Divisors of Two or More Figures. Addition and Subtraction of Easy Fractions. Multiplication by Mixed Numbers. Simple Denominate Numbers. Roman Notation.

Fifth Year

Chapters VI. and VII. — Fractions. Decimals of Three Places. Bills. Denominate Numbers. Simple Measurements.

Sixth Year

Chapters VIII. and IX. — Decimals. Bills. Denominate Numbers. Surfaces and Volumes. Percentage and Interest.

Seventh Year

Chapters XI. and XII. — Percentage and Interest. Commercial and Bank Discount. Cause and Effect. Partnership. Bonds and Stocks. Exchange. Longitude and Time. Surfaces and Volumes.

Eighth Year

Chapters XIII. and XIV. — Partial Payments. Equation of Payments. Annual Interest. Metric System. Evolution and Involution. Surfaces and Volumes.

While all of the above topics are generally included in an eight years' course, it may be considered advisable to omit some of them, and to take up, instead, during the seventh and eighth years, the constructive geometry work of Chapter XVI. Among the topics that may be dropped without injury to the pupil are Bonds and Stocks, Exchange, Partial Payments, and Equation of Payments.

Grammar School Algebra. — Chapter X., consisting of a dozen pages, is devoted to the subject of easy equations of one unknown quantity, as a preliminary to the employment of the equation in so much of the subsequent work in arithmetic as is rendered more simple by this mode of treatment. To teachers desirous of dispensing with rules, sample solutions of type examples, etc., the algebraic method of solving the so-called "problems" in percentage, interest, discount, etc., is strongly recommended.

In Chapter XV., intended chiefly for schools having a nine years' course, the algebraic work is extended to cover simple equations containing two or more unknown quantities, and pure and affected quadratic equations of one unknown quantity.

No attempt has been made in these two chapters to treat algebra as a science; the aim has been to make grammar-school pupils acquainted, to some slight extent, with the great instrument of mathematical investigation, — the equation.

Constructive Geometry. — Progressive teachers will appreciate the importance of supplementing the concrete geometrical instruction now given in the drawing and mensuration work. Chapter XVI. contains a series of problems in construction so arranged as to enable pupils to obtain for themselves a working knowledge of all the most important facts of geometry. Applications of the facts thus ascertained, are made to the mensuration of surfaces and volumes, the calculation of heights and distances, etc. No attempt is made to anticipate the work of the high-school by teaching geometry as a science.

be required. It is a pedagogical mistake to insist that all of the pupils of a class should set down a number of figures that are not needed by the brighter ones. As an occasional exercise, it may be advisable to have scholars give all the work required to solve a problem, and to make a written explanation of each step in the solution; but it should be the teacher's aim to have the majority of the examples done with as great rapidity as is consistent with absolute correctness. It will be found that, as a rule, the quickest workers are the most accurate.

Many of the slate problems can be treated by some classes as "sight" examples, each pupil reading the question for himself from the book, and writing the answer at a given signal without putting down any of the work.

Use of Books. — It is generally recommended that books be placed in pupils' hands as early as the third school year. Since many children are unable at this stage to read with sufficient intelligence to understand the terms of a problem, this work should be done under the teacher's direction, the latter reading the questions while the pupils follow from their books. In later years, the problems should be solved by the pupils from the books with practically no assistance whatever from the teacher.

Conduct of the Recitation. — Many thoughtful educators consider it advisable to divide an arithmetic class into two sections, for some purposes, even where its members are nearly equal in attainments. The members of one division of such a class may work examples from their books while the others write the answers to oral problems given by the teacher, etc.

Where a class is thus taught in two divisions, the members of each should sit in alternate rows, extending from the front of the room to the rear. Seated in this way, a pupil is doing a different kind of work from those on the right and the left, and he would not have the temptation of a neighbor's slate to lead him to compare answers.

As an economy of time, explanations of new subjects might be given to the whole class; but much of the arithmetic work should be done in "sections," one of which is under the immediate direction of the teacher, the other being employed in "seat" work. In the case of pupils of the more advanced classes, "seat" work should consist largely of "problems" solved without assistance. Especial pains have been taken to so grade the problems as to have none beyond the capacity of the average pupil that is willing to try to understand its terms. It is not necessary that all the members of a division should work the same problems at a given time, nor the same number of problems, nor that a new topic should be postponed until all of the previous problems have been solved.

Whenever it is possible, all of the members of the division working under the teacher's immediate direction should take part in all the work done. In mental arithmetic, for instance, while only a few may be called upon for explanations, all of the pupils should write the answers to each question. The same is true of much of the sight work, the approximations, some of the special drills, etc.

Drills and Sight Work. — To secure reasonable rapidity, it is necessary to have regular systematic drills. They should be employed daily, if possible, in the earlier years, but should never last longer than five or ten minutes. Various kinds are suggested, such as sight addition drills, in Arts. 3, 11, 24, 26, etc.; subtraction, in Arts. 19, 50, 53, etc.; multiplication, in Arts. 71, 109, etc.; division, in Arts. 199, 202, etc.; counting by 2's, 3's, etc., in Art. 61; carrying, in Art. 53, etc. For the young pupil, those are the most valuable in which the figures are in his sight, and in the position they occupy in an example; see Arts. 3, 34, 164, etc.

Many teachers prepare cards, each of which contains one of the combinations taught in their respective grades. Showing one of these cards, the teacher requires an immediate answer

from a pupil. If his reply is correct, a new card is shown to the next pupil, and so on. Other teachers write a number of combinations on the blackboard, and point to them at random, requiring prompt answers. When drills remain on the board for any considerable time, some children learn to know the results of a combination by its location on the board, so that frequent changes in the arrangement of the drills are, therefore, advisable. The drills in Arts. 111, 112, and 115 furnish a great deal of work with the occasional change of a single figure.

For the higher classes, each chapter contains appropriate drills, which are subsequently used in oral problems. It happens only too frequently that as children go forward in school they lose much of the readiness in oral and written work they possessed in the lower grades, owing to the neglect of their teachers to continue to require quick, accurate review work in the operations previously taught. These special drills follow the plan of the combinations of the earlier chapters, but gradually grow more difficult. They should first be used as sight exercises, either from the books or from the blackboard.

To secure valuable results from drill exercises, the utmost possible promptness in answers should be insisted upon.

Definitions, Principles, and Rules. — Young children should not memorize rules or definitions. They should learn to add by adding, after being first shown by the teacher how to perform the operation. Those not previously taught by the Grube method should be given no reason for "carrying." In teaching such children to write numbers of two or three figures, there is nothing gained by discussing the local value of the digits. During the earlier years, instruction in the art of arithmetic should be given with the least possible amount of science. While principles may be incidentally brought to the view of the children at times, there should be no cross-examination thereon. It may be shown, for instance, that subtraction is the reverse of addition, and that multiplication is a short method of combining equal

numbers, etc.; but care should be taken in the case of pupils below about the fifth school year not to dwell long on this side of the instruction. By that time, pupils should be able to add, subtract, multiply, and divide whole numbers; to add and subtract simple mixed numbers, and to use a mixed number as a multiplier or a multiplicand; to solve easy problems, with small numbers, involving the foregoing operations and others containing the more commonly used denominate units. Whether or not they can explain the principles underlying the operations is of next to no importance, if they can do the work with reasonable accuracy and rapidity.

When decimal fractions are taken up, the principles of Arabic notation should be developed; and about the same time, or somewhat later, the principles upon which are founded the operations in the fundamental processes, can be briefly discussed.

Definitions should in all cases be made by the pupils, their mistakes being brought out by the teacher through appropriate questions, criticisms, etc. Systematic work under this head should be deferred until at least the seventh year.

The use of unnecessary rules in the higher grades is to be deprecated. When, for instance, a pupil understands that *per cent* means *hundredths*, that seven per cent means seven hundredths, it should not be necessary to tell him that 7 per cent of 143 is obtained by multiplying 143 by .07. It should be a fair assumption that his previous work in the multiplication of common and of decimal fractions has enabled him to see that 7 per cent of 143 is $\frac{7}{100}$ of 143 or $143 \times .07$, without information other than the meaning of the term "per cent."

When a pupil is able to calculate that 15% of 120 is 18, he should be allowed to try to work out for himself, without a rule, the solution of this problem: 18 is what per cent of 120? or of this: 18 is 15% of what number? These questions should present no more difficulty in the seventh year than the following examples in the fifth: (*a*) Find the cost of $\frac{3}{20}$ ton of hay at $12 per ton. (*b*) When hay is worth $12 per ton, what part of a

ton can be bought for \$1.80? (*c*) If $\frac{3}{20}$ ton of hay costs \$1.80, what is the value of a ton?

When, however, it becomes necessary to assist pupils in the solution of problems of this class, it is more profitable to furnish them with a general method by the use of the equation, than with any special plan suited only to the type under immediate discussion.

In the appendix to the Manual will be found the usual definitions, principles, and rules, for the teacher to use in such a way as her experience shows to be best for her pupils. The rules given are based somewhat on the older methods, rather than on those recommended by the author. He would prefer to omit entirely those relating to percentage, interest, and the like as being unnecessary, but that they are called for by many successful teachers, who prefer to continue the use of methods which they have found to produce satisfactory results.

Language. — While the use of correct language should be insisted upon in all lessons, children should not be required in arithmetic to give all answers in "complete sentences." Especially in the drills, it is important that the results be expressed in the fewest possible words.

Analyses. — Sparing use of analyses is recommended for beginners. If a pupil solves a problem correctly, the natural inference should be that his method is correct, even if he be unable to state it in words. When a pupil gives the analysis of a problem, he should be permitted to express himself in his own way. Set forms should not be used under any circumstances.

Objective Illustrations. — The chief reason for the use of objects in the study of arithmetic is to enable pupils to work without them. While counters, weights and measures, diagrams, or the like are necessary at the beginning of some topics, it is important to discontinue their use as soon as the scholar is able to proceed without their aid.

Approximate Answers. — An important drill is furnished in the "approximations." (See Arts. 521, 669, 719, etc.) Pupils should be required in much of their written work to estimate the result before beginning to solve a problem with the pencil. Besides preventing an absurd answer, this practice will also have the effect of causing a pupil to see what processes are necessary. In too many instances, work is commenced upon a problem before the conditions are grasped by the youthful scholar; which will be less likely to occur in the case of one who has carefully "estimated" the answer. The pupil will frequently find, also, that he can obtain the correct result without using his pencil at all.

Indicating Operations. — It is a good practice to require pupils to indicate by signs all of the processes necessary to the solution of a problem, before performing any of the operations. This frequently enables a scholar to shorten his work by cancellation, etc. In the case of problems whose solution requires tedious processes, some teachers do not require their pupils to do more than to indicate the operations. It is to be feared that much of the lack of facility in adding, multiplying, etc., found in the pupils of the higher classes is due to this desire to make work pleasant. Instead of becoming more expert in the fundamental operations, scholars in their eighth year frequently add, subtract, multiply, and divide more slowly and less accurately than in their fourth year of school.

Paper *vs.* **Slates.** — To the use of slates may be traced very much of the poor work now done in arithmetic. A child that finds the sum of two or more numbers by drawing on his slate the number of strokes represented by each, and then counting the total, will have to adopt some other method if his work is done on material that does not permit the easy obliteration of the tell-tale marks. When the teacher has an opportunity to see the number of attempts made by some of her pupils to obtain the correct quo-

tient figures in a long division example, she may realize the importance of such drills as will enable them to arrive more readily at the correct result.

The unnecessary work now done by many pupils will be very much lessened if they find themselves compelled to dispense with the "rubbing out" they have an opportunity to indulge in when slates are employed. The additional expense caused by the introduction of paper will almost inevitably lead to better results in arithmetic. The arrangement of the work will be looked after; pupils will not be required, nor will they be permitted, to waste material in writing out the operations that can be performed mentally; the least common denominator will be determined by inspection; problems will be shortened by the greater use of cancellation, etc., etc. Better writing of figures and neater arrangement of problems will be likely to accompany the use of material that will be kept by the teacher for the inspection of the school authorities. The endless writing of tables and the long, tedious examples now given to keep troublesome pupils from bothering a teacher that wishes to write up her records, will, to some extent, be discontinued when slates are no longer used.

III

EARLY ARITHMETIC TEACHING

Counting. — While the majority of children are able, upon entering school, to repeat the names of the first ten or more numbers, they are not always able to count things. The first duty of the teacher is to secure correct notions of the first nine numbers, and this can best be done by the employment of objects, such as beans, splints, shoe-pegs, blocks, etc. A numeral frame is very useful for this purpose.

In counting, it is very important to have the child understand that the *second* splint is not *two* splints. This may be made clear to a child by having him put on his desk one bean, then near it two beans, three beans in another place, etc. After the pupil can count understandingly to nine, he should be taught the figures. The notation and numeration of numbers of two or more figures will be discussed in later chapters.

Primary Arithmetic. — After children have learned to count readily, experts disagree as to the best method of procedure. Many excellent teachers believe that work should be commenced at once upon numeration and notation, followed by the fundamental operations in the usual order. Some of the advocates of this method favor the completion of each topic before proceeding to the next; that is, numeration and notation are taught at least to billions; then addition is taken up, beginning with small numbers and gradually increasing to examples containing numbers of eight or nine figures. Subtraction, multiplication, and division are each studied to this extent before the next is commenced.

The more intelligent advocates of teaching operations at the

outset, recognize the fact that it is neither necessary nor advisable to defer the addition of small numbers until children are able to write those of three or more periods, nor to postpone finding the sum of $\frac{1}{2}$ and $\frac{1}{4}$ until after the properties of numbers have been studied in the fifth school year. Their plan is to follow such simple examples in the addition of small numbers as involve no carrying, by corresponding ones in subtraction. More difficult examples in both of these operations come next, followed by simple ones in multiplication and division. Easy work in fractions is introduced at an early stage, and problems involving the more common denominate units are brought in from time to time.

The Grube Method. — A growing number of educators believe that early arithmetical instruction should be based upon the study of numbers, rather than upon that of processes, — that the former should be the prominent feature of the early instruction, and the latter incidental, at least for the first two years.

This method, called after its inventor, Grube, requires the teaching of all of the processes in the case of each number before proceeding to the next. Thus, when the number 4 is studied, the pupil measures it by all numbers smaller than itself. Using 4 beans, he measures by 1, by arranging them as follows: 0 0 0 0. In this way he sees that $1+1+1+1=4$; that there are 4 ones in 4, or $1 \times 4 = 4$; that $4-1-1-1=1$; that $4 \div 1 = 4$.

Measuring by 2, 00 00, he sees that $2+2=4$, $2 \times 2 = 4$, $4-2=2$, $4 \div 2 = 2$.

Measuring by 3, 000 0, he sees that $3+1=4$, $1+3=4$; $4-3=1$, $4-1=3$; that $(1 \times 3)+1=4$, and that $4 \div 3 = 1$ and 1 over.

The pupil then answers questions given by the teacher, first using the counters and afterwards without them: —

Four is how many more than 3? Than 1? Than 2? Three is how many less than 4? Two is how many less? One is how many less?

How many ones in 4? How many twos? Threes? One-half of 4 is what? Two is $\frac{1}{2}$ of what number?

Problems containing the foregoing combinations are then given in great variety by the teacher until all of the facts about the number 4 in its relation with the smaller numbers are fully mastered.

In teaching any number, no larger number must appear in any way whatever. During the study of 4, it is not permissible to ask 4 twos, or that 4 is 1 less than what, etc., etc.

The work proceeds slowly and thoroughly, at least a year being devoted to the numbers below 10. The second year is given to the numbers from 10 to 20, and the third year to those from 20 to 100. This is probably as far as the method is carried in this country.

In the greater number of the schools using this method, systematic instruction in the fundamental processes is commenced by the beginning of the third year; while in some, the Grube method is used for oral work, and the teaching of slate addition is carried on at the same time, even during the first year.

Slate Problems. — When, instead of receiving oral instruction for some time, children are taught processes from the outset, it frequently happens that many of them show little ability in solving problems. While some attention should be given in the early years to this side of arithmetic, it should not be permitted to retard too much the advancement of pupils. Many of them have to leave school soon, and they should be taught as rapidly as is consistent with real progress to perform accurately the ordinary operations in whole numbers, simple fractions, and decimals. Being familiar with these tools, greater maturity will, of itself, show which is to be used in such questions as are likely to come up in ordinary avocations.

The teacher should exercise much care to give only such problems as can readily be understood by the pupil, and which do not contain too many conditions or numbers that bewilder

the learner. While a beginner will have no difficulty in determining whether to add or subtract in a mental problem suited to his capacity, the same kind of problem with larger figures will give him much difficulty. For this reason, the earlier slate problems should be the merest trifle beyond his ability to solve mentally. In his attempt to work them out in his head, he will determine whether addition or subtraction is needed, etc.

Problems in all grades should be "miscellaneous," and pupils should be allowed as far as possible to determine for themselves what operation is necessary to solve any given one.

IV

NOTES ON CHAPTER ONE

THE hints given as to the work of this chapter are intended chiefly for the guidance of teachers of young children that are beginning slate work in the fundamental processes without much preliminary oral instruction. Pupils that have been taught for two years by the Grube method should not be required to spend unnecessary time on the simpler portions of the work.

Art. **4.** — In teaching notation of numbers of two figures to young children that have not been previously taught by the Grube method, it is not advisable to lay stress on the local value of the tens' figure. Show them how to read and write 10, 11, 12, etc., to 20; then 30, 40, 50, etc., to 90. After this, there is but little difficulty.

7. By working an example for the pupils, teach them to place under each column its sum. As their tendency is to begin working at the left, be careful to see that they always commence to add at the right.

9. The problems will present no difficulty, as they involve only addition.

11. These sight exercises may first be employed as drills to teach children to use in blackboard addition as few words as possible. The first figure should not be named, — only the sum of the first and the second, then this total added to the third. In subsequent drills upon these combinations, each pupil should, in turn, give the sum of any set indicated by the teacher. The work should be done rapidly to be of value.

13. The making of original problems by the pupils should be a feature of every grade.

15 and **16.** Subtraction is here introduced by the "building-up" method. Pupils find it easier to ascertain the difference between two numbers by going forward from the smaller to the larger, than by "taking away" one from the other.

17 consists of sight exercises in the form of addition, leading to the subtraction exercises in Art. 19.

21. While in adding, the use of the word *and* is considered unnecessary; in subtracting, it is used just before the figure that is to be written.

For some advantages obtained by employing the "building-up" method, see Art. 384, where it is used to obtain in one operation the difference between 1000 and $643 + 287 + 25$. In Art. 385, it is used to find a remainder in long division without writing the product of the divisor by the quotient.

23. Here begins the real problem work, as the pupil has now to determine for the first time in slate examples whether the result is to be reached by addition or subtraction. When the pupils are able to solve one of these problems without using the pencil, it should be repeated, but with such a change in one of the numbers as will render necessary the use of the slate. For the 10 cents in the first example, for instance, 14 cents or 24 cents may be substituted.

As many pupils attend rather to the numbers in a problem than to its terms, some may subtract when they should add, especially as this seems the natural operation when only two numbers are involved. It is important that they should be led to see that the size of the numbers does not change the nature of the example, and that they can easily determine whether addition or subtraction is required, by considering what operation

they would employ in a similar example containing very small figures.

It is not advisable as a regular thing to follow an oral problem by a written one of exactly the same nature, as this tends to make children inattentive to the terms of the latter when they already know from the oral problem what operation is required.

28. It is inadvisable to waste time in endeavoring to make clear to very young children the reason for "carrying."

37. Teachers should require pupils to write the proper sign before working an example, as this tends to make them listen more carefully in order to determine whether addition or subtraction is involved. In some problems that are too simple to need the use of the pencil, changes may be made in the numbers employed; great care, however, should be taken not to use numbers so large as to confuse the pupils.

38. Have children understand that when a number contains the word "hundred," it should consist of three figures. Do not explain.

54. These exercises are intended to lead up to the subtraction with "borrowing" in the next article. Perhaps the following would be a better arrangement:

$$\begin{array}{r} ? \\ +29 \\ \hline 41 \end{array} \qquad \begin{array}{r} ? \\ +37 \\ \hline 50 \end{array} \qquad \begin{array}{r} ? \\ +17 \\ \hline 25 \end{array} \qquad \begin{array}{r} ? \\ +86 \\ \hline 90 \end{array} \qquad \begin{array}{r} ? \\ +\ 75 \\ \hline 100 \end{array} \qquad \begin{array}{r} ? \\ +\ 90 \\ \hline 150 \end{array}$$

As children are generally taught to begin with the bottom figure in addition, they will naturally say in the first example, 9 and 2 are 11, writing the 2 in its place, etc.

55. Subtraction with "borrowing" is generally taught in one of three ways. The "building-up" method given in the text is the most readily taken hold of by young pupils.

$$\begin{array}{r} {}^{11} \\ 4\cancel{1} \\ {}^{8} \\ \underline{\cancel{2}9} \end{array}$$

By the second method, the child is instructed that whenever he increases by ten any figure of the minuend, he must add 1 to the next figure of the subtrahend. Seeing that he cannot take 9 from 1, he says 9 from 11 leaves 2; 1 (to carry) and 2 are 3, 3 from 4 leaves 1.

While this method is just as logical as the next, it is not so easily "explained," and, for this reason, is not so much favored by many teachers of the present day.

$$\begin{array}{r} {}^{8\,11} \\ \cancel{4}\cancel{1} \\ \underline{29} \end{array}$$

The third method consists of diminishing the next left-hand figure of the minuend after "borrowing." Where the minuend contains ciphers, this method is particularly confusing to beginners, especially where they are forbidden, as should be the case, to write the changes that are made in the figures of the minuend.

Except in the addition of long columns, children should be required from the beginning of slate work to abstain from counting, writing "carrying" figures, and the like. The guide figures introduced into the foregoing explanations of methods of subtracting should not be used by pupils.

61. As a change from sight work, and to increase the pupils' readiness in the solution of mental examples, these drills are useful. Not requiring any preliminary writing on the board, they can be taken up at any time the class is unoccupied for a few minutes — waiting for the signal to go home, for example.

The pupils all stand; the teacher announces the number to be added, 2 for instance, and begins by saying 1 herself. The first pupil says 3, then sits; the next, 5; and so on. After 39, or some other convenient number, is reached, the teacher begins by saying 2, and the pupils, in order, give 4, 6, 8, etc., to 40.

The intelligent teacher will be careful to suit these drills to the capacity of her pupils. She will not weary beginners by spending too much time on the more difficult drills with 7, 8, and 9; nor will she waste the time of older scholars by dwelling on the addition by twos.

The same kind of work may be employed as subtraction drills.

Subtract by twos:

40,	38,	36,	etc.
39,	37,	35,	etc.

By threes:

40,	37,	34,	etc.
39,	36,	33,	etc.
38,	35,	32,	etc.

By fours:

40,	36,	32,	etc.
39,	35,	31,	etc.
38,	34,	30,	etc.
37,	33,	29,	etc.

V

NOTES ON CHAPTER TWO

74. Slate multiplication is commenced as soon as the table of 2 times is learned. The first examples contain no carrying.

76. Division tables should not be memorized.

81. Do not permit children to prefix an unnecessary cipher in the quotient of $100 \div 2$; that is, do not have the answer written 050.

84. Many scholars think that when a slate problem contains a very small number and a large one, they must either multiply or divide. Examples 1–4 are given with simple numbers to show them that the nature of the operation depends entirely upon the conditions of a problem. While pupils should not be required to use a pencil to solve a problem that can be solved mentally, it would help the class to have these four examples worked on the board as an indication that in the subsequent examples there may be needed any one of the four operations learned thus far, and to serve as a model in their arrangement of the other problems.

While many teachers require the pupils to write the denomination of each addend, of the subtrahend and the minuend, of the multiplicand, and of the dividend, it is scarcely necessary. In later life it is not done; and confusion is sometimes produced in the minds of young scholars by attempting to make them understand why, for example, 60 pints divided by 2 will sometimes give a quotient of 30 pints, and at other times, as in the 6th

problem, an apparent quotient of 30 quarts. It will be found more satisfactory, even if less scientific, to have the denomination written only with the result.

Although no formal instruction in finding halves and thirds of numbers has as yet been given, the average pupil will be able to solve problems 10, 11, and 14.

85. Lay no stress on the local value of the figures. Practice will enable the children to read and write correctly numbers of four figures. Teach the pupil to write the comma when the word "thousand" is said and after the number of thousands, the comma to be followed always by three figures.

97. Children should be led to see that 12×2 is the same as $12 + 12$; so that when they come to 15×2, they will have no difficulty in deducing the rule for writing 0 and carrying 1 when they multiply the 5 by 2.

98. Give the pupils time to find for themselves the quotient of $30 \div 2$. If it becomes necessary to show some of them how to work the example, do not elaborate the meaning of the 1 (ten) remainder when the tens' figure, 3, is divided by 2. An experienced mathematician, in dividing 9752 by 2, does not say 2 into 9 thousand 4 thousand times with a remainder of 1 thousand, 2 into 1700 8 hundred times with a remainder of 1 hundred, etc.

In dividing 30 by 2, children should not be permitted to write the first remainder, 1, before the 0, to indicate that 2 is to be divided into 10 for the second quotient figure. Children learn to work just as well without these unnecessary scaffolds.

104. While these drill exercises introduce a multiplier greater than 2, they contain no combinations, except 3×3, other than those found in the preceding work. After working these examples, the pupils will have learned that twice 9 is equal to 9 twos,— that when he knows the table of 2's, he knows a portion at least of the table of 3's, 4's, etc., to 9's.

111. When the teacher places the pointer on a number in one of the two outer spaces of the first circle, the pupil promptly gives the result obtained by adding to it the number contained in the inmost space. When this last number has been combined with all the others, it is replaced by a different number.

112. These drills are useful to impress upon a child the fact that when he knows, for instance, that 6 and 5 are 11, he should also know that 6 and 15 are 21, that 6 and 25 are 31, etc. They may also be employed as subtraction drills.

115. Division drills are necessary to enable pupils to acquire facility in obtaining quotients and remainders. When pupils are dividing by 2, the numbers from, say, 9 to 19 are written on the board with 2 underneath.

$$\begin{array}{l} 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15,\ 16,\ 17,\ 18,\ 19 \\ \div 2. \end{array}$$

When the pointer is placed at the 9, the pupil answers 4 and 1; when placed at 14, he answers 7; at 17, 8 and 1; etc. Other divisors may be employed, but care should be taken not to have any quotient figure but 1 or 2 at this time, as pupils have not yet learned the table of 3's. Thus, when 6 is used as a divisor, the teacher should not use a dividend greater than 17. When the three-times table is known, numbers from 12 to 29 may be written.

Facility in division will come only by practice, and it may be necessary for the teacher to supplement the examples of the book by others of her own.

118. Do not fail to keep up practice in addition and subtraction.

119. Subtraction examples in which the subtrahend is given before the minuend should occasionally be used.

121. Do not worry a pupil by attempting to explain, through problem 9, the difference between division and partition. Let him write $\begin{array}{l} 2)\underline{50} \\ \;\;\,25 \text{ pounds} \end{array}$ without taking advantage of the opportunity to show him that he should have an abstract quotient when the divisor and dividend are both concrete.

140. The analysis of problem 3 should not be required. A pupil that obtains from problem 2 the knowledge that 18 five-dollar bills amount to $90 will probably get the correct answer to the next problem, even though he may have to use 5 as a multiplier instead of the 18 that the more common form of the analysis would require. The other form should not be presented at this stage.

143. Children should be permitted to determine for themselves the method of obtaining the half of 36. It may require a little longer time than to show them, but the time will not be wasted.

147. Roman notation is not of much importance. Most children learn sufficient about it from the numbers affixed to their reading lessons.

157. Teachers should not endeavor to show by drawings that a quart measure is twice as large as a pint. If a pint measure is represented by a rectangle, each side of the rectangle indicating the quart should be only about $1\frac{1}{4}$ times that of the former in order to preserve the correct ratio, and children are not mathematicians enough to understand that where one of two similar solids has its corresponding dimensions $1\frac{1}{4}$ times those of the other, the volume of the former is double that of the latter. Use the measures themselves, borrowing them, if necessary, from a neighboring store.

159. A few problems involving more than one operation are here introduced. Avoid, if possible, giving help; and do not

require the scholars to perform unnecessary work, or to follow the same mode of solution or arrangement. In solving the first, some may write on their slates only two numbers, viz. 15 and 35. Others may set down 15, 15, and 20, etc. Do not teach yet how to multiply by a mixed number.

VI

NOTES ON CHAPTER THREE

While the teaching of formal definitions should find no place in the arithmetical instruction of the earlier years, the teacher should not hesitate to employ such technical terms as are called for by the work of the grade. Pupils gradually learn to understand what is meant by multiplier, quotient, remainder, etc., even where no attempt is made to explain the signification of the words. They will also become able to use each correctly, even if they cannot state its exact meaning in language that will satisfy a critical mathematician.

164. Sight exercises in division should be extended to cover dividends that are not multiples of the divisor. The slate examples in division supplied thus far have no remainders, as children find it more agreeable in the earlier stages of this work to have the answer a whole number. The partial dividends, however, do not always exactly contain the divisor, hence the need of such drills as will enable the pupil to determine rapidly the quotient figure and the remainder. Until Art. 176 is reached, this remainder need not be given by the pupils in the form of a fraction. See Art. 115.

168. In making "original problems," the pupil should strive to be original. No problem should be accepted as satisfactory that is substantially the same as one already furnished by another pupil. If, for example, the following is given to illustrate 12×5: "What will be the cost of 5 yards of ribbon at 12 cents a yard?" the teacher should not be satisfied with — "How

much will be paid for 5 pounds of cheese at the rate of 12 cents per pound?"

174. While the problems are gradually becoming more difficult, some of them can be done by bright pupils without using the pencil. In these cases, require that only the answers should be written. See previous notes on problem work. (Arts. 23, 84, and 159.)

178. Children should be permitted to follow their own plan of finding the product of 26 by $1\frac{1}{2}$. Some may do the work by simply placing 13 under 26. The regular method should not be taught until, perhaps, the 25th example, as the previous ones can be done by the children without assistance.

$$\begin{array}{r} 26 \\ \underline{+13} \\ 39 \end{array}$$

At this point, however, the systematic way of multiplying by a mixed number may be presented, which should be followed in such subsequent examples as are not so simple as to make this amount of writing unnecessary, as is the case in the 26th.

$$\begin{array}{r} 124 \\ \underline{2\frac{1}{4}} \\ 31 \\ \underline{248} \\ 279 \end{array}$$

In finding $\frac{1}{4}$ of 124, the pupil should not be permitted to write the multiplicand, 124, in some other part of his slate, and 4 as a divisor in front of it. No other writing of figures should be allowed than is given above. A little practice will enable scholars to perform this division and other similar operations, without always bringing into close contact the numbers to be handled.

In some European countries, the multiplier is placed at the right of the multiplicand, instead of being written underneath. An example like the 26th would be worked in that case without writing 760 a second time.

$$\begin{array}{l} 760 \times 1\frac{1}{5} \\ \underline{152} \\ 912 \end{array}$$

To small children, however, it would be confusing to be required to learn two methods of working examples so nearly alike; hence the advisability of uniformly following the plan originally given, of first finding the fractional part, and then multiplying by the whole number.

$$\begin{array}{r} 760 \\ \underline{1\frac{1}{5}} \\ 152 \\ \underline{760} \\ \mathbf{912} \end{array}$$

180. The arrangement of work should begin to receive some attention. In solving the second problem, some children will find the cost of ½ pound of tea on one portion of the slate, and then write this amount, 35¢, on another part, with 25¢ underneath. They should be led to see how to avoid doing unnecessary work.

$$\begin{array}{r} 2\underline{)70} \\ 35 \\ \underline{+25} \\ 60¢ \end{array}$$

186. Some short examples in the addition and the subtraction of horizontal columns are given, to accustom children to handle numbers that are not arranged for work in the usual way. The addition example could be used to explain the reason for "carrying," but the explanation should be deferred for the present.

191. Examples in division should occasionally be presented to pupils in the form used in the second column. When children recognize $\frac{27}{7}$ as an example in division, they need no rule for the reduction of an improper fraction to a whole or to a mixed number.

197. Do not furnish the pupils with a method of solving the 9th example that is suited to a seventh year class in denominate numbers. Leave them to their own resources as much as possible.

202. More drill examples are needed than are furnished in the book.

203. To secure good work in division, much practice must be given. Many more examples than are here supplied may be needed by some classes.

213. While it is convenient to write the subtrahend under the minuend, pupils should gradually accustom themselves to perform the fundamental operations with numbers in other than the usual positions.

215. Children should be encouraged to avoid unnecessary writing. They should be led to see that after finding on one part of the slate that $\frac{1}{7}$ of 119 is 17, they should not place this number in another place in order to multiply by 5.

$$\begin{array}{r} 7)\underline{119} \\ 17 \\ \underline{\times 5} \\ \text{etc.} \end{array}$$

220–223. These drills are intended to lead up to the use of larger numbers in the oral work of the pupils.

224. It is not advisable to begin formal instructions in fractions at this stage of school life. There is no need of defining the word "fraction" for the present. Every member of the class will be able to tell what is the sum of $\frac{1}{2}$ and $\frac{1}{2}$, especially if the question is put in the form of a problem.

It will be necessary, perhaps, to explain that $4 \times \frac{1}{2}$ is another way of expressing $\frac{1}{2}$ of 4; that $\frac{1}{2} \times 10$ means 10 halves. $1 \div \frac{1}{2}$ will also require translation into the form, "How many halves in 1?" Pupils may be led to see this by being asked to indicate by signs and figures the example, "How many twos are there in eighteen?" The drills in the use of fractional divisors need not be made prominent for the present.

230. Accustom children to writing the decimal point in the product, as soon as it is reached in multiplying. Reasons should not be dwelt upon.

231. The above applies to placing the decimal point in the quotient.

238. Unless pupils have been carefully trained to give only *reasonable* answers to slate problems, there will be some who will obtain 171 as the sum of $13\frac{1}{2}$ and $4\frac{1}{2}$. They will first write 1 as the equivalent of $\frac{1}{2} + \frac{1}{2}$; and to this they will prefix 17, obtained by adding 13 and 4. The special training in number received by pupils taught by the Grube method prevents to a great extent the absurd mistakes found in the answers

given by pupils, even of high-school classes, to simple problems. When the early arithmetical instruction is largely given to work in the fundamental processes, the teacher should make liberal use of oral problems, to give the requisite knowledge of number that will enable a pupil to know when his answer is very much out of the way. Systematic instruction in finding "approximate" results is supplied in later chapters.

239. These examples are intended to lead up to finding the difference between a whole number and a mixed number.

240. Pupils will find little difficulty in working out these examples if they are left to themselves.

241. When the addition and the subtraction of mixed numbers containing halves are readily performed, the teacher will find comparatively little trouble with the work under Arts. 241–245. Encourage pupils to make diagrams; or, if necessary, to divide circles into quarters, and to use these parts in performing the required operations with the fractions.

To find, for instance, the sum of $\frac{3}{4}+\frac{3}{4}$, it may be advisable to permit some scholars to arrange the six quarter-circles in such a way as to make a whole circle and a half-circle.

246. As children are more accustomed to dealing with halves and quarters than with thirds, a little more illustrative work may be needed in Arts. 246–250, than was required in the previous work in the addition and the subtraction of mixed numbers.

VII

NOTES ON CHAPTER FOUR

253–258. In the last chapter, pupils were required to add only fractions containing the same denominator; in this chapter, an addition or a subtraction example may contain fractions whose denominators are different. For the present, however, it will not be necessary to call attention to the need of reducing fractions to a common denominator. The average scholar can solve these examples without assistance, if he has been able to work out those found in Chapter III.

259. While these problems are becoming more difficult, they are still well within the powers of a pupil that is really anxious to solve them. When, however, they are found to be beyond the capacity of many members of the class, the teacher may first use them as "sight" problems, with some slight changes in the figures.

If, for instance, after a pupil that reads the first from his book declares that he is unable to obtain the answer mentally, the teacher may give it as follows:

A sailor has 10 yards of cloth. He uses 4 yards for a coat and 2 yards for a vest. How many yards has he left?

In the second, $1\frac{1}{2}$ pounds may be substituted for $1\frac{1}{4}$ pounds; in the third, 3 packages instead of 4; 20 dozen in the fourth, instead of $3\frac{1}{2}$ dozen.

Slate work on these problems should not be permitted until so many have been solved in this way that the pupil has time to forget what operations have been used in each. This will

require him to study the conditions of the different problems, instead of relying upon his memory.

266. When the formal analysis of oral problems is made a feature of the work, it is important that the statements be not so long as to be tedious.

In the first, for example, the following would be sufficient, after the pupil has stated the problem:

"If 8 ounces of tea cost 40 cents, 1 ounce will cost 5 cents, and 5 ounces will cost 25 cents."

While the customary order has been followed in the systematic treatment of the various topics, pupils are called upon in the earlier chapters of *Mathematics for Common Schools* to solve many problems that are frequently deferred in other books to a later stage of their arithmetical instruction. While scholars readily solve this class of problems, they are not always able to state in technical language the reasons for the various processes employed in obtaining the answers. A child who sees that division is used to ascertain the number of ten-cent pies that can be purchased for forty cents, cannot be made to understand thus early in his school life that the same process is used to find what part of such a pie can be bought for five cents. A correct statement by the pupil of his method of reaching the result, should usually be accepted as satisfactory. Even in the more simple questions, set forms of analysis should be carefully avoided.

268. To prevent misunderstanding, parentheses have been employed even when not required by arithmetical usage. The quantities within the parentheses must be added, multiplied, etc., before being operated upon by the quantity outside. The third example becomes 30×3; the fourth, $80 \div 4$; the fifth, $\frac{1}{4}$ of 80; the eighth, $70 \div 7$, etc.

269. These may be used as slate examples, if they are found too difficult for "sight" work.

271. Some of these questions may not require the use of a pencil; Nos. 6, 7, 8, 11, and 19, for instance.

272. The answers to the first ten examples should be given at sight.

273. Use 49 to 57, inclusive, as "sight" examples; also as many as possible of those in the next section.

274. When the divisor ends in one or more ciphers, the latter are set off by a vertical bar, and also a corresponding number of figures from the right of the dividend. To keep the pupil from omitting these figures from the remainder, it is advisable to require him to write the partial remainder as above, before he begins to divide. Then, using 8 as a divisor, he writes the quotient figures in their places, and completes the partial remainder by prefixing 2 to the 1 that was originally brought down.

$$8|0\overline{)434|1} \quad \frac{1}{80}$$

$$8|0\overline{)434|1} \quad 54\tfrac{21}{80}$$

It being the usual practice in abstract examples in division to refrain from reducing the fractional part of the quotient to lowest terms, the above method may be used in examples where both the divisor and the dividend terminate in a cipher. Some teachers prefer, however, in this case, to cancel the cipher in each, and to give the quotient of $4340 \div 80$ as $54\frac{2}{8}$.

$$8|0\overline{)434|0} \quad 54\tfrac{20}{80}$$

$$8\cancel{0}\overline{)434\cancel{0}} \quad 54\tfrac{2}{8}$$

277. Employ in "sight" work.

278. The foot-rule and the yardstick should be used by the children. They should ascertain, for instance, the length of their slates in inches, the length of the blackboard in yards or in feet, the height of the blackboard in feet, the dimensions of the room, etc.

280. It will be sufficient to accustom pupils to placing the product by the tens' figure one place to the left without giving the reason therefor. Neatness in the arrangement of the work, and the careful writing of figures, will prevent some mistakes.

282. In short division, the scholar has been taught to place the first figure of the quotient under the last figure of its partial dividend, and to write under each succeeding figure of the dividend its corresponding quotient figure. When his work is neatly arranged, he seldom omits ciphers, nor does he often obtain two quotient figures from one partial dividend.

$$\begin{array}{r} 4\underline{)12361} \\ 3090\frac{1}{4} \end{array}$$

To obtain the benefit of this experience, the pupil should be taught in long division to write the quotient over the dividend. By doing this, he will not be tempted, as are some beginners that place the quotient at the right, to give $23\frac{1}{21}$ as the answer to the above example; nor will he be likely to think that 252 contains 21, 111 times. This last result is obtained by assuming that the second partial dividend, 42, contains the divisor 1 time, with a remainder of 21. This latter is then made a partial dividend, with the above result.

$$\begin{array}{r} 2030\frac{1}{21} \\ 21\overline{)42631} \\ \underline{42} \\ 63 \\ \underline{63} \\ 1 \end{array}$$

285. While the pupil may write 16 as a multiplier in the 5th problem, he should be required to multiply by 30, in order to shorten the work. The multiplication by 30 should be performed, also, without rewriting the numbers so as to place 30 under 16.

$$\begin{array}{r} 30 \\ \underline{\times 16} \end{array}$$

286–290. The special drills will be found of great value in giving pupils a knowledge of numbers; and many oral problems employing these and similar combinations should be made by the teacher. Oral problems containing large numbers should, as a rule, require but one operation for their solution.

In the oral addition of numbers of two figures, the pupil should not commence, as in slate work, with the units' figures. The special drills of the last chapter should have taught him to think immediately of 80 when he sees $40 + 40$, $60 + 20$, $50 + 30$, etc. The next step in this work should contain such combinations as $47 + 40$, $63 + 20$, $54 + 30$, etc. In adding 54 and 30, the pupil should be taught to first see the *eighty*, then the *four*. The sum of 27 and 32 (the third step) should be obtained by joining 27 and 30 to make 57, and adding 2 to this result to obtain 59. If the pupil begins with the units, 7 and 2, he is likely to forget the tens' figures. When the addition work is readily performed, the pupil finds little trouble with the rest. There being no carrying, he will readily obtain the product of 32 by 3, and the others given in Art. 288, especially if he begins the multiplication at the tens' figure. After he becomes expert in adding and multiplying, he will experience no difficulty in subtracting and dividing.

294. The teacher should not encourage unnecessary work, by permitting children to write the sum of $12\frac{7}{8} + 6\frac{1}{8}$ as $18\frac{8}{8} = 18 + 1 = 19$. If, however, it be deemed advisable in the 4th example, for instance, that the fractions should be expressed with the same denominator, care must be taken to prevent pupils from making such mistakes as using the sign of equality between $\frac{1}{4}$ and $\frac{2}{8}$ in such a way as to represent that $50\frac{1}{4}$ is the equivalent of $\frac{2}{8}$. A vertical line drawn between the two sets of fractions will serve to separate the original example from the auxiliary portion. (See Art. 310.)

$$\begin{array}{r|r} 50\frac{1}{4} & \frac{2}{8} \\ 16\frac{3}{8} & \frac{3}{8} \\ \hline 66\frac{5}{8} & \frac{5}{8} \end{array}$$

304. As some children merely look for the figures of a problem without paying attention to its terms, an occasional one is given in which some or all of the numbers are expressed in words.

306. In making out a bill, it is convenient to be able to write the cost of 196 lb. at 4¢ per lb. without using another sheet of

paper and placing the 4 under 196. In working these examples, the pupil is expected to write only one figure of the product at a time. It is not intended that all of these twenty-five examples should be done before proceeding with the subsequent work. A few of them should be used from time to time throughout the term.

307. The last sentence applies also to these examples. A few of the easier ones should first be given. After more practice in long division, the more difficult ones may be taken up.

310. Whenever it becomes necessary, in the opinion of the teacher, to permit the rewriting of the fractions with a common denominator, she should, as soon as possible, have her pupils write the common denominator only once, as above. When the common denominator is written under each numerator, it is likely to be confusing to children, not to speak of the danger of its being added in occasionally with the numerators.

	8
$49\frac{7}{8}$	7
$20\frac{1}{2}$	4
$70\frac{3}{8}$	$\frac{11}{8} = 1\frac{3}{8}$

$49\frac{7}{8}$	$\frac{7}{8}$
$20\frac{1}{2}$	$\frac{4}{8}$
	$\frac{11}{8}$

312. See Art. 268.

316. Where the multiplier ends with ciphers, some teachers think that time is saved by omitting the ciphers from the partial products. The ciphers at the right of the multiplier are written beyond the multiplicand, and are brought down at the end of the work.

$$\begin{array}{r} 76 \\ \underline{1300} \\ 228 \\ \underline{76} \\ 98800 \end{array}$$

Other teachers prefer to place the numbers as is generally done in multiplication, writing a cipher under each one in the multiplicand as its partial product, and writing the partial products by 3 and 1 under these figures, respectively. This method will be found to give more satisfactory results later on, when pupils have such multipliers as $20\frac{1}{2}$, $300\frac{1}{4}$, etc., in which a fraction follows the ciphers.

$$\begin{array}{r} 76 \\ \underline{1300} \\ 22800 \\ \underline{76} \\ 98800 \end{array}$$

319. See Art. 310. When an addition example consists of more than two mixed numbers with fractions of different denominators, it may be advisable to permit young children to write out the successive operations in the manner here indicated.

$$
\begin{array}{r|l}
 & 6 \\
\hline
5\frac{1}{2} & 3 \\
7\frac{2}{3} & 4 \\
9\frac{1}{6} & 1 \\
\hline
22\frac{1}{3} & \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}
\end{array}
$$

Many of the fifty examples on this page should be used as "sight" work from the blackboard, the pupils writing only the results. Nos. 1–6, 8–9, 13–16, 23–24, 26, 31–37, 41–43, 49–50 can be treated in this way after they have been worked out on the slate, if not in the first instance.

321. Until children obtain some knowledge of numbers, their progress in long division is very slow. In dividing 918 by 17, for instance, a pupil that is not properly instructed will sometimes take 1 as the first figure of the quotient. When, after subtracting, he obtains a remainder of 74, he may realize that he is wrong without being able to determine just how far astray he is. In this case he tries 2 as the quotient figure, ascertaining the product of 17×2 in a corner of his slate, and then transferring the 34 to its proper position under the first two figures of the dividend. Another subtraction follows, with a resulting remainder, again, perhaps, recognized as too great; and so on.

The object of these drills is to enable the scholar to reach at once a close approximation to the correct quotient figure. Their use may be commenced in some such way as the following:

The teacher writes on the blackboard a convenient number of those found among the first twenty, arranging them as shown below, with the divisor preceding the dividend. Under these she places the corresponding ones from the second and third sets, respectively.

20)160	60)360	90)450	50)300	30)270
19)160	59)360	89)450	49)300	29)270
21)160	61)360	91)450	51)300	31)270

Placing the pointer on those in the first row, successively, she receives the quotients promptly. She then asks for the quotient of the first in the third row, 21)160. If the pupil announces 8 as the result, he should be required to give mentally the product of 21×8, which he will find to be too great. He is thus led to see that the quotient is 7, with a remainder. The other quotients in this row are then elicited. After a pupil discovers that 21 is not contained 8 times in 160, that 61 is not contained 6 times in 360, etc., he may be introduced to the second row. A little questioning will enable him to perceive that if $160 \div 20 = 8$, the quotient of $160 \div 19$ must be at least 8, with a remainder; that $360 \div 59$ gives a quotient of 6, with something over, etc. Regular practice with this particular set of drills will rob division by 19, 29, 39, etc., of some of its terrors to slow pupils, as they will be led to use 2, 3, 4, etc., as "trial divisors" instead of 1, 2, 3, etc., whereby they will be able to obtain their answers in a reasonable time.

After the children have become able to announce at once the quotients of all the drills in the first three sets, and other similar ones supplied by the teacher, they may take up the remaining ones by degrees. When there is a remainder, the pupils should not be required to calculate it.

324. The quotient of $2,800 \div 200$ may be made more obvious if the dividend is read 28 hundred, instead of two thousand eight hundred.

328. See Art. 274, as to writing the partial remainder before beginning to divide.

341. Do not give reasons for the location of the partial products. There is plenty of time for the science of arithmetic later on in the course.

343. Although the divisors contain three or four figures, these examples should not prove so difficult as many of those already

$$\begin{array}{r} 4 \\ 201\overline{)8643} \\ \underline{804} \\ 603 \\ \text{etc.} \end{array}$$

worked. A pupil that has learned from the previous drills that $800 \div 200 = 4$, will be able to see that 201 is contained 4 times in the first three figures of 8,643. The teacher should be careful to see that the first quotient figure is written in its proper place.

No. 36 may cause some hesitation until the pupil perceives that he has to divide 81 hundred and something by 9 hundred and something. No. 37 will become simple if handled in the same way. In No. 47, 98 hundred divided by 12 hundred will give the clue to the quotient; in Nos. 48 and 50, nine thousand and two thousand should be used for this purpose.

344. With such a multiplier as 209, some teachers write a series of ciphers to denote the product by 0. The method given in the text-book is the one generally followed in later school life, and is just as easily taught to beginners as the above.

$$\begin{array}{r} 456 \\ \underline{209} \\ 4104 \\ 000 \\ \underline{912} \end{array}$$

346. Where the multiplicands are small, as in nearly all of these examples, the product by the fraction should be determined "mentally" and written in its place. A pupil should not be encouraged to waste time by indicating on another part of his slate that 64 is to be divided by 8, and that this product is to be multiplied by 3, and doing all this work to reach a result that can be readily obtained without any writing whatever. In Nos. 78 and 88, such pupils as need to use the pencil in multiplying by the fraction should be permitted to do so. The teaching of the common method of multiplying by a mixed number is taken up at the beginning of the next chapter.

$$\begin{array}{r} 64 \\ \underline{1\frac{3}{8}} \\ 24 \\ \underline{64} \\ 88 \end{array} \qquad \begin{array}{r} 8\underline{)64} \\ 8 \\ \underline{3} \\ 24 \end{array}$$

VIII

NOTES ON CHAPTER FIVE

348. The denominators of fractional multipliers have heretofore been factors of the multiplicands, and the latter have been, as a rule, small numbers. With the introduction of larger numbers and the occasional use of multiplicands that are not multiples of the denominators of the fractions in the multiplier, it becomes necessary to furnish pupils with a general method of dealing with this class of examples. (See Arithmetic, Art. 347.)

349. In multiplying 27 by $13\frac{1}{3}$, some pupils may be tempted to follow the rule, and to multiply 27 by the numerator 1. In the first few examples this may be permitted, but the scholars should soon be taught to discontinue the practice, and to divide the multiplicand without rewriting it. (See Art. 178.)

$$\begin{array}{r} 27 \\ 13\frac{1}{3} \\ \hline 3\,)\,27 \\ \hline 9 \\ \text{etc.} \end{array}$$

350. In adding 56 and 17, the pupil should first combine 56 and 10 to make 66, and then add 7. (See Art. 286.)

351. Children taught subtraction by the "building-up" method will ascertain how many must be added to 19 to make 66, by saying 19 and 40 are 59, and 7 are 66; or 19 and 7 are 26, and 40 are 66. While the second plan is easier in some respects, it gives the 40 and the 7 of the result in the reverse order, which makes it necessary for the pupils to transpose them. In this respect, the first plan is more satisfactory.

When the other method of subtraction is practiced in slate work, 66 is first diminished by 10 and then by 9. To find the difference between 94 and 76, the pupil takes 70 from 94, leaving 24, and from this remainder takes 6.

352. In multiplying 24 by 4 the pupil begins at the tens. Four times 20 are 80, to which is added 4×4, making 96.

353. While nearly the whole class will learn to give answers mentally to the previous combinations, it may be necessary to use the division drills as "sight" work chiefly.

359. See Art. 319.

362. Oral problems involving several operations, or those of an unfamiliar type, should be solved from the book as "sight" work, and should be followed later on by similar questions answered without seeing the numbers. No. 5 is of the second kind; and it might be well to place it on the board, writing "2 thirds" and "1 third" to express the parts, instead of employing the fractional form or that given in the book. In No. 7, the quotient of $60 \div 40$ will be expressed by $1\frac{1}{2}$, instead of the $1\frac{20}{40}$ obtained by writing the remainder over the divisor. No. 5 should not be made an excuse for teaching a method of obtaining the cost of the whole when that of a part is given.

These examples are introduced to give variety to the work, to lay a foundation for subsequent systematic treatment of problems of this kind, and to give a pupil an opportunity to use his thinking powers. The way to deprive them of value is to "explain" how they should be done, or to require from the scholars too much analysis.

363. If the school does not own these measures, the teacher should endeavor to secure the loan of a quart, a peck, and a bushel, for a few hours, at least. Sawdust could be used to show pupils that the peck contains eight quarts, etc.

364. While many of the problems of this article resemble the previous oral problems, it may be advisable to solve a number of them as "sight" work, changing the numbers when necessary. The first may be read "How many 200-lb. barrels can be filled from 6,000 lb.?" In the second and third, the fractions may be omitted. The cost of the calico and of the ribbon in No. 4 may be made 10¢. Nos. 5 and 6 need no change, perhaps.

370–372. Do not waste time by endeavoring to use these examples to explain "carrying" or the local value of digits.

374. The answers should be written directly from the book. Do not permit scholars to copy the examples on their slates.

377. First, perform operations on the quantities enclosed within the parentheses.

384. Very little preliminary explanation will be needed. Place (*a*) on the blackboard, and ask a pupil to write the missing number in its place, one figure at a time, beginning with the units' figure.

$$(a)\quad \begin{array}{r} ? \\ 125 \\ \underline{632} \\ 99 \end{array} \qquad\qquad (b)\quad \begin{array}{r} ? \\ 125 \\ \underline{632} \\ 1000 \end{array}$$

Have another pupil work (*b*) in the same way. Nos. 1 to 5 may be used as a class exercise, each pupil writing only the answer on his paper, the examples being placed on the board.

385. In many German schools, children are not permitted in long division to write the partial products. Examples 6–23 are given to train pupils to omit these products when the quotient contains but one figure. After a few of them are worked on the board, the answers to the others may be written by all the pupils, as suggested in the preceding article. In writing the answers, the pupils should first set down the quotient figure, then the divisor as the denominator of a fraction, and lastly the remainder as a numerator. (See Art. 563, p. 55.)

386–388. These examples should be placed on the board, and the pupils should write the results one figure at a time.

397–401. See Art. 321.

405–406. See Arts. 306 and 307.

407. Prove the correctness of the grand total by comparing the total of the 6th column with that of the 11th row.

412. Permit the pupils to use their own method of working these examples, and avoid giving unnecessary assistance.

413–414. Example 1 should be omitted where pupils do not receive marks that are thus averaged. No. 2 may also be omitted if the word "average" is not understood by the pupils.

424–426. See Arts. 286–290, page 34.

429. In Examples 1, 2, 5, 9, etc., it will hardly be necessary to inform the pupils that 1 is not considered a factor of a number.

IX

NOTES ON CHAPTER SIX

The previous work in mixed numbers should make the pupils reasonably familiar with the addition and subtraction of fractions having small denominators. In this chapter, the work is extended to cover the addition and subtraction of fractions whose common denominator is determinable by inspection. For the present, the teacher should be satisfied if her pupils acquire reasonable facility in performing the various operations, even if they are unable to formulate, in the language of experienced mathematicians, the reasons for the different steps. The children should be required to use correctly and intelligently such technical terms as are required by the work of the chapter; but they should not be compelled to memorize any definitions that convey to them no meaning. They should incidentally learn what is meant by numerator, denominator, common denominator, multiple, etc., by hearing the teacher employ these words from time to time, rather than by commencing with what is to them an unintelligible jumble of words.

451. While systematic work in fractions belongs properly to the next chapter, the teacher should not hesitate to call $\frac{12}{8}$, $\frac{15}{4}$, etc., "improper fractions," and to ask a pupil to state how they are changed to whole or to mixed numbers.

453. Do not, for the present, formulate the rule for changing a fraction to an equivalent one with higher terms.

458. The meaning of "lowest terms" is given in No. 6. Leave the rule for the next chapter. After a pupil has rea-

soned out in his own way that 18 hours is $\frac{3}{4}$ day, in No. 15, the teacher may explain that 18 hours can be written $\frac{18}{24}$ day, which is reducible to the answer given above.

463. Have pupils see that $\frac{5}{6}$ is larger than either $\frac{5}{7}$ or $\frac{5}{8}$, because 1 sixth is larger than a seventh or an eighth; and this for the reason that the fewer the number of equal divisions made in a unit, the larger is each portion. Do not require scholars to change these fractions to equivalent ones having a common denominator.

467. For finding the difference between two mixed numbers when the fraction in the subtrahend is greater than that in the minuend, the method given in the text-book is the one generally employed. The teacher should always consider herself at liberty to use any other way of performing this and other operations, but she should not willingly adopt any method that is more tedious. Children should not, for instance, be required to change mixed numbers to improper fractions, and then to reduce these to a common denominator in order to subtract one from the other.

469. Pupils should now be required to pay more and more attention to the arrangement of the problem work, without, however, being permitted to use unnecessary figures or to waste time. In some good schools, the full written analysis of a problem is occasionally used as an exercise in composition.

When the pupils find difficulty in determining the operations necessary to the solutions of problems, the latter should be used as "sight" work. The alterations in the figures needed to simplify a problem should now be made by a pupil, instead of by the teacher, as recommended in previous chapters. The scholar that reads No. 1, for instance, might change $5\frac{3}{4}$ and $4\frac{3}{8}$ yards, to 5 and 4, respectively. No. 2 can be solved as it stands. In No. 3, \$150 might be substituted for \$140.40, and \$2 for \$1.80.

Work of this kind should gradually lead the pupil to form the habit of using some similar method of ascertaining for himself how to manage a problem.

471. Written in this form: $3\underline{)93\frac{3}{4}}$, No. 11 should give the children no trouble. If, however, they hesitate when the fraction is reached, the difficulty may be cleared up by making a concrete problem: Divide \$$93\frac{3}{4}$ equally among 3 persons. What is the share of each?

Under no circumstances should these dividends be changed to improper fractions.

484. In nearly all of the previous multiplication work involving mixed numbers, the latter have been used as multipliers. In No. 35, the mixed number appears as a multiplicand. The first six of these examples and the last two should be used as sight work, the answers being written directly from the book. When the pupil reaches one that needs to be worked out in full, say No. 41, he should not be permitted to use $18\frac{7}{9}$ as a multiplier, as it is important that he should learn the proper method of working both classes of examples.

$$\begin{array}{r} 18\frac{7}{9} \\ \underline{13} \\ 9\underline{)91} \\ 10\frac{1}{9} \\ 54 \\ \underline{18} \end{array}$$

489. Many scholars will carelessly give 20 halves as the result obtained by dividing 500 halves by 25 halves. To prevent the possibility of a mistake of this kind, some teachers multiply the divisor and the dividend by the least common multiple of the denominators of the fractions. While this method produces exactly the same figures as the one given in the text-book, it is probably less likely to be followed by the error mentioned above.

$$\begin{array}{rr} 18\frac{3}{4}) & 1387\frac{1}{2} \\ \underline{\times 4} & \underline{\times 4} \\ 75) & 5550 \end{array}$$

497. Some teachers may prefer to write the example as is here given, although using 5 as the multiplier. Other teachers "analyze" as follows: At 1¢ per lb., 157 pounds of sugar would cost \$1.57; at 5¢ per lb., the

$$\begin{array}{r} \$.05 \\ \underline{\times 157} \\ \$7.85 \end{array}$$

$1.57 cost is 5 times $1.57. Business men pay no attention
$\underline{\times 5}$ to these fine-spun distinctions; they use as a multiplier
$7.85 the most convenient number, and write the dollar sign and the period in the product alone.

500. In analyzing problems of this kind, it is better, perhaps, to emphasize the fact that multiplication is employed in obtaining the result. Thus, 32 base-balls @ 25¢ = 32 times $\$\frac{1}{4}$ = 32 quarters = $8

502. In No. 28, the price of 11 yards can be found by taking 11 times $\$\frac{3}{4}$, or 33 quarters, etc. No. 24 is rendered easier by saying that 24 bushels at 1 quarter per bushel would cost 6 dollars; and that at 3 quarters per bushel the cost would be 3 times 6 dollars, etc. Pupils should be encouraged to use the method best adapted to the particular example under consideration.

509. See Art. 306.

510. In finding, for example, the number of 50-cent knives that can be purchased for $20, it may be advisable to make the division idea prominent. The analysis can take some such form as this: There can be bought as many knives as one half-dollar is contained times in 20 dollars — or, as there are half-dollars in 20 dollars.

Later problems involving division of fractions cause less trouble if the appropriate operation is always kept before the pupils, regardless of the method employed to shorten the solution of questions of certain types. These short methods should, however, be used.

To ascertain the number of 2-dollar knives obtainable for $24, the scholar turns naturally to division; and he should learn to see that he actually divides when he obtains 48 as the number of 50-cent knives that can be purchased for the same

money. In the latter case, the numbers given are 24 and $\frac{1}{2}$, from which 48 can result only when $\frac{1}{2}$ is used as a divisor.

Many pupils that give the correct answer when $24 \div \frac{1}{2}$ is placed upon the blackboard as a sight example will think that 12 is the quotient of $\frac{1}{2}\overline{)24}$. For purposes of drill, this last form should occasionally be employed in sight work, as should be the third form of division, $\frac{24}{\frac{1}{2}}$.

511. Example 6: There can be bought as many bars of soap as there are quarters in \$$3\frac{1}{4}$, or 13 bars. Example 8: As many yards as there are quarters in \$$5\frac{3}{4}$. Example 9: As many bushels as there are quarters in \$$10\frac{3}{4}$.

While set forms of analysis should not be required in any grades, older pupils should be led to use such as are most likely to lead to an intelligent appreciation of mathematical principles. From the beginning of about the fifth year, the science of arithmetic should begin to receive some attention, but not so much as to lessen to too great an extent the time that should be devoted to arithmetic as an art.

516. These examples are introduced to lead up to division of Federal money. From their previous experience, the scholars will readily work the first example, for instance, by changing it to the form $\$24\frac{1}{2} \div \$\frac{1}{2} = \frac{49}{2} \div \frac{1}{2} = 49$. No. 2 becomes $\$12\frac{1}{4} \div \$\frac{1}{4}$; No. 3, $\$26 \div \$\frac{1}{8} = \frac{208}{8} \div \frac{1}{8}$; etc.

Without laying much stress upon the terms "abstract" and "concrete," the teacher should bring her pupils to understand that the quotient of the first example is 49, — not 49 dollars.

517. While giving the answers to these exercises, the children should be able to state, after proper questioning, that the dividend must be of the same denomination as the divisor. In 2 ft. ÷ 8 in., instead of changing the divisor to $\frac{2}{3}$ ft., they naturally reduce the dividend to 24 inches, even if in 2 ft. ÷ 6 in. they may have used $\frac{1}{2}$ ft. as the divisor.

518. Changing the dividend to cents, No. 1 becomes 400 cents ÷ 10 cents, or 400 ÷ 10. To No. 11, many will give 50 as the result, unless previously well taught. In No. 12, the denomination of both terms being the same, the problem becomes $3 \div \frac{1}{4}$, or 12 quarters ÷ 1 quarter, rather than 300 ÷ 25. Nos. 13–20 are more readily worked by reducing each divisor to a fraction of a dollar. Pupils should understand that the answer is the same whether the dividend is changed to the same denomination as the divisor, or *vice versa*.

519. Some teachers write this example .36)27.00. It will be found safer to make the terms whole numbers by changing both to cents.

520. The method suggested for the first example, 11000 ÷ 275, is the more general one, although longer, perhaps, than $110 \div 2\frac{3}{4}$. Do not permit long division in No. 2. In No. 3, after writing 14000 ÷ 560, the pupils should strike out a cipher from each term: this should be insisted upon whenever the divisor ends in a cipher. In No. 4, either $74\frac{1}{2} \div \frac{1}{2}$ or $745\cancel{0} \div 5\cancel{0}$ should be accepted. If the work in No. 7 takes the form 75)$27.00, the answer should be $.36; if the pupil writes 75)2700¢, his answer should be 36 cents. The first form is the one employed in the work of preceding chapters, and no change should be suggested.

521. These drills in obtaining approximate results are intended to lead the pupil to such an examination of his answer as will prevent his being satisfied with one very much out of the way. It should not be expected that the same approximation will be obtained by all the members of a class.

1. 4200 ÷ 200.
2. $\frac{1}{2} \times 48$.
3. 12000 ÷ 2000.
4. $\$2 \times 99$, or $\$1.95 \times 100$, or $\$2 \times 100$.
5. 30 + 38.
6. 175 ÷ 25.
7. 19×10.
8. 87 − 50.
9. $5 \times 5 \times 5$.

526. Formal instruction in denominate numbers should be deferred for a year or more. The average scholar will be able to solve all these problems if left to himself.

528–532. See notes on previous drills of this kind, Arts. 286 and 350. Special exercises in multiplication are regularly given by some teachers in the following manner:

2, 12, 22, 32, 42, 52, 62, 72, 82, 92
$\times 5$

A horizontal or a vertical row of numbers ending in 2, for instance, is written on the board with, say, 5 as a multiplier. Attention is called to the fact that 2×5 is 10, so that all of these products must end in 0.

The pupils are also reminded that when the multiplicand is a number of two figures, 1 must be carried to the product of 5 times the tens' figure. When the teacher points to 12, the pupil says 5, 6, 60 — the first number (5) being the product of the multiplier and the tens' figure of the multiplicand; the second, (6) being this product increased by the carrying figure 1; the third being the result, which has been completed by annexing the units' figure (0) of the first product (2×5).

Pointing to 52, the pupil says 25, 26, 260; to 92, he says 45, 46, 460. After sufficient drill with 5 as a multiplier, it is replaced successively by 6, 7, 8, etc. The row of multiplicands is also changed to 3, 13, 23, etc.; 4, 14, 24, etc.; when the previous row has been employed with all of the multipliers, say from 5 to 12. With sufficient practice of this kind, pupils become able to give the product of any number of two figures, by multipliers to 12, with great readiness.

Some teachers, however, prefer in oral multiplication to use the method previously suggested of commencing to multiply at the tens' figure of the multiplicand. In finding 5 times 38, the pupil takes the latter number as it is given, thirty — eight, and multiplies in the same order, obtaining 150 and 40, or 190.

534–535. Long-division drills. See Arts. 321 and 397–401.

538. See Art. 563, p. 55, and Arithmetic, Art. 385.

540. See Art. 384.

543. In problems of this kind, writing the given numbers in the places called for by the conditions of each example helps the pupil in his solution. After writing No. 5, as here indi-

1.	2.	5.
$\begin{array}{r} 68 \\ 43 \\ \underline{?} \\ 150 \end{array}$	$24\overline{)264}$ with quotient ?	$\begin{array}{r} ? \\ \underline{-89} \\ 92 \end{array}$

cated, he can see that addition is the required operation more readily than if he endeavors to determine it from the words of the problem.

546. Teachers should not weary pupils by giving too many items in the earlier bills. It is useful to employ occasionally such quantities and prices as will not require the use of a separate piece of paper to perform the necessary multiplications. In No. 1, for instance, the pupil should be compelled to fill out the cost of each item without recourse to his slate. If he does not know the product of 16×5, he should multiply one figure at a time, writing the result in its proper place. Except, possibly, No. 2, the other bills called for under this section should be made out in the way suggested for No. 1, the use of a slate or other paper not being permitted.

The form given in the text-book is the one generally followed by business men. The first two vertical lines are kept to enclose the day of the month (see Arithmetic, Art. 642). The total cost of each item is placed in the first columns of dollars and cents, the amount of the bill being placed in the last columns, and on the line below the last item. When a single article is sold, its cost is placed directly in the first columns of dollars and

cents, and is not written in the column of prices. Unnecessary words, — *at* or @, for instance, *per yd.*, *lb.*, etc., — are never used; nor are commas employed after the names of the articles. It is now customary to omit the period after the date and after the name of the seller. The names of the articles are generally commenced with capitals, and the quantities are written with small letters. The heading given is the one most frequently used, though other forms are common; such as

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Pupils should write the cost of $10\frac{1}{4}$ lb. of chicken @ 30¢, in No. 4, as $3.08. Fractions of cents should not appear in the results; those below $\frac{1}{2}$¢ being rejected, and those of $\frac{1}{2}$¢ and higher being considered 1¢.

547–551. When pupils have become familiar with the notation and numeration of decimals, the remaining decimal work of this chapter should not require much discussion.

554. After working Nos. 1–4, pupils should be left to themselves to arrange No. 5. Nearly all of them will place the numbers in their proper places. Neither require nor permit unnecessary ciphers to be employed to fill out all the numbers to three decimal places.

555. The above suggestions apply here.

559. In the product of .36 by 3, pupils will naturally place the decimal point where it belongs, as they will in example No. 42. Before working No. 43, they should be required to deduce the rule for "pointing off." All unnecessary ciphers should be canceled in the product, the answer to No. 94 being read by the pupil as 960, not 960 and no tenths — 960.0.

560. These exercises should lead scholars to see that the number of decimal places in the multiplicand cancels a corresponding number of ciphers in the multiplier.

561. In giving the quotient of No. 11, a pupil may write $9\frac{32}{100}$. If called upon to read this answer, he will see that 9.32 expresses the same result. He will readily understand that $\frac{86}{1000}$ is also written .086. After a few examples, he can state the rule.

From this point, the teacher may change the method of dividing by a number ending in one or more ciphers. Instead of marking off, by a line, a corresponding number of figures from the right of the dividend, the pupil can locate the decimal point in the proper place. In No. 17, the decimal point will be moved one place to the left, to divide by 10; moving it two places to the left in the dividend of No. 18 will give its quotient; etc.

563. The rule for "pointing off" should be deduced from the sight exercises of Art. 562. In dividing 8 and 64 hundredths by 2, the pupil will, without prompting, obtain 4 and 32 hundredths. When he comes to No. 5, $8.4 \div 5$, he can be led to see that this example is the equivalent of No. 4, $8.40 \div 5$, in which the quotient is 1.68. Nos. 9 and 10 also require the annexation of a cipher at the right of the dividend.

When the scholars understand that the quotient must contain the same number of decimal places as the dividend, including any ciphers that may have been annexed to the latter, they should be taught the method a business man would employ. The latter, in dividing 120 by 64, does not find it necessary to write ciphers in the dividend, and then to count the number thus annexed, in order to determine the position of the decimal point in the quotient. He sees at once that the result is 1 and a decimal, and he places the point after the 1, before he writes the next quotient figure.

In a short-division example, the pupils should write the decimal point in the quotient when they reach it in the dividend, placing it under the latter. In long division, the decimal point in the quotient is placed over the point in the dividend. While the scholars have been warned in their early work in short division against writing 02 as the quotient of 8)16, they will see the need of the prefixed cipher in the answer, .02, to 8).16. From the inspection of a few examples of this kind, they will understand that each figure of the dividend after the decimal point requires a quotient figure (or cipher).

In the long-division examples worked out in Art. 563 of the Arithmetic, the partial products are omitted, to show how some European countries shorten work of this kind. The horizontal lines given in the text-book are not used.

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    1.875
64)120.
    560
     480
      320
        0
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After the pupil writes the quotient figure 1, he subtracts by the "building-up" method. The second remainder, 48, is obtained by saying 8 fours are 32, and 8 (writing it) are 40; 8 sixes are 48, and 4 (carried) are 52, and 4 (writing it) are 56. See Arithmetic, Art. 385.

564. Teachers should not forget that systematic instruction in decimal fractions belongs to the sixth school year. They should be content if their pupils learn to place correctly the decimal point in the quotient.

Nos. 21–30 should be considered rather as examples in division, than as examples in the reduction of common fractions to decimal ones. By $\frac{1}{4}$ is meant $1 \div 4$; and to solve it, the pupil may write 4)1.00, as in No. 11. He should learn by degrees, however, that it is not necessary to write all the ciphers in the dividend in order to obtain the result. The answer to No. 22 can be derived by a bright scholar from 8)1.0 just as well as from 8)1.000. He may desire the first cipher as a starting-point, but he finds the others superfluous.

565. These problems contain a few simple applications of the decimal work learned thus far. In No. 1, the value of the franc may be given in the fractional form also, $19\frac{3}{10}$¢, and the problem worked fractionally and decimally.

Approximate sight results might be asked before written work is begun. Taking the franc as about 20¢, or $\$\frac{1}{5}$, the pupil should say that the answer to No. 1 is less than $250. Assuming 40 inches, or $1\frac{1}{9}$ yd. as the length of the meter, 1800 meters would be equal to 2000 yards. No. 33 can be solved without using the pencil. In No. 38, the first result may be in the form of a common fraction, $\frac{1}{8}$ peck, to be changed, as in No. 22, to .125 peck.

569. The remarks made in Art. 564 as to the formal teaching of decimal fractions is equally applicable to the subject of measurements. At one time, all instruction in mensuration was deferred until the last year of the common-school course. At present, this subject is generally taken up in connection with the systematic work in denominate numbers; but there is no good reason why pupils compelled to leave school by the end of the fifth year of the course should not receive so much practice in finding the areas of rectangles as they have time for and can readily understand.

In most city schools, the children of this grade know from their lessons in form and drawing what is meant by a square and a rectangle. If pupils are not familiar with these terms, they should be explained.

That scholars may obtain a good idea of a square inch, they should be required to cut out a number of square pieces of paper, each side measuring an inch. These squares should then be used in determining the number of square inches in the two rectangles next drawn; 2 inches by 1 inch, and 3 inches by 2 inches. Children that determine the areas of these rectangles by covering them with their paper squares will have a better knowledge of 2 square inches and 6 square inches than if they

merely divide the rectangles by lines as suggested in the text-book.

The larger rectangles, 6×3 and 4×4, may be cut up into inch squares by drawing lines, and the rule for obtaining the area of each deduced from an examination of the figures. In the first, the pupil will see that he has 3 rows of squares, 6 to a row (or 6 rows, 3 to a row), making 6×3, or 18, squares. The rule should take in his mind some such form as this: the number of square inches in a rectangle is equal to the product of the number of inches in one dimension multiplied by the number of inches in the other; but he should not be required to give it expression. The teacher should take care that he does not think that "inches by inches give square inches."

570. As has already been said, the chief use, in arithmetical instruction, of objects, diagrams, etc., is to enable pupils to work without them. After the scholars understand how to obtain the area of a rectangle, they should cease to draw the figure and to subdivide it into squares.

It will be noted that the answers to the first 20 examples are to be given in square inches. In Nos. 11–20 each dimension should be reduced to inches before the multiplication is performed.

X

NOTES ON CHAPTER SEVEN

At this point regular fraction work should begin. From time to time, as occasion offers, the meanings of the technical terms should be elicited from the pupils; but the teacher should neither accept a memorized definition that is not thoroughly understood, nor should she require absolute correctness in the phraseology of a definition made by a scholar.

577. While the denominators of the fractions should generally be small; and while common denominators should, as a rule, be determinable by inspection, it is necessary, nevertheless, that the children be taught how to handle such other fractions as they may occasionally meet. It is not necessary that they should grasp the exact meaning of $\frac{387}{504}$ in the answer to No. 4, although proper teaching may enable them to see later on that this fraction approximates $\frac{400}{500}$, $\frac{4}{5}$.

In these earlier examples, the inspection method of determining the common denominator is continued.

580. Besides being necessary as a preliminary to subsequent work in fractions, expertness in determining the factors of a number is useful in enabling pupils to shorten their work by cancellation. The teacher should use these and similar exercises again and again, for a few minutes at a time, until her scholars can give the answers with great rapidity.

581. The pupils will need to learn the difference between the three factors of 12, and three divisors of 12. The factors will be 2, 2, and 3, because their product, $2 \times 2 \times 3$ equals 12. The divisors of 12 are 2, 3, 4, and 6.

These exercises, as well as those in Arts. 582–585, are not so valuable as to demand the reviews suggested for those in Art. 580.

In finding three (or more) factors of a number, the scholar should commence with the smallest. The first of the three factors of 8 is 2; dividing 8 by this factor, 4 is obtained, of which 2 and 2 are the factors.

The three factors of 18 are $2 \times 3 \times 3$; of 20, are $2 \times 2 \times 5$; of 27, $3 \times 3 \times 3$; etc.; etc.

582. It is customary to define a prime number as one that has no factor except itself and unity. The omission of the last four words will not mislead any person, as there could be no prime numbers if 1 were considered a factor. When the factors of a number, say 20, are asked for, no one gives $1 \times 1 \times 1 \times 1 \times 1 \times 2 \times 2 \times 5$ as the answer, or says that 20 has eight (or more) factors.

588. In reducing these fractions to lowest terms, it is not necessary that the pupils should use the greatest common divisor. See Arithmetic, Art. 592. On the other hand, they should not waste time in dividing each term by 5, if 25 is a common divisor.

589. Pupils should not be permitted to forget these tests of the divisibility of numbers. To those given in the text-book, there may be added that when a number divisible by 3 is even, it is also divisible by 6.

While a teacher should know that 1001, with, of course, its multiples, — 2002, 6006, 15015, etc., — is divisible by 7, 11, and 13, she should not burden her scholars with the information; nor should she dwell upon the test of divisibility by 8.

591. Beginners should be taught only one method of finding the greatest common divisor, and the one here given is applicable to all kinds of numbers. Teachers should not bewilder young pupils by endeavoring to make them understand the principles upon which this method is based.

595. Many teachers prefer to permit their pupils to write down all of the denominators, and then to strike out any one that is repeated or that is the factor of any other. They think the pupil is less likely to make a mistake by following this plan. In no case should scholars be permitted to begin work before rejecting or striking out the unnecessary numbers.

$$\begin{array}{r|l} 2 & \underline{\cancel{3}-9-\cancel{7}-\cancel{14}-\cancel{6}-14-\cancel{2}-12} \\ & \;\;9 \qquad\qquad\;\; 7 \qquad 6 \end{array}$$

etc.

605–606. Pupils that have had regular drills in the combinations given in the previous chapters will be able to take the extra step required by these examples. See Arts. 286–290 and 350–352.

607. A scholar that can find mentally the cost of 47 articles at 25¢ each should be able to give the product of 47×25 or 25×36 without using the pencil, and the teacher should give him a chance to determine for himself the method of doing it.

609. Such questions as $18\frac{2}{3} \div 2\frac{2}{3}$ can be worked by the method given in the last chapter; viz., 56 thirds $\div$ 8 thirds $= 56 \div 8 = 7$. Those contained in the 4th column should not be used until the pupils have had formal instruction in division of fractions. When they are taken up, the method followed should be that given above, the fractions being reduced to a common denominator, etc. $\frac{1}{2} \div \frac{2}{3} =$ 3 sixths $\div$ 4 sixths $= 3 \div 4 = \frac{3}{4}$; $\frac{2}{3} \div \frac{3}{4} =$ 8 twelfths $\div$ 9 twelfths $= \frac{8}{9}$; etc. See Arithmetic, Art. 639, note.

610. No. 9: For \$1.25 I can buy 5 times as many pounds as for 25¢, or 15 pounds. No. 16: For 18¢ there can be bought $\frac{18}{24}$ lb., or $\frac{3}{4}$ lb., or 12 oz.

613. In giving answers to these exercises, pupils should be permitted to write the fraction first and then the whole number.

The object of these exercises is to accustom the scholars to dispense with writing unnecessary reductions in adding and subtracting simple fractions.

614. Another method of finding the difference between $11\frac{3}{8}$ and $6\frac{7}{8}$ is to take $6\frac{7}{8}$ from 7, obtaining $\frac{1}{8}$, and to add to this the difference between 7 and $11\frac{3}{8}$, or $4\frac{3}{8}$.

In some classes of sight exercises, those given in Arts. 587, 594, and 605–609, for instance, the pupils should not take pens to write the result until told to do so by the teacher, after sufficient time has been given to obtain the answer. In the exercises of Arts. 613 and 614, the pupils should be permitted to take their pens at once, and to write each part of the result as soon as it has been obtained. Arts. 584, 650–654, 699, etc., also contain exercises of this kind.

$$\begin{array}{r} 495\frac{17}{36} \\ 36)17837 \\ 343 \\ 197 \\ 17 \end{array}$$

616. See Art. 563. The first quotient figure, 4, is written. The pupil then says 4 sixes are 24 and **4** (writing it) are 28; 4 threes are 12, and 2 (to carry) are 14, and **3** (writing it) are 17. This gives the first remainder, 34. The next figure, 3, is then brought down, and 9 is written in the quotient. The product of 9 times 36 is subtracted from 343 as given above, to obtain the next remainder, 19. The pupil says 9 sixes are 54 and **9** (writing it) are 63; 9 threes are 27 and 6 (to carry) are 33, and **1** (writing it) are 34.

618. While pupils should be encouraged to shorten their work by cancellation, the slower children should not be censured when they overlook some cases in which it is possible to employ this expedient. In these examples, however, all the scholars should be required to indicate the operations, and then to cancel.

619. It is not supposed that pupils should write answers to these questions, as is generally done in the case of the oral problems. These exercises are intended to lead up to the rule for multiplication of fractions. Diagrams should be drawn on the blackboard by the pupils, to illustrate the answers, but the teacher should refrain as much as possible from "explaining."

The board work should be done chiefly by the more backward members of the class rather than by the brighter ones. In illustrating fractions by diagrams, the unit employed should generally be a circle, the part dealt with being distinguished by shading.

The zealous teacher should not become discouraged at the inability of some members of the class to thoroughly grasp the mathematical principles involved in this and other operations. Even the ability to handle fractions mechanically will be of great use in after life, and all the pupils can be taught at least this much.

624. No. 31 reduces to $9\frac{3}{4} \div 3$, which can readily be worked by the pupils without assistance. No. 34, $40\frac{1}{2} \div 8$, may be difficult for some; but the teacher should not offer help too soon. The second term of No. 36 is easily obtained; and No. 40 will give no trouble.

625. These exercises are to be used in the same way as those in Art. 619.

Many children find it difficult to understand that 4 fourths ÷ 3 fourths = $1\frac{1}{3}$. They think that the answer should be $1\frac{1}{4}$, reasoning it out in some such way as this: 3 fourths into 4 fourths goes 1 time and 1 fourth over. They fail to recollect that the remainder in division is written over the divisor, which would give them $1\frac{\text{1 fourth}}{\text{3 fourths}}$, or $1\frac{1}{3}$. If they have been well taught previously, they may remember that 4 fourths ÷ 3 fourths $= 4 \div 3 = 1\frac{1}{3}$. Even a fairly bright pupil, when asked how much tea at $\$\frac{3}{4}$ per lb. can be bought for \$1, will sometimes reply, "A pound and a quarter." When he is told that his answer is correct if he means by it a pound of tea and a silver quarter, he sees the mistake and changes the result to "a pound and a third."

626. If $1 \div \frac{2}{3}$ is made concrete, a pupil can more easily show by a diagram that the result is $1\frac{1}{2}$. A problem of this kind may

be given: If it requires $\frac{2}{3}$ yd. of material for an apron, how many aprons can be made from 1 yard? A rectangle is drawn to represent the yard of material, and it is divided into thirds. Underneath, a rectangle two-thirds as long is drawn to represent the quantity required for an apron. When the pupil compares the two rectangles, he sees that the portion remaining after one apron is made will supply sufficient material for one-half of another.

627. While it is generally better in oral work to divide one fraction by another by reducing both to a common denominator, it will be found simpler in written work to have pupils invert the divisor.

631. "Invert the divisor, and proceed as in multiplication," is the rule generally followed.

634. No. 33 can be shortened by writing it as follows, before beginning work: $\frac{20}{1} \times \frac{3}{4} \times \frac{8}{7}$, the divisor being inverted. No. 34 should be treated in the same way: $(20 \div \frac{7}{8}) \times \frac{3}{4} = \frac{20}{1} \times \frac{8}{7} \times \frac{3}{4}$. The divisor of No. 35 consists of two fractions, both of which should be inverted: $20 \div (\frac{7}{8} \times \frac{3}{4}) = \frac{20}{1} \times \frac{8}{7} \times \frac{4}{3}$. This method should be followed with Nos. 36, 41, and 42. The first of these becomes $\frac{20}{1} \times \frac{8}{7} \times \frac{4}{3}$; the next, $\frac{17}{3} \times \frac{9}{1} \times \frac{38}{5} \times \frac{5}{23} \times \frac{6}{5}$; and the next,

$$\frac{11}{2} \times \frac{31}{4} \times \frac{19}{6} \times \frac{25}{4} \times \frac{4}{11} \times \frac{5}{21} \times \frac{1}{31}.$$

635. Each teacher must determine for herself what method of analysis should be encouraged in such questions as Nos. 4, 8, 13, and 14. While set forms should be avoided, children need direction in the solution of problems of this kind.

In solving No. 4, for instance, the greater number of teachers prefer to have pupils first find the cost of $\frac{1}{3}$ yd. When this method is followed, care must be taken that all the pupils understand why $\frac{1}{3}$ yd. costs one-half of 20 cents. This may be made clearer to some by writing the fractions in this way: If 2 thirds

(or parts) cost 20 cents, what will 1 third (or part) cost? A diagram similar to that given in Arithmetic, Art. 636, may help others to understand the method.

Other successful teachers think the written work is benefited by treating these examples as problems in division. They lead their children to determine in each case what operation is involved, by requiring them to consider what they would do if the fraction were a whole number. In No. 1, for example, the cost of 16 balls at $3 each would be $3 × 16. In No. 2, the pupil would say, "If I paid $12 for base-balls at $3 each, the number of balls would equal 12 ÷ 3. I must, therefore, divide." He mentally inverts the divisor, $\frac{3}{4}$, then cancels, etc.

636. The scholars should be allowed sufficient time to work these out in their own way.

639. No. 4: $24\frac{1}{2} \div 3\frac{1}{2} = \frac{49}{2} \div \frac{7}{2} = 49 \div 7$. Some pupils will see that time is lost in No. 6 by finding the cost of a pound. No. 7 is an example in division: $1\frac{2}{3} \div 2\frac{1}{2} = \frac{5}{3} \div \frac{5}{2} = \frac{5}{3} \times \frac{2}{5}$; or $\frac{10}{6} \div \frac{15}{6} = 10 \div 15$, etc. In No. 16, 36 hats will cost 3 times $7.

642. See Art. 546.

649. In multiplying by 25, the pupil is generally told to annex two ciphers and to divide by 4. In mental work especially, the annexation of the ciphers confuses some scholars by giving them a larger dividend than is really required. The product of 25 times 19 may be obtained more easily by taking one-fourth of 19, or $4\frac{3}{4}$, and changing this quotient to 475, than by finding one-fourth of 1900. In No. 9, the pupil should see that at $100 per bbl., the pork would cost $5600, and that at $12.50 per bbl. ($\frac{1}{8}$ of $100), it would cost $\frac{1}{8}$ of $5600.

650. In No. 1, divide 837 by 4, and for the 1 remainder affix 25 to the quotient. In No. 4, annex two ciphers to the quotient of 508 by 4. In No. 9, affix 250 to the quotient of 837 ÷ 4.

In multiplying 6281 by $12\frac{1}{2}$, No. 18, divide 6281 by 8, obtaining 785, and annex $12\frac{1}{2}$ for the 1 remainder, making the result $78{,}512\frac{1}{2}$.

654. When the divisor is a whole number, time should not be wasted in changing a mixed number dividend to an improper fraction. Nos. 64, 65, and 66 resemble those already worked. In No. 69, after obtaining the quotient 14, there will be a remainder $2\frac{1}{2}$, which is changed to $\frac{5}{2}$ and divided by 5, giving $\frac{1}{2}$ as the result. In No. 69, the remainder, $5\frac{1}{4}$, is changed to $\frac{21}{4}$, which gives $\frac{21}{32}$ when it is divided by 8.

656. Some mistakes would be avoided if pupils would learn to ask themselves if the answer they have obtained is a reasonable one. Permit the scholars to work out all these examples without giving them a rule for "pointing off."

669. See Art. 521.

3. 6×6.
4. $300 \div 12$.
5. 86×1.
6. $36 \div 4$.
7. $800 \div 100$.
8. 8×8.
9. 7×11.
10. $64 \times \frac{3}{8}$.

670.

3. 25×12.
4. $36 \div 6$.
5. $86 \div 1$.
6. 32×5.
7. $800 \times .1$.
8. 8×8.
9. 7×12.
10. $64 \div \frac{1}{4}$.

671.

2. $\$2\frac{1}{2} \times 200$.
3. 25¢ $\times 4$.
4. $\$12 \times 400$.
5. $\$2 \times 8$.
6. 25¢ $\times 800$.
7. $\$2\frac{1}{2} \times 20$.
8. 60¢ $\times 1000$.
9. $\$5000 \times 7$.
10. $\$1 \times 6$.

677. Teachers should carefully avoid giving unnecessary "rules." There is no good reason why an average pupil should not be able to determine for himself how to ascertain what part of $15 a man has spent when he has spent $5. While the introduction of fractions into such an example makes it more difficult for the scholar to give the answer off-hand, his instruction up to this time should have taught him that the same process is to be employed. A pupil should be required to depend upon himself to at least a reasonable extent.

678. As a preliminary to the work in denominate numbers in the next three pages, the teacher should place on the board a few such examples as the following, to which the scholars should give answers at sight:

$$\begin{array}{r} 1\tfrac{1}{2}\text{ qt.} \\ \underline{+\,1\tfrac{1}{2}\text{ qt.}} \end{array} \qquad \begin{array}{r} 1\text{ qt. }1\text{ pt.} \\ \underline{+\,1\text{ qt. }1\text{ pt.}} \end{array} \qquad \begin{array}{r} 3\text{ qt.}\phantom{1\text{ pt.}} \\ \underline{-\,1\text{ qt. }1\text{ pt.}} \end{array} \qquad \begin{array}{r} 1\text{ qt. }1\text{ pt.} \\ \underline{\phantom{1\text{ qt. }1\text{ pt}}\times 2} \end{array}$$

$$2)\underline{3\text{ qt.}} \qquad\qquad 1\text{ qt. }1\text{ pt.})\underline{3\text{ qt.}}$$

Nearly every member of the class will be able to obtain the results in a moment, without any suggestions from the teacher. If the examples are left on the board, the pupils can refer to them for aid in working some of those found in the text-book.

The teacher that wishes to develop power in her scholars should be careful not to give a particle more assistance than is necessary. She should permit the children to deduce from the above examples the rules necessary to solve the others, being patient if the pupils are somewhat slow in doing this work. When, however, a circuitous method has been employed, she should lead the class to see how the work can be improved by the use of a shorter way.

680. It may be necessary to take up again, for purposes of review, the preliminary exercises of the previous chapter. See Art. 569, pp. 56 and 57.

681. As the table of square measure is not introduced until the next chapter, it will be necessary to reduce to yards the dimensions that are given in feet or inches.

2. 18 yd. by 21 yd.	**8.** 18 yd. by 2 yd.
3. 2 yd. by 3 yd.	**9.** 16 yd. by 15 yd.
7. 9 yd. by 32 yd.	**10.** $1\frac{1}{2}$ yd. by 24 yd.

682. No. 14. 14 yd. by $\frac{2}{3}$ yd.

No. 15. 8 pieces, each 36 yd. long and $\frac{32}{36}$ yd., or $\frac{8}{9}$ yd. wide.

No. 18. See Arithmetic, Art. 818, problem 20. A modification of this diagram, showing four squares instead of four rectangles will be the drawing required, except that the squares above and below need not necessarily occupy the positions there indicated.

XI

NOTES ON CHAPTER EIGHT

With this chapter begins the regular work in decimal fractions, and the pupils should now be taught the principles underlying the various operations.

685. While pupils may know that $\frac{23}{8}$ means that 23 is to be divided by 8, it may be well to lead them again to see that $\frac{3}{4}$ is the same as $3 \div 4$, or $\frac{1}{4}$ of 3. After they understand that every common fraction may be considered an "indicated division," they will understand that the decimal fraction obtained by performing this operation is the equivalent of the common fraction whose denominator is used as a divisor and whose numerator is used as a dividend. See Arts. 563 and 564.

686. As previous work in decimals has been confined chiefly to three places, some review and extension of the notation and numeration exercises of Arts. 547–551 may be necessary.

687. After writing each of these decimals in the form of a common fraction, a scholar should be able to determine at a glance whether or not it can be reduced to lower terms. This reduction is possible when the decimal is an even number or terminates in a 5.

While it is inadvisable to waste time in calculating the greatest common divisor, pupils should be encouraged to use large divisors; 4 rather than 2, when possible, and 25 rather than 5.

688. The common fractions contained in these exercises are such as do not require much calculating to change them to deci-

mals. The scholars should be able to write the numbers in vertical columns directly from the text-book, making the necessary reductions mentally.

In reducing $\frac{1}{16}$ to a decimal, it may be easier for some to consider it $\frac{1}{4}$ of $\frac{1}{4}$, or $\frac{1}{4}$ of .25. The reduction of $\frac{23}{50}$ is simplified by multiplying each term by 2, making it $\frac{46}{100}$, or .46, instead of dividing 23 by 50, etc., etc.

690. Nos. 62, 64, 66, and 68 may be worked by using the common fraction given, and also by reducing this to a decimal before performing the multiplication.

691. See Art. 563, p. 55, and Art. 616.

692. The teacher should not permit the employment of long division in these examples. In No. 92, the children can see that changing the dividend to .18756 divides it by 100, and that .18756 ÷ 3 is the same as 18.756 by 300. See Arithmetic, Art. 668.

694. Ciphers at the right of a decimal should be rejected, excepting, perhaps, the final 0 in cents. See Nos. 3, 4, and 10.

2. \$.95 × 7.6.
3. \$2.80 × 48.6.
4. \$21.30 × 39.25.
5. \$.68 × 18.75.
6. \$22 × 108.745.
7. \$.75 × 148.6.
8. \$.13$\frac{1}{4}$ × (2376 ÷ 12).
9. \$35 × 4.5.
10. \$13.50 × [(28 × 12) ÷ 144].

While it is inadvisable to confuse children by too many short methods in the earlier stages, they should be encouraged in examples like the foregoing to use as a multiplier the number that will make the work easier, and to employ a common fraction instead of a decimal whenever the use of the former would lighten their labor. In No. 1, for instance, the result is obtained with fewer figures by multiplying 24.4 by 6$\frac{1}{4}$, instead of 6.25 × 24.4.

695. The operation should first be indicated.

11. $\dfrac{\$.90 \times 38648}{60}$. 13. $\dfrac{\$.36 \times 48576}{32}$.

12. $\dfrac{\$5 \times 18964}{2000}$. 14. $\dfrac{\$1.83 \times 69104}{2 \times 56}$.

etc., etc.

703–704. See notes on previous special drills, Arts. 286 and 350.

705. See Arts. 528 and 649. In multiplying 46 by $33\frac{1}{3}$, divide 46 by 3, which gives $15\frac{1}{3}$, and substitute $33\frac{1}{3}$ for the fraction in the quotient, thus obtaining the result, $1533\frac{1}{3}$.

706. $975 \div 25 = 9\frac{3}{4} \div \frac{1}{4} = 9\frac{3}{4} \times 4$. $433\frac{1}{3} \div 33\frac{1}{3} = 4\frac{1}{3} \div \frac{1}{3} = 4\frac{1}{3} \times 3$.

708. Use first as "sight" problems, if the pupils find the numbers too large to be carried in the mind. By degrees, however, they should acquire the power to solve problems of this kind without seeing the figures, especially when the operations are not numerous or involved.

709. In such examples as Nos. 1, 9, 10, 11, and the like, many children fail to comprehend the form of analysis generally given. While they get some facility in applying the method, they do not understand the underlying principle. In finding a number, $\frac{5}{6}$ of which is 180, they learn to divide by 5 and to multiply the quotient by 6, and to repeat the customary formula, without knowing the reasons for the different operations. There are only four fundamental processes in arithmetic, and children should be taught to determine for themselves which to use in a given example that is within their experience, rather than to depend upon a rule which they do not fully understand, and which they are likely to forget or to misapply. See Art. 635. A few diagrams are here introduced, to be used by the teacher that does

not wish her pupils to obtain in No. 1, for instance, the length of the room by dividing 15 by $\frac{5}{6}$. The five spaces in the width are each 3 ft., which will make the length 18 ft. When a scholar understands this from the diagram, he can understand that when $\frac{5}{6}$ of a number is 15, $\frac{1}{6}$ is $15 \div 5$, or 3.

While, for purposes of drill, many "abstract" examples of the same kind are brought together in one place, care has been taken in the problems to avoid having two consecutive ones alike in character. Problem work to be of value should not be permitted to become mechanical. Pupils should need to study each problem to determine the method of solving it.

710. See Arithmetic, Art. 384.

716. By placing the multiplier at the right of the multiplicand, the pupil can use the latter as the first partial product, instead of writing it again, as he would be compelled to do if the multiplier were placed in its usual position.

717. Some teachers might prefer to place the product by 8 above the multiplicand, as being the form to which the scholars are more accustomed; but in such an example as No. 26, the latter part of the work can be shortened only by placing the product by the units' figure under the other.

$$\begin{array}{r} 19744 \\ \underline{2468 \times 18} \\ 44424 \end{array}\ \textit{Ans.}$$

$$\begin{array}{l} 576 \times 14 \times 21 \\ \underline{\ \ 2304} \\ \ \ 8064 \\ \underline{16128} \\ 169344 \end{array}\ \textit{Ans.}$$

No. 28 should be worked as is here shown. Annexing two ciphers to the multiplicand and multiplying by $\frac{2}{3}$ gives the product by $66\frac{2}{3}$.

$$\begin{array}{r} 48600 \\ \underline{2} \\ 3\overline{)97200} \\ 32400 \\ \underline{226800} \\ 2300400 \end{array}$$

719–720. See Art. 521.

1. 24 lb. @ \$ $\frac{1}{2}$.
2. 24 horses @ \$125.
3. 64 yd. @ \$ $\frac{7}{8}$.
4. 485 bu. @ \$1.

5. 96 lb. @ \$$\frac{1}{4}$.	**18.** $60 \div \frac{5}{8}$.
6. 840 yd. @ \$$\frac{1}{8}$.	**19.** $64 \div \frac{2}{3}$.
7. 360 yd. @ \$$\frac{2}{3}$.	**20.** $28 \div 1\frac{3}{4}$.
8. 48 cwt. @ \$$\frac{5}{8}$.	**21.** 17×4.
9. 92 hats @ \$$1\frac{1}{2}$.	**22.** $256 \times \frac{1}{4}$.
10. 128 lb. @ \$$\frac{1}{8}$.	**23.** 25×16.
11. $27 \div \frac{1}{4}$.	**24.** 6×6.
12. $300 \div 1\frac{1}{2}$.	**25.** 86×1.
13. $24 \div \frac{3}{8}$.	**26.** 33×5.
14. $15 \div \frac{1}{8}$.	**27.** $800 \times \frac{1}{8}$.
15. $60 \div 2\frac{1}{2}$.	**28.** 8×8.
16. $32 \div \frac{1}{8}$.	**29.** 7×11.
17. $70 \div \frac{7}{8}$.	**30.** $64 \times \frac{3}{8}$.

721. The decimals should be reduced to common fractions whenever the work is rendered easier by the change.

1. $360 \times \frac{1}{4}$.	**7.** $72 \times \frac{1}{4}$.	**13.** $84 \times \frac{1}{4}$.
2. $560 \times \frac{1}{8}$.	**8.** $84 \times \frac{3}{4}$.	**14.** 15×6.
3. $240 \times \frac{3}{8}$.	**9.** $96 \times \frac{1}{20}$.	**15.** 4×4.
etc.	etc.	etc.

722. The pupil should employ such method as is best adapted to the particular example:

1. $240 \div \frac{1}{2}$.	**9.** $48 \div \frac{1}{200}$.	**17.** $65 \div \frac{1}{8}$.
2. $360 \div \frac{3}{4}$.	**10.** $72 \div \frac{1}{40}$.	**18.** $840 \div 8$.
3. $45 \div \frac{1}{8}$.	**11.** $92000 \div 2$.	**19.** $11 \div \frac{1}{16}$.
etc.	etc.	etc.

723. Nos. 1 to 8 are intended to furnish practice in sight cancellation. In Nos. 13 to 16, the reduction of the multiplier to an improper fraction will simplify the work for some pupils.

726. Whenever possible, the least common denominator should be determined by inspection.

735. Do not give "rules." See Art. 678.

736–737. These exercises are introduced to accustom the pupils to add and to subtract simple mixed numbers without rewriting the fractions reduced to a common denominator.

740. In these and other similar examples, the teacher should not anticipate the work of the higher grades by systematic instruction in advanced topics. All that should be done with respect to these problems is to show the pupils that, when a solution involves multiplication and division, time may frequently be saved by means of cancellation. The pupils should be permitted to work out No. 1 at length, if they wish; after which they should be required to indicate the work by signs, and then to cancel. Division should, of course, be indicated by writing the divisor as a denominator.

Some excellent teachers require their scholars before beginning work on a problem to indicate by signs all the operations necessary to its solution, thereby compelling them to study the conditions thoroughly at the outset. Too many pupils commence to add, subtract, etc., without fully realizing what is required in a given example.

742–744. See Art. 678.

745. The pupils should write the dimensions on each diagram, changing them, when necessary, to the denomination required in the answer.

746. The formal study of percentage belongs to the next year of the course, and teachers should not dwell too much on this topic. After the pupils understand the meaning of the term *per cent*, they should be able to work the examples given. Other technical terms, definitions, etc., should be omitted for the present.

753. The pupils will readily see that the words "Bought of," used in Arts. 546 and 642, are inappropriate in bills for wor!

done. No. 5 may be made out in the form here shown or similar to the one given in Art. 546. See Art. 642 for a bill for goods bought at different times; or use the heading given in this article.

754. What has been said about percentage in Art. 746, is applicable to this topic. Such children as hear their parents talk of savings-banks, etc., know sufficient about interest for the purposes of this chapter. No rules should be given.

756. The pupils should deduce their own rule for calculating the area of a right-angled triangle.

758. In Art. 653 the pupils have been taught to multiply $18\frac{3}{8}$ by 6 in one line; in Art. 654, they have learned how to divide $18\frac{3}{8}$ by 2, which is the same as finding $\frac{1}{2}$ of $18\frac{3}{8}$, so that nothing new is here presented.

763–764. Although these examples are not strictly practical, they are useful in giving the pupils the facility necessary to perform readily operations involving fractions or decimals. While it is not necessary to work them all, the scholars should by this time have acquired such expertness in the fundamental operations as to be able to obtain the results in a very short time.

765. See Arithmetic, Art. 591.

XII

NOTES ON CHAPTER NINE

The technical terms used in denominate number work should now be regularly employed by teacher and pupil, but set definitions should not be memorized. The scholars should be required to arrange their work properly, and to perform the various operations with as few figures as are consistent with accuracy.

767. In reducing 16 gal. 1 qt. to quarts, the pupil should write 65 qt. at once. He multiplies by 4, saying 4 sixes are 24, and 1 are 25 — writing the 5, etc. In reducing $31\frac{1}{2}$ gal. to quarts, the work should occupy but a single line. See Arithmetic, Art. 653.

770. No special rule should be given in Nos. 33, 34, and 35 for the reduction of a fractional or a decimal denominate unit.

773. A pupil should be permitted to work such examples as No. 2 in his own way. They do not occur frequently enough in practice to make it advisable to give them special treatment; but the teacher should suggest, as in other exercises, the advisability of shortening the work by indicating operations and cancelling. Thus,

$$12 \text{ min. } 30 \text{ sec.} = 12\tfrac{1}{2} \text{ min.} = \frac{12\frac{1}{2}}{60} \text{ hr.} = \frac{12\frac{1}{2}}{60 \times 24} \text{ da., etc.}$$

5. $\dfrac{9}{60 \times 24}$ da.

6. 750 lb. $= \frac{750}{2000}$ T. $= \frac{3}{8}$ T.; $\$5 \times 5\frac{3}{8} = \$26.87\frac{1}{2}$, or $26.88. *Ans.*

7. No. of tons $= \$18.76 \div \$5 = 3.752$; $.752$ T. $= (.752 \times 2000)$ lb. $= 1504$ lb. *Ans.* 3 T. 1504 lb.

8. 7 T. 296 lb. $= 14296$ lb.; $(\$35.74 \div 14296) \times 18748 =$ *Ans.*

9. 9 T. 1568 lb. $= 19568$ lb.; $\$48.92 \div 19568 =$ cost per lb. $\$73.11 \div (\$48.92 \div 19568)$, or $(\$73.11 \times 19568) \div \$48.92 =$ number of pounds. Reduce to tons, etc.

774. By this time, the pupils should know how to add compound numbers, so that the chief duty of the teacher should be to see that the operation is not spun out too much. A scholar of this grade should not find the total number of ounces in **1** by adding each column separately; he should say 27, 36, 39 oz., or 2 lb. 7 oz., without writing anything but the 7 oz., which is put in its proper column and 2 lb. carried.

In **4**, the addition of the units' column of minutes gives a sum of 15. Since minutes are changed to hours by dividing by 60, which ends in a cipher, the units' figure of the remainder will be 5, so that this figure may be written in the total. Carrying one, the sum of the tens' column is 11, which contains 6 once with a remainder of 5. This is written in its place, making 55 minutes, and 1 hour is carried. The two columns of hours are added in one operation—21, 38, 43, or 1 day 19 hours. **6** should be treated in the same way, no side work being permitted.

In **7**, the pounds are reduced to tons by dividing by 2000, so that the sum of the units', tens', and hundreds' columns of pounds may be written in the total, the sum of the thousands' column being divided by 2 to reduce to tons.

775. Nothing should be written but the results. In **27**, the addition of 1 ton to 1552 lb. will change only one figure of the latter, and this change can be carried in the head. In **29**, 320 rods should be added to 15 rods mentally and 24 rods deducted from the sum, only the answer being written.

779. In dividing 5 bu. by 4, **79**, the answer is not to be given as $1\frac{1}{4}$ bu.; the division should be continued through pecks. The result in **88** should contain weeks, days, hours, and minutes.

784. While these drills seem somewhat difficult for mental work, they should not be too severe for children that have been studying arithmetic for over five years, especially if the previous drills have been faithfully attended to. The ability of many children to handle numbers seems to decrease after the fourth school year, the greater portion of the subsequent instruction being given to new topics to the neglect of continued practice in the fundamental processes. The conscientious teacher should remember that the bulk of the mathematical work of most of her scholars after they leave school will not extend much beyond what has been learned in the first four years.

The ability to handle at sight or mentally such numbers as are here given, will be of use to the scholars in various ways. The average pupil attends to only one figure at a time; and he is frequently unable, after a simple addition or multiplication, to see that his answer is very far astray. Practice with such drills as these, and in the sight approximations, will enable him to test his work in such a way as to detect any very serious error.

Scholars find it easier to add or subtract such numbers as 163, 8610, etc., when they are read "one, sixty-three;" "eighty-six, ten;" etc. Following the order in which the figures are read seems the most natural way in mental work. When a pupil is asked to find the sum of 163 and 137, he is less likely to make mistakes if he proceeds in this way: 263, 293, 300; adding to the first number — 163 — 100, 30, and 7 in the order in which the figures are repeated to him.

786. In multiplying 21 by 15, 41 by 14, etc., the scholar generally finds it easier to commence with the tens: 15 twenties are 300, 15 ones are 15 — 315; 14 forties are 560, and 14 are 574.

$48 \times 16\frac{2}{3}$ becomes $\frac{1}{6}$ of 48 hundred; $32 \times 37\frac{1}{2} = \frac{3}{8}$ of 32 hundred, etc.

787. These exercises present rather more difficulty, and are probably not so useful, on the whole, as the others. For this reason, they should be employed as sight work chiefly.

788. In $13\frac{3}{4}\times 5$, multiply 13 first by 5, and then $\frac{3}{4}$, obtaining $65+3\frac{3}{4}$, or $68\frac{3}{4}$. In dividing 24 by $2\frac{2}{3}$, reduce both to thirds — 72 thirds $\div$ 8 thirds $= 72 \div 8 = 9$.

790. The teacher should not neglect such addition exercises as are scattered throughout the book.

791. It happens occasionally in multiplying by a mixed number, that the units' figure of the integer and the numerator of the fraction are the same. In such a case, a few figures will be saved by following the method given in the text-book, instead of writing again the product by 3 as shown above.

$$\begin{array}{r} 4846 \\ \times\, 3\frac{3}{5} \\ \hline 5\,)\,14538 \\ \hline 2907\frac{3}{5} \\ 14538 \\ \hline \text{etc.} \end{array}$$

792. The product by 100 may be placed above the number, if desired. In multiplying by 1000, the multiplicand is subtracted from 1000 times itself. To find the product of 9832 by 990, multiply by 99, and annex a cipher to the result. Taking one-fourth of 268400 gives the answer to **21**.

$$\begin{array}{r} 2761 \times 999 \\ 2761000 \quad\quad \\ \hline 2758239 \quad \textit{Ans.} \end{array}$$

800. The pupils should find for themselves in **5** the number of square inches in a square foot, etc. A drawing is asked in the first part of **14**, so that children will see that the dimensions are not 4×6. The short method of finding the area of the fence in **15**, by multiplying 900 by 10, should not be given yet: the scholars should be permitted, for the present, to calculate the area of one part at a time. In **16**, it is suggested that the area of the walk be ascertained by subtracting from the whole area (250×200) sq. ft., the area of the part left for the garden (230×180) sq. ft.; but the scholars should be encouraged to calculate the surface of the walk in another way, such as by taking the two ends as measuring each 250 ft. by 10 ft., and the sides as 180 ft. each by 10 ft. The number of square feet in the sidewalk of **17** will be $(270\times 220)-(250\times 200)$; or $(270\times 10)+(270\times 10)+(200$

$\times 10) + (200 \times 10)$. For **20**, a modification of the diagram in Problem 20, Art. 818, is desired.

801. To show pupils what is required in **21**, a pasteboard box, without a cover, may be opened out as is represented in Problem 2, Art. 871, the upper rectangle (the bottom of the box) representing the ceiling.

802. **3.** The sixth dose will be taken at 7 o'clock, the second at 3 o'clock, the fourth at 5 o'clock. **4.** He works 6 days. **6.** A fence 6 ft. long will require 2 posts; a 12-foot fence will require 3 posts; a fence 120 ft. long will require 21 posts.

803. In finding the time between two dates, the first date is excluded except when the contrary is expressly specified.

804. **11.** 30 days + 19 days. **12.** 0 days in October + 30 days in November + 30 days in December.

805. **1.** In February, there are $(29 - 6)$ days, or 23 days. **3** and **4.** Leap year. **11.** Jan. 8, 15, 22, 29; Feb. 5, 12, 19, 26 are Sundays. The man works 30 days in January and 28 in February, less 8 Sundays and 1 holiday.

807–809. See Arts. 746 and 754.

808. **3–13** should be worked as "sight" exercises, $\frac{1}{4}$ being used for 25%, $\frac{1}{8}$ for $12\frac{1}{2}$%, etc.

810. First compute the interest for one year.

1. \$3.60 for a year; $\frac{1}{6}$ of \$3.60 for 2 months.
2. \$3.60 for a year; $\frac{1}{6}$ of \$3.60 for 60 days.
3. \$5.00 for a year; $\$5 \times 2\frac{1}{2}$ for 2 yr. 6 mo.
4. \$6.00 for a year; $\frac{1}{12}$ of \$6 for 30 days.
5. \$9.00 for a year; $\frac{1}{4}$ of \$9 for 90 days.

812. To take advantage of any opportunities for cancellation that may be offered, this method is given. It will afterwards be found useful in calculating the principal, the rate, or the time.

Pupils should not at this stage be taught more than one method of finding interest, and that the most direct and the most obvious one.

813. 2. In changing 2 mo. 12 da. to the fraction of a year, it is not necessary to reduce to the lowest terms. Change the time to 72 days, and write 360 underneath, $\frac{72}{360}$; the necessary reduction can be made later in the cancellation. 6. Write 21 months as $\frac{21}{12}$ years, canceling afterwards.

The 100 in the denominator of an interest example should seldom be canceled, except as a whole or by 10.

815. 1. 6 hr. 17 min. 5 sec. = 22625 sec.; 3 hr. 15 min. 25 sec. = 11725 sec. *Ans.* $\frac{11725}{22625} = \frac{469}{905}$.

2. 3 mi. 96 rd. $\times 3\frac{1}{3} =$ 9 mi. 288 rd. + 1 mi. 32 rd. = 11 mi. *Ans.*

4. A furnished $\frac{1}{2}$ of the money, and should receive $\frac{1}{2}$ of \$1500, or \$750; B should receive $\frac{1}{3}$ of \$1500, or \$500; C should receive $\frac{1}{6}$ of \$1500.

5. If 5 T. 1000 lb., or 11000 lb., cost \$30.25, 1 lb. will cost \$30.25 ÷ 11000; and 7 T. 320 lb., or 14320 lb., will cost (\$30.25 ÷ 11000) × 14320. Cancel. $\frac{\$30.25 \times 14320}{11000}$

6. 25¢ $\times 8\frac{36}{60} =$ 25¢ $\times 8\frac{3}{5}$. *Ans.*

7. 2 yr. 7 mo. 8 da. = 31 mo. 8 da. = $31\frac{8}{30}$ mo. = $31\frac{4}{15}$ mo. \$45 $\times 31\frac{4}{15} =$ *Ans.*

10. 360 yd. @ 30¢ cost \$108. The number of square yards $= 360 \times \frac{27}{36} = 360 \times \frac{3}{4} = 270$, on which the duty at 8¢ per sq. yd. will be 8¢ × 270, or \$21.60. The duty on the value will be 50% of \$108, or $\frac{1}{2}$ of \$108, or \$54; the total duty being \$21.60 + \$54 = \$75.60. *Ans.*

816. 3. 5 bbl., 300 lb. each, @ 5¢ per lb.

4. Interest on \$200 for 6 mo. @ 6%.

5. 12 men take 24 days; how long will 24 men take?

6. What decimal of 640 acres is 320 acres?

7. 20 thousand bricks @ \$20 per M.

8. 5600 lb. @ 87¢ per bu. of 56 lb.

9. 10 lb. cost \$8; find cost of 21 lb.

10. Freight on 20 hundred lb. @ 70¢ per cwt.

817. 2. The wall 8 yd. $\times$ 4 yd. contains 32 sq. yd.; the door is $\frac{8}{3}$ yd. by $1\frac{1}{2}$ yd., and contains 4 sq. yd.; 32 sq. yd. $-$ 4 sq. yd. $=$ 28 sq. yd. *Ans.*

3. Number of square inches in the surface of the widest face $= 8 \times 4$; in the surface of one side $= 8 \times 2$; in the surface of end $= 4 \times 2$.

4. $(288 \times 96) \div (8 \times 4)$.

5. $[(24 \times 12) \times (8 \times 12)] \div [8 \times 2]$.

6. See No. 20. Make four rectangles adjoining each other, each 8 inches high — the first and the third being 4 inches wide; and the second and the fourth, 2 inches wide. Above and below the second, and connected with it, draw rectangles 2 inches wide and 4 inches high. These two rectangles may be drawn above and below either of the other rectangles, the above dimension being used if drawn above and below the fourth; if drawn above and below the first or the third, they will be 4 inches wide and 2 inches high. The pupils should be permitted to make the diagram in their own way, and they should be encouraged to make one that differs from one drawn by a desk-mate.

8. The number of rolls will be $(45 \times 36) \div (24 \times 1\frac{1}{2})$. Cancel.

818. The scholars should make this table without any assistance. To obtain the number of acres in a square mile, indicate the number of square rods in a square mile, 320×320, and divide by the number of square rods in an acre, 160.

14. The number of yards $= (5 + 3 + 4 + 7 + 3 + 6 + 12 + 10) \times 5\frac{1}{2}$.

15. Original dimensions 12 rods $\times$ 13 rods, making area 156 sq. rd. Present area $=$ 156 sq. rd. $-$ (15 + 21) sq. rd.

19. $[\frac{1}{2}$ of $(80 \times 60\frac{1}{2})] \div 4840 =$ *Ans.*

819. 1. 43 yd. $= (43 \div 5\frac{1}{2})$ rd. $= (43 \div \frac{11}{2})$ rd. $= (43 \times \frac{2}{11})$ rd. $= \frac{86}{11}$ rd. $= 7\frac{9}{11}$ rd.

2. 43 yd. $= 7\frac{9}{11}$ rd. $\frac{9}{11}$ rd. $= (\frac{9}{11} \times 5\frac{1}{2})$ yd. $= (\frac{9}{11} \times \frac{11}{2})$ yd. $= 4\frac{1}{2}$ yd. *Ans.* 7 rd. $4\frac{1}{2}$ yd.

3. 43 yd. = 7 rd. $4\frac{1}{2}$ yd. = 7 rd. 4 yd. $1\frac{1}{2}$ ft.

4. 43 yd. = 7 rd. 4 yd. $1\frac{1}{2}$ ft. = 7 rd. 4 yd. 1 ft. 6 in.

824. 34. Carrying 1 to the column of yards, the total becomes 8 yd. or 1 rd. $2\frac{1}{2}$ yd. Changing $\frac{1}{2}$ yd. to 1 ft. 6 in., and adding this to 17 rd. 2 yd. 1 ft. 6 in., the accompanying answer is obtained.

4 rd.	3 yd.	1 ft.	
9 rd.	4 yd.	2 ft.	
3 rd.		1 ft.	6 in.
17 rd.	$2\frac{1}{2}$ yd.	1 ft.	6 in.
17 rd.	2 yd.	1 ft.	6 in.
	$+\frac{1}{2}$ yd.	= 1 ft.	6 in.
17 rd.	3 yd.	*Ans.*	

38. 8 rd. 0 yd. 1 ft. − 2 rd. 0 yd. 2 ft. = 5 rd. $4\frac{1}{2}$ yd. 2 ft. = 5 rd. 4 yd. 2 ft. + 1 ft. 6 in. = 5 rd. 5 yd. 6 in. *Ans.*

40. 5 rd. 4 yd. 2 ft. × 4 = 23 rd. $1\frac{1}{2}$ yd. 2 ft. = 23 rd. 1 yd. 2 ft. + 1 ft. 6 in. = 23 rd. 2 yd. 6 in.

825. 10. The other dimensions would be 8 ft. and 4 ft., or 16 ft. and 2 ft.

14. Number of cubic yards $= \frac{18}{3} \times \frac{55}{3} \times \frac{6}{3}$. Cancel.

15. $\frac{1}{2}$ (yd.) × 2 (yd.) × width (yd.) = 1 (cu. yd.); or $\frac{1}{2} \times 2 \times x = 1$; $x = 1$; 1 yd. *Ans.*

16. A gallon contains 231 cu. in.; a cubic foot contains 1728 cu. in. 1 cu. ft. = (1728 ÷ 231) gal. = $7\frac{111}{231}$ gal. = about $7\frac{1}{2}$ gal. *Ans.*

17. About $1\frac{1}{4}$ cu. ft. *Ans.*

18. Number of gallons $(21 \times 15 \times 22) \div 231$.

19. The decimal in the denominator is removed one place to the right, and a cipher is annexed to 64 in the numerator.

$$\frac{36 \times 28 \times 64 \odot 0.}{2150 \odot 4.}$$

NOTE. — 2150.4 cu. in. is used instead of 2150.42 cu. in., the more correct equivalent, because the former is divisible by 6, 7, 8, etc.

25. $[\$6.40 \times (40 \times 16\frac{1}{2}) \times 4 \times 3] \div 24\frac{3}{4}$. Cancel.

826. 3. At $7\frac{1}{2}$ gal. to cu. ft., a tank of 150 gal. will contain $(150 \div 7\frac{1}{2})$ cu. ft. $= 20$ cu. ft. The dimensions will be 2 ft. $\times$ 2 ft. $\times$ 5 ft., or 4 ft. $\times$ 1 ft. $\times$ 5 ft., etc., etc.

4. At $1\frac{1}{4}$ cu. ft. to a bushel, the bin will contain $1\frac{1}{4}$ cu. ft. $\times 100 = 125$ cu. ft. The dimensions will be 5 ft. $\times$ 5 ft. $\times$ 5 ft., or 5 ft. by 10 ft. by $2\frac{1}{2}$ ft., etc., etc.

5. 1000 bricks will build $(1000 \div 20)$ cu. ft. A wall 1 ft. thick can be 10 ft. long and 2 ft. high, or 5 ft. long and 4 ft. high, etc.

6. 10 yd. $\times$ 5 yd. $\times$ 2 yd. (30 ft. $\times$ 15 ft. $\times$ 6 ft.), 4 yd. $\times$ 5 yd. $\times$ 5 yd., etc.

7. A gallon weighs about 8 lb.; a pint about 1 lb.

8. A cubic foot of iron weighs about 7 times 64 pounds.

9. About $\frac{1}{2}$ of \$800.

10. About 4 years' interest.

831. 5. See Arithmetic, Art. 642.

6. See Arts. 829–830.

7. Three inches square $= (3 \times 3)$ sq. in.

832. 1. See Art. 1022, No. 15.

4. Including Sept. 19, the time is $(28 + 30 + 31 + 30 + 31 + 31 + 19)$ days.

833. 6. The written analysis of an arithmetic example should be required occasionally as an exercise in composition.

7. 7000 gr. $\times 2\frac{7}{8}$ = number of grains in 2 lb. 14 oz. Dividing by 480 grains, the number of Troy ounces is obtained — $(7000 \times 2\frac{7}{8}) \div 480$. Multiplying \$1.80 by this number, the cost of the urn is ascertained — $(\$1.80 \times 7000 \times 2\frac{7}{8}) \div 480$.

835. 8. Since the denominator of a fraction indicates the number of parts into which a thing is divided, a larger denomina-

tor indicates a greater number of parts, and, therefore, smaller ones.

839. 1. If three-fifths of a bbl. cost \$2.13, six-fifths will cost twice \$2.13.

8. Commission at 1% would amount to \$3; at $\frac{1}{2}$%, it is \$1.50.

18. $\frac{3}{4} = 18¢$; $\frac{1}{4} = 6¢$; $\frac{1}{8}$, or $\frac{1}{2}$ of $\frac{1}{4}$, $= \frac{1}{2}$ of 6¢.

841. Add without re-writing the fractions reduced to a common denominator.

845. 10. 75% of $\frac{6}{10}$, or $\frac{3}{4}$ of $\frac{6}{10}$, or $\frac{9}{20}$, is sold for \$1710. Factory is worth $\$1710 \div \frac{9}{20}$.

13. First piece contains $(20 \times \frac{3}{4})$ sq. yd., or 15 sq. yd. Width of second piece in yards $= 15 \div 12$.

16. 8 men and 5 boys $=$ 8 men $+ 2\frac{1}{2}$ men $= 10\frac{1}{2}$ men. If 7 men do a piece of work in $10\frac{1}{2}$ days, 1 man will do it in $10\frac{1}{2}$ days $\times 7$, and $10\frac{1}{2}$ men will do it in $(10\frac{1}{2}$ days $\times 7) \div 10\frac{1}{2}$. Cancel.

17. Each of the six square faces of a cube contains (6×6) sq. in., or 36 sq. in.; the whole surface will be, therefore, 36 sq. in. $\times 6$. Each face contains $(\frac{1}{2} \times \frac{1}{2})$ sq. ft. $= \frac{1}{4}$ sq. ft.; whole surface $=$ $\frac{1}{4}$ sq. ft. $\times 6$.

Contents in cu. in. $= 6 \times 6 \times 6$; in cu. ft. $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$.

18. $\frac{27}{45}$ in water and $\frac{10}{45}$ in mud $= \frac{37}{45}$, leaving $\frac{8}{45}$ above water, or 5 ft. Length of post $=$ 5 ft. $\div \frac{8}{45}$.

19. $[(10 \times 9) + (12 \times 10) + (8 \times 11) + (6 \times 12) + (2 \times 13) + (1 \times 14)] \div [10 + 12 + 8 + 6 + 2 + 1]$.

21. From Oct. 25 to Dec. 31, inclusive, there are $7 + 30 + 31$, or 68 days; Oct. 30 is Sunday; also Nov. 6, 13, 20, 27; Dec. 4, 11, 18, and 25 — 9 Sundays, Election Day, and Thanksgiving to be deducted, or 11 days, leaving 57 days, at \$$3\frac{1}{2}$ per day.

23. 12 lb. tea cost \$2.80 + \$2.00, or \$4.80; value per lb. 40¢.

24. House and lot, or $3\frac{1}{2}$ lots + 1 lot, or $4\frac{1}{2}$ lots = \$8100; 1 lot = \$8100 ÷ $4\frac{1}{2}$ = \$1800; house = \$1800 × $3\frac{1}{2}$.

25. $$\frac{\$18 \times (20 \times 12) \times (15 \times 12) \times (6 \times 12)}{1000 \times 8 \times 4 \times 2}.$$

27. 36 yd. 8 in. = 1304 in.; 13 yd. 1 ft. 9 in. = 489 in.; quantity left = 1304 in. − 489 in. = 815 in.; fraction left = $\frac{815}{1304}$ = $\frac{5}{8}$; decimal left = .625; per cent left = $62\frac{1}{2}$. *Ans.*

28. Assessed value = 80% of \$30000 = \$24000. Taxes on 24 thousand dollars = \$21.60 × 24.

846. **2.** See Arithmetic, Art. 1251; angles *E*, *F*, *G*, and *H*; and *M*, *N*, *O*, and *P*.

3. Art. 1251; angles *A* and *B*, *C* and *D*.

4. Angles *I* and *J*, *K* and *L*. The scholars should understand that two lines can be perpendicular without one being a horizontal line and the other a vertical line.

5. The size of an angle does not depend upon the length of the lines that form the angle. Two short lines may meet at a very obtuse angle, and two long lines may form a very acute angle.

13. If the pupils have in their drawing lessons constructed triangles by means of compasses, these may be used; otherwise, let them manage as best they can, no great accuracy being required.

15. Children are accustomed to seeing an isosceles triangle in only one position: they should learn that if a triangle has two equal sides, it is isosceles, no matter whether the unequal side is vertical, horizontal, or oblique.

16–22. Accustom the scholars to the occasional employment of an oblique line as a base in constructing squares, rectangles, etc. See Arithmetic, Art. 1265. A card may be used to make a square corner.

24. See Arithmetic, Art. 929, No. 8, for a rectangle, a rhombus, and a rhomboid, having equal bases and equal altitudes. No. 5 shows three rhomboids of equal bases and equal altitudes, but differing in shape.

25. See Art. 929, No. 8.

847. **1.** ($\frac{1}{2}$ of 15×20) sq. in. The length of the third side does not enter into the computation.

6. Let the scholars find the area of the rectangle, 66 ft. by 63 ft., and the two triangles, 31 ft. each by 63 ft., and find the sum of the areas. Then lead them to see that bringing the right-hand triangle to the left of the rhombus would make a rectangle 97 ft. by 63 ft., whose area is the sum above found.

7. Find the area in square meters, saying nothing more about the meter than that it is largely used on the continent of Europe, and is a little longer than a yard.

8–10. Give no rules yet for calculating the areas of trapezoids and trapeziums. Let the pupils ascertain the areas of the figures from the data supplied.

XIII

NOTES ON CHAPTER TEN

The formal study of algebra belongs to the high-school; but some so-called arithmetical problems are so much simplified by the use of the equation that it is a mistake for a teacher not to avail herself of this means of lightening her pupils' burdens.

In beginning this part of her mathematical instruction, the teacher should not bewilder her scholars with definitions. The necessary terms should be employed as occasion requires, and without any explanation beyond that which is absolutely necessary.

849. Very young pupils can give answers to most of these questions; so that there will be no need, for the present, at least, of introducing a number of axioms to enable the scholar to obtain a result that he can reach without them.

850. Pupils will learn how to work these problems by working a number of them. They may need to be told that x stands for $1x$; and that, as a rule, only abstract numbers are used in the equations, the denomination — dollars, marbles, etc. — being supplied afterwards.

While the scholars should be required to furnish rather full solutions of the earlier problems, they should be permitted to shorten the work by degrees, writing only whatever may be necessary.

4. $x+2x=54$.
5. $x+5x=78$.
6. $7x+5x=156$.
7. $9x-3x=66$.
8. $x+2x+6x=27000$.
9. $x+5x=72$.
10. $x+2x+3x=54$.
11. $x+6x=42$.

12. $2x + 10x = 96$.

13. Let $x =$ the fourth; then $4x =$ the third, $12x =$ the second, and $24x =$ the first.

$$x + 4x + 12x + 24x = 41.$$

14. $x =$ the second, $2x =$ first, $9x =$ third.

15. $5x + 4x = 81$.

16. $24x = 456$.

17. $4x = 340$.

19. $3x + 4x = 175$.

20. Let $x =$ each boy's share; $2x =$ each girl's share.

$$2x + 4x = 240.$$

21. $x =$ number of days son worked; $2x =$ number father worked. $3x =$ son's earnings; $8x =$ father's earnings.

$$3x + 8x = 165.$$

22. $x =$ number of dimes; $2x =$ number of nickels; $6x =$ number of cents.

$$(10 \times x) + (5 \times 2x) + (1 \times 6x) = 78,$$

or

$$10x + 10x + 6x = 78.$$

23. $15x - 12x = 75$.

24. $x + 4x + x + 4x = 250$.

25. Let $x =$ cost of speller; then $3x =$ cost of reader.

26. Let $x =$ smaller; then $5x =$ larger.

27. Let $x =$ Susan's number; $2x =$ Mary's; $3x =$ Jane's.

851. 10: $\frac{1}{3}x$ is the same as $\frac{x}{3}$.

852. Pupils already know that $\frac{3}{4}$ means $3 \div 4$, so that they can understand that $\frac{3x}{4}$ means $3x \div 4$, or $\frac{1}{4}$ of $3x$. When $\frac{1}{4}$ of something ($3x$) is 24, the whole thing ($3x$) must be 4 times 24, or 96; that is, when $\frac{3x}{4} = 24$, $3x = 96$.

When $\frac{2y}{3} = 24$, $2y = 24 \times 3$, or 72.

When $\frac{4z}{5} = 20$, $4z = 20 \times 5$, or 100.

From these examples can be formulated the rule for disposing of a fraction in one term of an equation, which is, to multiply

both terms by the denominator of the fraction. In changing the first term of the equation, $\frac{3x}{4} = 24$, to $3x$, it has been multiplied by 4, so that the second term must also be multiplied by 4.

853. In solving these examples by the algebraic method of "clearing of fractions," attention may be called to its similarity to the arithmetical method. To find the value of y in **2**, the pupil multiplies 8 by 5 and divides the product by 2; as an example in arithmetic, he would divide 8 by $\frac{2}{5}$, that is, he would multiply 8 by $\frac{5}{2}$; the only difference being that by the latter method he would cancel.

While $\frac{2y}{5} = 8$ may be changed to $\frac{y}{5} = 4$ by dividing both terms by 2, beginners are usually advised to begin by "clearing of fractions," short methods being deferred to a later stage.

854. **6** may be written $\frac{3x}{5} + \frac{5x}{7} = 92$.

8. $2\frac{7}{8}x$ should be reduced to an improper fraction, making the equation, $\frac{23x}{8} = 115$. Make similar changes in **12**, **14**, **18**, and **20**.

855. **2.** $x + \frac{5x}{2} = 100$.

5. $\frac{x}{2} + \frac{x}{4} = \frac{267}{4}$; $2x + x = 267$.

6. $\frac{3x}{4} - \frac{3x}{5} = 15$.

9. Let $5x =$ numerator; $7x =$ denominator. $7x - 5x = 24$; $2x = 24$; $x = 12$. The numerator, $5x$, will be 5 times 12, or 60; the denominator will be 84; and the fraction, $\frac{60}{84}$. *Ans.*

10. Let $x =$ greater; $\frac{x}{7} =$ less.

$$x + \frac{x}{7} = 480.$$

Clearing of fractions, $7x + x = 3360,$

$$8x = 3360,$$

$$x = 420, \text{ the greater number,}$$

$$\frac{x}{7} = 60, \text{ the less.}$$

Or, let $x =$ less; $7x =$ greater.

$$x + 7x = 480,$$

$$8x = 480,$$

$$x = 60, \text{ the less,}$$

$$7x = 420, \text{ the greater.}$$

The employment of the latter plan does away with fractions in the original equation.

11. $30x - x = 522$, or $x - \frac{x}{30} = 522$.

13. Let $x =$ number of plums; $4x =$ number of peaches. Then $2x$ will be cost of plums, and $12x$ the cost of the peaches.

$$2x + 12x = 70.$$

15. $$x - \frac{3x}{7} = 80.$$

17. $$x - \frac{3x}{8} - \frac{x}{4} = 24.$$

18. $$x + 1\tfrac{1}{2}x + (1\tfrac{1}{2}x \times 3\tfrac{1}{3}) = 15.$$

$$x + \frac{3x}{2} + 5x = 15.$$

19. Let $x =$ price per yard of the 48-yard piece; $2x =$ price per yard of the 36-yard piece; $48x$ will be the total cost of one, and $72x$, of the other.

$$48x + 72x = 240.$$

20. $160x + 120x = 840$.

856. The pupils should be permitted to give these answers without assistance.

In Art. 857 is explained what is meant by "transposing."

858. While these exercises are so simple that they can be worked without a pencil, they should be used to show the steps generally taken in more complicated equations. In **1**, for instance, the work should take the form here indicated, only a single step being taken at a time. In **19**, the first step is to clear the equation of fractions by multiplying by 6; the second step is to transpose the unknown quantities to the left side of the equation, and the known quantities to the right; the third step is to combine the unknown quantities into one, and to make a similar combination of the known quantities; the last step is to find the value of x.

$$\begin{aligned} x + 37 &= 56 \\ x &= 56 - 37 \\ x &= 19 \end{aligned}$$

$$\begin{aligned} 2x - 6 &= 16 + \frac{x}{2} - \frac{x}{3} \\ 12x - 36 &= 96 + 3x - 2x \\ 12x - 3x + 2x &= 96 + 36 \\ 11x &= 132 \\ x &= 12 \end{aligned}$$

After a little more familiarity with exercises of this kind, the pupil can take short cuts with less danger of mistakes; for the present, however, it will be safer to proceed in the slower way.

859. 5. $x + (x + 75) + x + (x + 75) = 250.$

$$x + x + x + x = 250 - 75 - 75.$$

NOTE.—The parentheses used here are unnecessary. They are employed merely to show that $x + 75$ is one side of the field.

6. $x + (x + 8) = 86.$

9. $x + x + 72 = 96.$

7. $x + x + 318 = 2436.$

10. $x - \frac{x}{3} - \frac{x}{4} = 45.$

8. $x + \frac{x}{2} + 7 = 100.$

11. $x =$ one part; $2x - 6 =$ other part.

$$x + 2x - 6 = 45.$$

12. $x =$ John's money; $x + 5 =$ William's money.

$$3x + 15 + 5x = 103.$$

13. Let $x =$ price of a horse; $x - 80 =$ price of a cow; $4x =$ cost of four horses; $3x - 240 =$ cost of three cows.

$$4x + 3x - 240 = 635,$$
$$7x = 635 + 240 = 875,$$
$$x = 125, \text{ price, in dollars, of a horse;}$$
$$x - 80 = 45, \text{ price, in dollars, of a cow.}$$

Other pupils may solve the problems in this way:

$$x = \text{price of a cow; } x + 80 = \text{price of a horse.}$$
$$3x + 4x + 320 = 635,$$
$$7x = 635 - 320 = 315,$$
$$x = 45, \text{ price, in dollars, of a cow;}$$
$$x + 80 = 125, \text{ price, in dollars, of a horse.}$$

14. $x =$ number of dimes; $x + 11 =$ number of five-cent pieces; $10x =$ value of dimes (in cents); $5x + 55 =$ value of five-cent pieces.

$$10x + 5x + 55 = 100.$$

15. $x =$ greater; $x - 48 =$ less.

$$x + x - 48 = 100.$$

Or, $x =$ less; $x + 48 =$ greater.

$$x + x + 48 = 100.$$

17. $x =$ share of the first;

$x + 2400 =$ share of the second;

$x + 2400 + 2400 =$ share of the third.

$x + x + 2400 + x + 2400 + 2400 = 18000.$

18. Let $x =$ less; $x + 33 =$ greater.

$$x + 33 - 3x = 11.$$

Bringing known quantities to the left side of the equation, and the unknown quantities to the right,

$$33 - 11 = 3x - x,$$
$$22 = 2x,$$
$$11 = x.$$

Or, $$x - 3x = 11 - 33,$$
$$-2x = -22.$$

Changing signs of both terms,

$$2x = 22,$$
$$x = 11.$$

This problem may also be worked in this way:

$$x = \text{less};\ 3x + 11 = \text{greater}.$$
$$3x + 11 - x = 33.$$

19. $x =$ number of 5-cent stamps; $x + 15 =$ number of 2-cent stamps; $x + 30 =$ number of postal cards.

$$5x + 2x + 30 + x + 30 = 100.$$

20. $x =$ number of horses; $x + 17 =$ number of cows; $2x + 39$ $=$ number of sheep.

$$x + x + 17 + 2x + 39 = 88.$$

XIV

NOTES ON CHAPTER ELEVEN

With this chapter begins the regular work in percentage, and it is important that the pupils obtain, as soon as possible; a correct idea of what is meant by the term *per cent.*

Many of the various subdivisions of this topic found in some books, are taken up only incidentally, while others are omitted altogether, the aim being to give the pupils a foundation upon which they can subsequently build, rather than to scatter their energies over too great a diversity of subjects.

860. The reduction of a common fraction to a per cent, consists in changing the former to a decimal of two places. In reducing $\frac{1}{2}$ to a decimal, the result is .5, or 5 tenths; in changing $\frac{1}{2}$ to an equivalent per cent, the result is 50 per cent, or 50 hundredths. In reducing $\frac{1}{8}$ to a decimal, the answer is given in three places, .125, or 125 thousandths; in changing it to a per cent, the division is stopped at the second place, and the remainder written as a fraction, $12\frac{1}{2}$ per cent, or $12\frac{1}{2}$ hundredths.

The denominator of a per cent being always the same, 100, the comparative value of several per cents is known at sight. To compare $\frac{5}{8}$ and $\frac{3}{5}$ as common fractions, they must be changed to $\frac{25}{40}$ and $\frac{24}{40}$; if a further comparison is to be made between these and $\frac{7}{12}$, a new common denominator must be employed, and the fractions reduced to $\frac{75}{120}$, $\frac{72}{120}$, and $\frac{70}{120}$. Changing the fractions to decimals, 625 thousandths, 6 tenths, and $58\frac{1}{3}$ hundredths, simplifies the comparison; but it is still easier to determine their relative value when they are expressed as $62\frac{1}{2}$ per cent, 60 per cent, and $58\frac{1}{3}$ per cent.

The teacher must not be discouraged if the pupil fails to grasp at once the full meaning of percentage. Definitions will not help materially; much practice in working examples is necessary to give the knowledge desired.

863. Many children find it difficult to distinguish between $\frac{1}{2}\%$ and 50%. If the former is read in the business way, $\frac{1}{2}$ of one per cent, it may make the distinction plainer.

864. Per cents being generally given in two figures, scholars hesitate to give the correct answers: 300%, 250%, 125%, $1633\frac{1}{3}\%$, 420%, 910%.

865. While pupils will find $33\frac{1}{3}\%$ of 81 cows, by dividing 81 by 3, they should understand that they are really multiplying 81 by $33\frac{1}{3}$ hundredths, or 81 by $\frac{1}{3}$. In **6**, 6% of 150, or $\frac{6}{100}$ of 150, may be obtained by multiplying 150 by 6 and cutting off two ciphers; or by dividing 150 by 100, obtaining $1\frac{1}{2}$, and multiplying this quotient by 6; or by reducing 6% to $\frac{3}{50}$, and finding $\frac{3}{50}$ of 150. In **9**, the pupil should find 1% of $640 and take one-half of the result.

The scholars should be permitted to use their own method of solving these problems, the different analyses given by the pupils furnishing their class-mates an opportunity to select a simpler method.

866. Although every pupil may not be able to determine at once the shortest way of calculating a given example, no one should be allowed to work **3**, by multiplying by $33\frac{1}{3}$. When the multiplication by $\frac{1}{3}$ has been performed, the answer has been obtained, except as to the location of the decimal point, and the waste of time in multiplying by 3, repeating this product, and adding three columns should not be tolerated. No fault should be found with the average pupil for failing to recognize in **1**, that $6\frac{2}{3}\%$ is $\frac{1}{15}$; or that in **12**, $3\frac{1}{3}\%$ is $\frac{1}{30}$. The general method should be to multiply by the figures given to represent the per

cent, except in such cases as $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, 25%, $33\frac{1}{3}\%$, $37\frac{1}{2}\%$, 50%, and possibly a few others.

$$\begin{array}{r} \$1240 \\ \times .00\frac{1}{8} \\ \hline \$1.55 \end{array} \qquad \begin{array}{r} \$92.40 \\ \times .04\frac{1}{2} \\ \hline \end{array}$$

Where the given numbers are used, they should be made the multipliers and expressed as hundredths. Nothing is gained in **5**, by reducing the $\frac{1}{2}$ to a decimal; although in **13**, writing $5\frac{1}{2}\%$, .055, might make it easier for some.

898. The rule generally given of finding the percentage, by multiplying by the rate expressed as hundredths, is here modified to the extent of using the common fraction to express hundredths, instead of the decimal, as being more in conformity with early algebraic methods.

The teacher that prefers to ascertain the base or the rate by the older arithmetical method, will omit **30–41**.

30. $\frac{x}{100} \times 65 = \frac{13x}{20}$. *Ans.*

31. $\frac{13x}{20} = 26$; etc.

32. $\frac{1}{4}$ of $x = \frac{x}{4}$. *Ans.*

33. $\frac{x}{4} = 42$; etc.

34. $x + \frac{x}{5}$. *Ans.*

35. $x + \frac{x}{5} = 132$; etc.

37. $x - \frac{x}{3} = 78$; etc.

38. $\frac{x}{100}$ of $\frac{2}{3} = \frac{x}{150}$. *Ans.*

39. $\frac{x}{150} = \frac{10}{3}$; etc.

40. $\frac{1}{800}$ of $x = \frac{x}{800}$. *Ans.*

41. $\frac{x}{800} = 23$; etc.

42. Let $x =$ rate. Then $\frac{x}{100}$ of $65 = 26$, or $\frac{x}{100} \times 65 = 26$.

In an equation containing quantities to be multiplied, the multiplication should be performed before the equation is cleared of fractions. This equation becomes $\frac{65x}{100} = 26$, or $\frac{13x}{20} = 26$, it being immaterial whether canceling be done or not.

43. After a little experience with this class of examples, the equation may be written at once, in the order in which the terms are given:

$$24 = \frac{18}{100} \text{ of } x, \text{ or } \frac{18x}{100} = 24.$$

44. $\frac{250}{100}$ of $x = 180$, or $\frac{5x}{2} = 180$.

45. $x + \frac{x}{4} = 85$.

46. $\frac{3}{5} = \frac{x}{100}$ of $\frac{4}{5}$, or $\frac{x}{125} = \frac{3}{5}$.

While the algebraic method is of no advantage to the bright scholar, it makes the employment of rules unnecessary in the case of the ordinary pupil.

48. $x \times \frac{5\frac{1}{2}}{100} \times 1 = \frac{11x}{200}$.

49. $\frac{11x}{200} = 44$.

50. $\frac{3}{5} \times \frac{5}{8}$.

51. $\frac{88x}{100} = 33$.

52. $\frac{1}{8}$ of $800 = 100$; $\frac{1}{8}\%$ of $800 = 1$.

53. $\$175 + \frac{1}{4}$ of $\$175$.

54. $2\frac{1}{2}\%$ of $x = 12.50$; that is, $\frac{x}{40} = 12.50$.

55. $6\frac{1}{4} \times .16$.

56. $3\frac{1}{3} = \frac{x}{100}$ of $\frac{2}{3}$; that is, $\frac{x}{150} = \frac{10}{3}$.

57. $\frac{x}{100}$ of $389.50 = 124.64$; $\frac{389.50x}{100} = 124.64$;
$389.5x = 12464$; $3895x = 124640$.

58. $\frac{95x}{100} = 174.04$; $95x = 17404$.

59. $x + \frac{16x}{100} = 1276$.

60. $984 = \frac{133\frac{1}{3}}{100}$ of x; that is, $\frac{4x}{3} = 984$.

62. $\frac{x}{4} = 386.75$.

65. $x =$ cost of oats; $x + \frac{2x}{100} = 1071$.

Divide the cost of the oats by 30¢ to find the number of bushels.

68. Assessed value $= \frac{2}{3}$ of \$48000 = \$32000. Taxes on 32 thousand dollars = $\$18.50 \times 32$.

869. In giving answers to these and to all other exercises, no "guessing" should be allowed. The pupil should be permitted to obtain the correct result in his own way — that is, no inflexible rule should be given him to follow — but he should be able to get the answer, using the algebraic method if that seems to him the easiest, as it may be in some instances.

The examples are not arranged by "cases," so that each will have to be understood before it can be worked.

The careless pupil will probably give the wrong answer to **13**; saying 6, instead of 600; he will be likely too, in **14**, to use the larger number as a divisor, and to obtain $44\frac{4}{9}\%$ instead of 225%. These mistakes are less likely to occur if he uses equations — $3 = \frac{x}{200}$ and $\frac{9x}{100} = 20\frac{1}{4}$. Even those scholars that have solved in their arithmetic work of the lower grades, examples similar to **15**, will have new light thrown on their method by using the equation, $x + \frac{x}{4} = 20$. In mental work, however, the first term should be made $\frac{5x}{4}$, to reduce it in size, so that it can be more easily remembered. **24** is simplified by changing the fractions to whole numbers — $\frac{9}{12}$ is what per cent of $\frac{10}{12}$, 9 is what per cent of 10 — before beginning to calculate the rate. In **25**, $1\frac{1}{2}$ and $6\frac{2}{3}$ become $\frac{3}{2}$ and $\frac{20}{3}$, $\frac{9}{6}$ and $\frac{40}{6}$, 9 and 40.

870. **1–5** can be worked by the pupils without any explanation; **6–20** present more difficulty. The beginner in algebra

desires to start at once with his x, without any preliminary calculations; and the usual method of treating these examples requires him first to ascertain the gain or the loss before commencing his equation. The formula employed in the first five examples is:

$$\text{Cost} \times \frac{\text{rate}}{100} = \text{gain or loss.}$$

When the pupil knows any two of these three terms, he can calculate the third; and **6–15** furnish data from which the necessary two items can be obtained. The pupil must, however, be careful in **11**, for instance, not to *subtract* the loss from the selling price to obtain the cost.

In the following equations, $cost \times \frac{x}{100}$ is made equal to the *gain* or the *loss*. No canceling has been done.

6. $\frac{600x}{100} = 18$; $6x = 18.$

7. $\frac{1203x}{100} = 401.$

8. $\frac{86.20x}{100} = 12.93$, or $\frac{8620x}{100} = 1293.$

9. $\frac{908.40x}{100} = 181.68.$

10. $\frac{84x}{100} = 5.25$; $84x = 525.$

11. $\frac{84x}{100} = 5.25.$

12. $\frac{125x}{100} = 25.$

13. $\frac{875x}{100} = 43.75$; $875x = 4375.$

14. $\frac{934.56x}{100} = 116.82.$

15. $\frac{1012.50x}{100} = 168.75.$

In **16–20**, the cost is represented by x.

16. $x + \frac{x}{4} = 468.75.$

18. $x + \frac{x}{3} = 1646.08.$

17. $x - \frac{x}{5} = 73.84.$

19. $x - \frac{15x}{100} = 204.$

20. $x - \frac{4x}{100} = 66.30.$

21. Gain $= \frac{1}{5}$ of \$275.

22. $x\%$ of $60 = 15.$

23. $x + \frac{x}{5} = 960.$

24. $x\%$ of $32 = 16.$

25. $x\%$ of $175 = 25.$

26. $x\%$ of $200 = 25.$

27. $33\frac{1}{3}$¢ $+ \frac{1}{3}$ of $33\frac{1}{3}$¢.

28. $x - \frac{16x}{100} = 33.60.$

29. Gain $= 2\frac{1}{2}\%$ of

30. $x - \frac{x}{10} - \frac{x}{4} - \frac{3x}{10} = 70.$

31. $6000 = x\%$ of 16000.

32. $6000 = x\%$ of 24000.

33. $1600 + 2\frac{7}{8}\%$ of 1600.

34. $4200 - 3\frac{1}{2}\%$ of 4200.

871. In **1**, the 30 cu. yd. are reduced to cubic feet by multiplying by 27. Instead of performing the different multiplications, they are merely indicated, so that work may be saved by canceling.

Although **2** should be a simple problem for a bright pupil, it is apt to prove puzzling unless an x is introduced. A pasteboard box may be used to represent the walls and the ceiling of a room, the sides and the top being then opened out to permit of its representation on the blackboard.

3. The area in square feet $= \frac{1}{2}$ of 132×110. This is reduced to acres by dividing by $9 \times 30\frac{1}{4} \times 160$.

$$\frac{132 \times 110 \times 4}{2 \times 9 \times 121 \times 160} = Ans.$$

4. Number of strips = 6 yd. $\div$ 27 in. = 6 yd. $\div \frac{3}{4}$ yd. $= 6 \times \frac{4}{3}$.

7. The "development" of the fence will be represented by four adjoining rectangles, each marked 6 ft. high, the lengths being 25 ft., 100 ft., 25 ft., and 100 ft., respectively, the whole forming a rectangle 6 ft. $\times$ 250 ft.

8. A board's area in square feet $= 12 \times \frac{1}{2} = 6$. Dividing number of square feet in the fence by 6, gives the number of boards.

9. The cost of a square foot is obtained by dividing \$181.50 by $(160 \times 30\frac{1}{4} \times 9)$; this, multiplied by (300×200), gives the cost of the plot.

$$\frac{\$181.50 \times 4 \times 300 \times 200}{160 \times 121 \times 9}.$$

The amount received for the lots will be $\$160 \times 6$.

10. Number of cakes $= (320 \times 160) \div (4 \times 2)$.

11. Number of cubic feet $= 320 \times 160 \times 1\frac{1}{2}$.

12. $(320 \times 160 \times 1\frac{1}{2}) \div (15 \times 32)$.

13. Number of square feet originally $= 640 \times 440$. For building purposes, there will be four pieces, each measuring 300 ft. by 200 ft.

14. The difference between the above areas will represent the number of square feet in the streets.

872. Many of these exercises can be used for mental and sight work. For methods of solution, see Art. 870.

873. As a preliminary to the formal study of interest, the teacher will need to see that her pupils understand what is meant by the term. She can explain that a person borrowing money should pay for its use, just as a person who rents a house, etc.

874. In changing 4 mo. 10 da. to the fraction of a year, many teachers prefer to reduce the time to days and to write the result over 360, $\frac{130}{360}$, leaving the reduction to lowest terms for the subsequent cancellation. In the same way, 1 yr. 5 mo. 15 da. is changed to $(360 + 150 + 15)$ da., or 525 da. $= \frac{525}{360}$ yr. The reduction to days is done very rapidly.

875. 1. $\$750 \times \frac{6}{100} \times \frac{5}{2}$.
2. $\$84.75 \times \frac{4}{100} \times \frac{16}{60}$.
3. $\$308.25 \times \frac{5}{100} \times \frac{20}{360}$.
4. $\$464.75 \times \frac{6}{100} \times \frac{252}{360}$.
5. $\$360 \times \frac{5}{100} \times \frac{33}{360}$.
6. $\$94.43 \times \frac{7}{100} \times \frac{63}{360}$.
7. $\$400 \times \frac{9}{200} \times \frac{391}{360}$.
etc., etc.

877. The teacher should explain that a person that owes money, frequently gives a note as an acknowledgment of the debt, etc.

878. There is no general method applicable to these problems.

1. Interest for a year is $12, or $1 per month, which gives $19 for 1 yr. 7 mo.

2. $3.60 per year is 1¢ per day, 33¢ for 33 da.

3. $6 per year, or $9 for $1\frac{1}{2}$ yr.

4. $6 per year, or $15 in 2 yr. 6 mo.

5. If $50 produces $6 in 2 yr., it will produce $3 in 1 yr.; rate, therefore, is 6%.

6. $18 per year is $1 for 20 da., or $\frac{1}{18}$ yr.

8. 4% per year = 1% for 90 da.; 1% of $150 = $1.50.

9. 5% per year = $\frac{1}{2}$% for 36 da.; $\frac{1}{2}$% of $240 = $\frac{1}{2}$ of $2.40.

11. $1 is 100% of $1; at 5% per year it will take 20 yr. to make 100%.

12. At 6% it will take $16\frac{2}{3}$ yr., or 16 yr. 8 mo., to make 100%.

13. Disregarding $14.90, it will take 25 yr. at 4% to make 100%.

14. $\frac{1}{2}$% per month = 8% for 16 mo.; 8% of $90 = $7.20.

15. 5% for 360 da. = 1% for 72 da.

16. 360 da. ÷ $4\frac{1}{2}$ = 720 da. ÷ 9 = 80 da.

17. 5% for 1 yr. = 1% for 72 da.; 1% of $75 = 75 cents.

18. 1% of $63.	20. $\frac{1}{2}$% of $840.	22. 1% of $275.
19. 1% of $570.	21. 1% of $150.	23. 2% of $360.

879. 1. 30 rd. 5 yd. 1 ft. = 511 ft.; 8 rd. 4 yd. 2 ft. = 146 ft.; $\frac{146}{511} = \frac{2}{7}$. *Ans.*

2. Number of feet deep = (36 × 5) ÷ (6 × 4).

3. 3 mi. 96 rd. $=$ 1056 rd.; 3 hr. 16 min. $= 3\frac{4}{15}$ hr.; 1056 rd. $\times 3\frac{4}{15} =$ 1056 rd. $\times \frac{49}{15} = \frac{17248}{5}$ rd. $= 3449\frac{3}{5}$ rd. $=$ 10 mi. $249\frac{3}{5}$ rd. *Ans.*

4. $(\frac{3}{4} + \frac{7}{8}) \times (\frac{9}{2} \times \frac{8}{27}) \div (\frac{3}{7} \times \frac{84}{5}) = \frac{13}{8} \times \frac{9}{2} \times \frac{8}{27} \times \frac{7}{3} \times \frac{5}{84} = \frac{65}{216}$. *Ans.*

8. The first two figures express 1800; the second two, 5.4.

22. $\$48.37 \div 8\frac{1}{2}$.

880. **2.** Provisions that will supply 450 men for 5 months will supply 5 times 450 men for 1 month, and will supply (5 times 450 men) $\div$ 9 for 9 months, or 250 men. The number that must be discharged $=$ 450 men $-$ 250 men $=$ 200 men. ***Ans.***

15. $x - \frac{15x}{100} - \frac{20x}{100} = 19500.$

16. D bought $\frac{7}{8} \times \frac{4}{5} \times \frac{5}{9} \times \frac{6}{7}$ of the ship.

18. $100x + 50x = 340 \times 75.$

19. $x =$ number distributed by each new man; $2x =$ number distributed by each experienced man.

$$16x + 32x = 36000.$$

20. $x - \frac{x}{4} = 1972.65.$

881. **10.** 100 cents $\div$ 1.13.

883. **3.** $\$1.10 + 15\%$ of $\$1.10$.

4. $\frac{9876}{x} = 87 + \frac{45}{x}.$

$$9876 = 87x + 45.$$

5. 640 is what per cent of $(640 + 560)$, etc.

6. 43 gal. 3 qt. 1 pt. $= 43\frac{7}{8}$ gal.; $\$70.20 \div 43\frac{7}{8} =$ *Ans.*

7. $(48 \times 32) \div (16 \times \frac{1}{2})$.

8. 20 is what per cent of 160? 20 is what per cent of 180?

10. Selling price per bbl. $= \frac{3450}{600} = \frac{23}{4}$; $4\frac{1}{6}\% = \frac{25}{600} = \frac{1}{24}.$

Let $x =$ cost per bbl. $x - \frac{x}{24} = \frac{23}{4}.$

11. $\frac{9.075x}{100} = 24.2.$

884. $425 + 99$ is 1 less than $425 + 100$; $425 + 999$ is 1 less than $425 + 1000$.

885. $565 - 99$ is 1 more than $565 - 100$; $1424 - 999$ is 1 more than $1424 - 1000$.

886. $24 \times 21 = (24 \times 20) + (24 \times 1)$. See Art. 786.

887. $16 \div .25 = 16 \div \frac{1}{4} = 16 \times 4$; $36 \div .75 = 36 \div \frac{3}{4} = 36 \times \frac{4}{3} = 12 \times 4$.

888. $7\frac{1}{2} \div \frac{3}{4} = \frac{15}{2} \div \frac{3}{4} = \frac{30}{4} \div \frac{3}{4} = 30 \div 3$. When the dividend is a whole number, it is frequently better to perform the division in the ordinary way: $63 \div 3\frac{1}{2} = 63 \div \frac{7}{2} = 63 \times \frac{2}{7} = 9 \times 2$.

889. **1.** $\$\frac{7}{8} \times 48$. **2.** $(48 \times \frac{3}{4})$ sq. yd. **3.** $(48 \div \frac{3}{4})$ yd. **7.** 9 into $83\frac{1}{4}$, 9 times and $2\frac{1}{4}$, or $\frac{9}{4}$, remaining; 9 into 9 quarters, $\frac{1}{4}$. *Ans.* $9\frac{1}{4}$. **10.** $\$1\frac{1}{4} \times 19$. **11.** $\$1\frac{7}{8} \times 120$. **12.** $(120 \div 1\frac{7}{8})$ yd. $= (120 \times \frac{8}{15})$ yd. $= (8 \times 8)$ yd. **14.** Dimensions of field $= 80$ rods $\times$ 80 rods. **16.** $95 \div 4\frac{3}{4} = 95 \div \frac{19}{4} = 95 \times \frac{4}{19} = 5 \times 4$. **17.** $4000 \div 2$. **19.** 3 T. will cost \$15; 480 lb. @ $\frac{1}{4}$¢ per lb. will cost \$1.20; total, \$16.20. **20.** For \$10, I can buy 2 tons; for 80¢, I can buy (80×4) lb., or 320 lb. **23.** $347 + 495 = (347 + 500) - 5$. **25.** One man can do $\frac{1}{24}$ in 1 day; the other can do $\frac{1}{48}$ in 1 day; both can do $\frac{1}{24} + \frac{1}{48}$ in 1 day, or $\frac{2}{48} + \frac{1}{48} = \frac{3}{48} = \frac{1}{16}$ in one day, thus requiring 16 days to do the whole work.

890. **1.** \$150, at 4%, for 3 years.

2. 12 cu. ft., at 60 lb. to cu. ft.

3. \$12 is 3% of what?

4. 250 is what per cent of 500?

5. 12 is what per cent of 4?

6. 20 M @ \$30 per M.

7. 4 bbl., 300 lb. each, @ 5¢.
8. $18 \div 4\frac{1}{2}$.
9. $84 \div 4$.
10. 75 @ \$79 (or \$80).

892. 4. Find $\frac{1}{11}$ and annex cipher.

13. See Art. 791.

893. See Arts. 792, 716, 717, 714.

26. Multiply by 36, by subtracting 4 times the number from 40 times the number; multiply by 45, by subtracting 5 times the multiplicand from 50 times the multiplicand.

895. See Art. 563, p. 55.

897. 2. $18x + 15x + 18x + 15x + (18 \times 15) = 930$.

6. Floor space $= (30 \times 24)$ sq. ft.
Air space $= (30 \times 24 \times 15)$ cu. ft.

7. $(30 \times 24) \div \frac{27}{36}$.

8. Reducing to yards: $[(10 \times 5) + (8 \times 5) + (10 \times 5) + (8 \times 5) + (10 \times 8)] \div 3$.

10. Commencing at lower right-hand corner: $(15 + 3 + 12 + 9 + 8 + 18 + 10 + 15 + 9 + 15)$ rd.

12. $[(22 \times 12) \times (14 \times 12) \times (9 \times 12)] \div 2150.4$.

15. Dimensions: 1000 yd., 2 yd., $3\frac{1}{2}$ yd.

16. Dimensions of pipe space: 1000 yd., $1\frac{1}{2}$ yd., $1\frac{1}{2}$ yd.

900. It may be necessary for the teacher to supplement the information given the pupils in connection with the demand notes in Art. 877. The present note is payable at a fixed time, and the place of payment is specified; but it does not bear interest. If, however, it is not paid at maturity it bears interest from that time at the legal rate.

While savings banks loan money only on good security, generally real estate, banks of deposit will advance money on a note, if the officers feel certain that it will be paid at maturity. When William Brown & Sons present the note for discount, they *endorse*

it by writing their name on its back. This transfers the ownership of the note to any subsequent holder, and also makes the endorsers liable for the amount in case the maker fails to pay it at maturity. The discounting bank thus has two parties upon whom to depend for the money.

The sum charged by the bank for this service is the interest on the face of the note for the time it has to run. This sum is called the *discount*, as it is deducted from the sum named in the note; and the difference — called the *avails* or *proceeds* — is given to the owner of the note.

When the above note is due, it is sent to the Park National Bank for collection. If Thomas Tierney, or some other person, does not pay the money before the close of banking hours, the note is *protested;* that is, a notary public certifies that payment has not been made, and notifies the endorsers, William Brown & Sons, of their liability.

901. In states in which days of grace are no longer allowed, the pupils should not employ them even in calculating discount on notes made in places that still have days of grace. Two answers are given to each problem in discount, one including days of grace; the other, enclosed in parentheses, in which days of grace are not employed.

902. These exercises are nothing more than examples in interest, except that in some states, three days are to be added to the time mentioned.

Deducting the discount from the face of the note gives the proceeds.

903. As will be seen in Art. 906, the exact number of days is taken for periods less than a year.

904. Pupils should be led to see that banks are entitled to interest only for the number of days they have to wait for repayment. A failure to understand this, leads to frequent mistakes. Many careless scholars find the difference in time between the

two dates named in the example, — from Feb. 27 to March 9, in 25, — disregarding entirely the time for which it is drawn.

Instead of explaining how to calculate the discount on the notes given in Art. 905, the teacher should permit the class to attempt to ascertain the result by themselves. In case of a failure to obtain the correct answer, a discussion of the matter will lead to a proper understanding of the principles involved.

906. 1900 is not a leap year. See Arithmetic, Art. 1303, Time Measure.

907. Find the total number of meters in the first twelve pieces, and ascertain their value at the price named. Do the same for the remaining four pieces. Reduce the total number of meters in the sixteen pieces to yards, by multiplying by 39.37 and dividing the product by 36.

908. See Arithmetic, Art. 758.

910. In $64\frac{1}{2} \times 11\frac{4}{7}$, the product by $\frac{4}{7}$ is found by multiplying the product of $\frac{1}{7}$, already ascertained, by 4.

912. **8.** A yard is 36 in. If ribbon is 36 in. long, its width must be $(144 \div 36)$ in. to contain 144 sq. in.

14. Some pupils will say without reflecting, 200% — not seeing that the profit is equal to the cost, 100%.

15. 1% of $1500.

20. $10 =$ what per cent of $(40 + 10)$?

27. The remainder $= 20\%$, or $\frac{1}{5} = \$2000$.

29. The thoughtful teacher must determine for herself just how much time she can afford to waste in giving the pupils a number of useless facts about taxes, brokerage, commissions, bonds, etc., etc. The time allotted to arithmetic should be spent chiefly in developing "power" in her scholars. If the latter can correctly apply mathematical principles in ordinary problems suited to their present experience, they will not find it difficult

in later life, after they understand the conditions, to solve such new problems as come up.

In this example, it will be sufficient to say that the "premium for insuring" means "cost of insuring."

913. 1. See Art. 685.

3. The pupils should attempt to frame the definitions asked.

5. (*a*) $\frac{7}{8}$ of $832 = Ans.$

(*b*) $832 = \frac{7}{8}$ of $x = \frac{7x}{8}$; etc.

6. $\frac{15x}{100} = 3750.$

8. The first boy gains $\frac{1}{4}$¢ on a 1¢ apple, or 25%; the second gains $\frac{1}{5}$¢, or 20%.

9. Sold (20×20) sq. rd. + 16 sq. rd.

15. The pupil should be able to state the rule.

19. $x + 2\frac{1}{2}x = 1050.$

21. $([(35 + 23 + 35 + 23) \times 13] + [35 \times 23]) \div 9.$

914. Use first as sight problems.

1. $\frac{2x}{3} - 10 = \frac{x}{4}.$

2. $x + \frac{5x}{7} = 24.$

3. $x + x + 5 = 31.$

4. If $\frac{2}{3}$ of A's money $= \frac{4}{5}$ of B's, A's money $= \frac{4}{5}$ of B's $\times \frac{3}{2}$ $= \frac{6}{5}$ B's. Let $x =$ B's, $\frac{6x}{5} =$ A's. $x + \frac{6x}{5} = 165$; $\frac{11x}{5} = 165$; dividing by 11, $\frac{x}{5} = 15$; $x = 75$. B's = \$75, A's = \$90.

8. After 2 days, there will be enough to feed 4 horses 4 days, or 1 horse 16 days, or 5 horses $3\frac{1}{5}$ days.

915. 7. An ounce avoirdupois contains 7000 gr. $\div$ 16; an ounce troy contains 480 gr.

8. 4 lb. 8 oz. avoirdupois = 7000 troy grains $\times\ 4\frac{1}{2}$.

9. The pure silver amounts to 192.9 gr. $\times$ .9; divide by 480 to reduce to ounces, and multiply 75¢ by the quotient.

$$\frac{75¢ \times 192.9 \times .9}{480}.$$

11. Number of square feet $= [(16 + 14 + 16 + 14) \times 8] + 16 \times 14$.

NOTE. — When the bottom of a tank is covered with sheet lead, the side strips will be $\frac{1}{32}$ in. less than 8 ft. high, etc., but this small difference may be neglected in these four problems.

12. Multiply the number of square feet by $\frac{1}{32}$, and divide the product by 12. Cancel.

21. Assessed value, $\frac{3}{4}$ of \$24000, or \$18000. Taxes $= 1\frac{3}{4}\%$ of \$18000.

917. 8. Dividend $= 2\frac{1}{2}\%$ of (\$50 $\times$ 65).

918. 1. Area of surface to be papered: $[(16 + 14 + 16 + 14) \times 10] - 174$; divided by area of roll, 24 ft. by $1\frac{1}{2}$ ft.

4. When he sells 31 gills, the grocer charges for 32 gills, or $1\frac{1}{31}$ times the correct quantity, thereby charging for $\frac{1}{32}$ more than he gives. In 2 hhd. of 58 gal. 1 qt. 1 pt. each, there are $117\frac{1}{4}$ gal., the dishonest gain on which is $\frac{1}{31}$ of $117\frac{1}{4}$ gal. worth \$.80 $\times\ \frac{1}{31}$ of $117\frac{1}{4}$.

6. 30% of cost $(x) = 21$.

919. 1. (*a*) \$48.50 + (\$48.50 $\times \frac{6}{100} \times \frac{63}{360}$) = \$48.50 + .51 = \$49.01. *Ans.*

Omitting days of grace, \$48.50 + (\$48.50 $\times \frac{6}{100} \times \frac{60}{360}$) = \$48.50 + $.48\frac{1}{2}$ = $\$48.98\frac{1}{2}$, say \$48.99. *Ans.*

These examples should be worked with days of grace or without days of grace, but not in both ways. See Art. 901. Days of grace were not abolished in New York until January 1895.

(*b*) With days of grace, this note is due Dec. 17. Term Dec. 1 to Dec. 17 = 16 days. Discount on \$49.01 for 16 days at 6% $= \$49.01 \times \frac{6}{100} \times \frac{16}{360} = 13¢$. Proceeds = \$49.01 − .13 = \$48.88 *Ans.*

Omitting days of grace, the note is due Dec. 14. Term = 13 days. Discount on \$48.99 for 13 days at 6% $= \$48.99 \times \frac{6}{100} \times \frac{13}{360} = 11¢$. Proceeds = \$48.99 − .11 = \$48.88. *Ans.*

2. With days of grace, the amount due at maturity will be $\$175 + (\$175 \times \frac{6}{100} \times \frac{93}{360}) = \$175 + 2.71 + = \$177.71+$. The term of discount = 93 days − 33 days = 60 days. Interest of \$177.71 for 60 days at 6% will be \$1.78 nearly. Proceeds = \$177.71 − 1.78 = \$175.93. More accurately, \$177.7125 − 1.7771 = \$175.9354 or \$175.94 −.

Without days of grace, the amount due at maturity will be $\$175 + (\$175 \times \frac{6}{100} \times \frac{90}{360}) = \$177.63 -$. Interest of this amount for 57 days = \$1.69 −. Proceeds = \$177.63 − 1.69 = \$175.94.

920. See Arithmetic, Arts. 821, 822.

921. **9.** 4)£183 14*s.* 8*d.*

13. Total number of days' work = 32 + 53 + 41 = 126. Value of 1 day's work = \$283.50 ÷ 126. Share of first man = (\$283.50 ÷ 126) × 32. Cancel.

14. Amount furnished, \$12000. The one furnishing \$3000, or $\frac{1}{4}$, is entitled to $\frac{1}{4}$ of \$1800; the second to $\frac{5}{12}$ of \$1800, etc.

15. After 10 days, there are rations for 1200 men for 30 days; which will last 1 man 30 days × 1200; and will last 1200 men + 300 men, (30 days × 1200) ÷ 1500 = *Ans.*

16. Train leaving B goes $1\frac{1}{2}$ times as fast as the other, so that meeting place will be $1\frac{1}{2}$ times as far from B as it is from A. If x represents distance from A, $1\frac{1}{2}x$ will represent distance from B, and $x + 1\frac{1}{2}x = 120$, or $x = 48$. Trains meet 48 mi. from A, or 72 mi. from B. The first train takes $\frac{48}{20}$ hr. to travel the distance, or 2 hr. 24 min.; second train requires the same time,

$\frac{72}{30}$ hr., or 2 hr. 24 min. Time of meeting $= 9$ A.M. $+ 2$ hr. 24 min. $=$ 24 min. past 11.

Or, let $x =$ time required to reach meeting point; then $20\,x$ $=$ distance travelled by one train, and $30\,x =$ distance travelled by the other, and $20\,x + 30\,x =$ whole distance $= 120$, or $x = 2\frac{2}{5}$. Time is $2\frac{2}{5}$ hr., etc.

17. $x + 3\,x + 6\,x + 10\,x = 900.$

18. Let $x =$ share of third; $x + 75 =$ share of second; and $x + 75 + 48 =$ share of first.

$$x + x + 75 + x + 75 + 48 = 540.$$

19. If 4 men need 105 hr., one man would need 420 hr., and 6 men would need 70 hr., or $(70 \div 10)$ da.

20. Let $x =$ number of dozen bought, $15\,x =$ cost in cents; $x - 1\frac{1}{4} =$ number of dozen sold, 16 times $(x - 1\frac{1}{4}) = 16\,x - 20$ $=$ selling price; $16\,x - 20 = 15\,x$, or $x = 20$. He bought 20 dozen, or 240 eggs.

21. The interest on \$250 for 8 mo. is the same as that on \$1 for 8 mo. $\times$ 250, and on \$400 for (8 mo. $\times$ 250) $\div$ 400, or 5 mo.

22. Provisions for 3000 men would last $1\frac{1}{3}$ times as long as for 4000 men, or 18 wk. $\times\ 1\frac{1}{3} =$ 24 wk. *Ans.*

Or, (18 wk. $\times$ 4000) $\div$ 3000.

24. Area of first plank in square feet, 20×1; of second, $24 \times x$. $24\,x = 20$.

$$x = \tfrac{20}{24}\ \textit{Ans.}\text{ in feet, or } (\tfrac{20}{24} \times 12)\text{ in.}$$

25. He can pay $\frac{2025}{3040}$ of his debts. Mr. Smith should receive \$576 $\times\ \frac{2025}{3040}$. Cancel.

922. **6.** See table, Arithmetic, Art. 795.

923. **10.** $\cdot23 \times 11 \times x = 2749.$

13. $48 \times 72 \times x = 2150.4 \times 40.$

$$x = \frac{2150.4 \times 40}{48 \times 72} = \textit{Ans.}\text{ in inches.}$$

22. 55 cts. $\times\ 6 \times 54 \times \frac{32}{36}$.

924. 1. The pupils should gradually become accustomed to business methods of obtaining results. In calculating the amount to be paid, a clerk writes the discount at once under the gross price. He takes $\frac{1}{20}$ by dividing by 2 and placing the first quotient figure one place to the left.

$583.40
29.17
$554.23

2. In the first example, a discount of 5% is made for prompt payment; the discount here allowed is a reduction from what is called the "list" price. Catalogues are issued by some merchants on which the prices named are not the ones regularly charged, but are much larger so as to mislead persons that do not know the rate of discount allowed. Information as to this rate is communicated to customers, and varies from time to time owing to fluctuations in the market, the "list" price seldom being changed. The list price is sometimes called the "gross" price, the "net" price being the one actually paid.

A bill for the Roman candles would be made out as follows:

16 gross Roman Candles,	$26.75,	$428. –
	Less 60%,	256.80
	Net,	$171.20

The product by .60 is written under the "gross" total, the first figure being written two places to the left.

The net cost can be directly obtained by multiplying $428 by .40.

7. Two, three, and even more discounts are very frequent in business. An article catalogued at $100 is sold, for instance, at $70, and customers informed that the discount is 30%. A later reduction in price is accompanied by a notice that a further discount of 10% will be allowed. This does not signify 30% + 10%, or 40%, from the "list" price; it means that the regular price of $70 is to be reduced $7, making the new price $63. A third discount of 5% means 5% of the last price, $63; etc., etc. In writing these discounts, the per cent mark is written only after the last.

11. On a bill of \$100, 40 and 10% gives a "net" amount of \$60 − \$6, or \$54; 30 and 20% gives \$70 − \$14, or \$56; the former being better for the buyer by \$2.

12. \$100 less $33\frac{1}{3}$ and 10% = \$60. The discount is \$40 on \$100, or 40%. The net is 60%.

13. $100 - 40 = 60$; 20% of $60 = 12$; discount = 40% + 12% = 52%. *Ans.*

14. Let $x =$ "list" price. After first discount of $\frac{1}{3}$ is deducted, there will remain $\frac{2x}{3}$. Deducting 10% of this, or $\frac{1}{10}$ of it, there will remain $\frac{9}{10}$ of it. $\frac{9}{10}$ of $\frac{2x}{3} = \frac{3x}{15} = 60$.

15. The first reduction is 100% − 20%, or 80% of the list; the second is 100% − 10%, or 90% of the former. 90% of 80% = $\frac{9}{10}$ of 80% = 72%. *Ans.*

16. 80% of 90% = 72%. *Ans.* The net price is the same for the same discounts in whatever order they are taken.

925. **3.** $\frac{x}{100}$ of 5000 (cents) = 5 (cents); $50x = 5$; etc.

11. Value at par, \$50 × 96 = \$4800. Discount = \$4800 − \$4476 = \$324 = x% of \$4800, *i.e.*, $324 = 48x$; etc.

15. \$500 × $\frac{8}{100}$ × $\frac{100}{365}$.

926. **3.** 50% + [10% of (100% − 50%)] = 50% + 5% = 55%. *Ans.*

4. 30% + [30% of (100% − 30%)] = 30% + (30% of 70%) = 30% + 21% = 51%. *Ans.*

5. $\frac{9}{10}$ of gross price (x) = 729; $\frac{1}{10}x = 81$; $x = 810$.

927. **21.** Cost per acre = \$40293 ÷ 396, at which price 112 acres were sold.

25. 40¢ × $\frac{30}{12}$ × $\frac{50}{12}$. Cancel.

26. Number of hours = $365 \div 4\frac{180}{320}$.

928. 2. $x \times \frac{5}{100} \times \frac{9}{4} = \frac{9x}{80} = 180$; $9x = 14400$; etc.

4. $4250 \times \frac{6}{100} \times x = 765.$ 6. $2020 \times \frac{6}{100} \times x = 606.$

5. $x \times \frac{4}{100} \times 3 = 240.$ 7. $6000 \times \frac{x}{100} \times \frac{5}{2} = 900.$

929. 1. The pupils should not be shown how to calculate these areas.

2. If any difficulty is experienced in finding the areas of these triangles, the pupils should be referred to **1**; after which they should be led to deduce the rule. Thus the area of the second triangle may be calculated from the figure in **1** by adding $\frac{1}{2}$ of (60×50) to $\frac{1}{2}$ of (60×50); that of the third by adding $\frac{1}{2}$ of (60×60) to $\frac{1}{2}$ of (60×40); and that of the fourth by adding $\frac{1}{2}$ of (60×70) to $\frac{1}{2}$ of (60×30). Each of these will be found equal to $\frac{1}{2}$ of (60×100).

3. The second rectangle is divided into three triangles, two of them right-angled. By deducting from the area of the rectangle the sum of the areas of the two right-angled triangles, they will obtain the area of the remaining triangle.

4. The area of each of these triangles can be ascertained by referring to the corresponding triangle of **3**. Let the scholars do this for themselves.

5. The areas of the oblique-angled triangles constituting the first and second quadrilaterals, can be calculated by the pupils that have benefited by the work in **4**. If they see that the area of each triangle of a parallelogram is equal to $\frac{1}{2}$ (base $\times$ altitude), the area of the latter is equal to base $\times$ altitude.

For definitions of quadrilaterals see Art. 1265.

6. The area of the first is equal to the area of the rectangle, (50×60), plus the area of the triangle, $\frac{1}{2}$ of (50×60); or 4500 sq. ft.

The second is made up of a rectangle and of two triangles; its area is also 4500 sq. ft.

The pupils should be led to see that if, in the fourth, the upper left triangle were cut off and placed below, and if the lower right triangle were cut off and placed above, as indicated by the dotted lines, the resultant figure would be a rectangle 60×75.

Cutting off both triangles in the third, and placing them above, will make a 60×75 rectangle.

The area of each trapezoid is equal to $[\frac{1}{2} \text{ of } (50 + 100)] \times 60$.

7. The area of each of these quadrilaterals equals $\frac{1}{2}$ of (30×100) $+ \frac{1}{2}$ of (40×100), or $[\frac{1}{2} \text{ of } (30 + 40)] \times 100$.

The first three quadrilaterals are trapeziums. The last is a trapezoid. Which are the parallel sides?

8. A strip of paper of any uniform width may be used. Carefully cut a rectangle by making square corners with a card. Using the base of the rectangle as a measure, place two dots on the lower edge of the strip to mark the extremities of the base of a parallelogram equal in length to the base of the rectangle, and above these, at any convenient distance to the right or to the left, two others to mark the extremities of the opposite side of the parallelogram. Draw lines forming the right and left sides, and cut along these lines. That the parallelogram is equal in area to the rectangle, may be shown by carefully drawing a perpendicular at one corner; cutting off the triangle thus made, and placing it, in a reversed position, on the opposite side of the parallelogram.

9. See 6, third and fourth trapezoid.

930. 2. Four faces will measure 6 ft. by $4\frac{1}{2}$ ft. each, and two will measure $4\frac{1}{2}$ ft. by $4\frac{1}{2}$ ft. each.

4. Dimensions of floor, 57 ft. by 18 ft., or 19 yd. by 6 yd.

5. Volume in cubic feet, $4 \times 4 \times 12$. Multiply by 1000 to get the weight in ounces of an equal volume of water. Multiply by 2.8 to get weight of marble in ounces. Divide by 16×2000 to reduce to tons.

$$\frac{4 \times 4 \times 12 \times 1000 \times 2.8}{16 \times 2000}$$

9. Outer dimensions, $14 \times 14 \times 14$, or 2744 cu. in. See if the same number of cubic inches of wood is obtained by calculating the volume of the wood in 6—2 pieces, 12×12, 1 in. thick; 2 pieces, 12×14, 1 in. thick; 2 pieces, 14×14, 1 in. thick.

10. A cube of water 2 ft. long contains $(2 \times 2 \times 2)$ cu. ft., or 8 cu. ft. At 1000 oz. to a cubic foot, it weighs $\frac{1000}{16}$ lb. $\times 8$, or 500 lb. The cube of iron weighs 8 times as much as an equal volume of water.

A cube of iron 1 ft. long also weighs 8 times as much as a corresponding cube of water, or 8 times $\frac{1000}{16}$ lb. $= 500$ lb., or $\frac{1}{4}$ ton.

A 3 ft. cube of iron contains 27 cu. ft., weighing 8 times as much as a corresponding cube of water, or 216 times 1000 oz. $= 6\frac{3}{4}$ tons.

XV

NOTES ON CHAPTER TWELVE

931. While problems requiring the pupil to find the principal, the rate, or the time have very little "practical" value, they can be so readily taught by the algebraic method that the time spent upon them need not be very great. A pupil that is able to calculate one of a series of related items is benefited by being required to calculate the others, if he is not compelled to resort to a series of ill-understood rules in order to obtain the results.

Although there is no real difference between the algebraic method and the arithmetical one, a great number of scholars fail to obtain a thorough understanding of the latter. They can work a number of examples, following a model solution at the head of the lesson; but they fail to grasp the underlying principles. By the algebraic method, x is used to represent the number of years or dollars, or the rate, instead of the 1 year, \$1, or 1%, of the other; but this method seems to require the formulation of a number of rules, as against practically none in the case of the other.

After pupils have learned to work examples by the algebraic method, they can be encouraged to discontinue the use of the x; but they should not be taught both methods at one time.

933. Represent the required item by x. Simplify the first member before proceeding to solve the equation.

1. $2000 \times \frac{x}{100} \times 3 = 300.$

2. $1800 \times \frac{4}{100} \times x = 144.$

3. $x \times \frac{9}{200} \times \frac{8}{12} = 2.88.$

4. $38 + \frac{38 \times x \times 2}{100} = 40.28.$

Or, $38 \times \frac{x}{100} \times 2 = 40.28 - 38 = 2.28.$

5. $140 \times \frac{7}{200} \times \frac{105}{360} = x = \mathit{Ans.}$

6. $x + \frac{x \times 4 \times 5}{100 \times 2} = 39.60.$

7. $460 + \frac{460 \times 7 \times x}{200} = 484.15.$

8. $39.60 + \frac{39.60 \times 4 \times 585}{100 \times 360} = x = \mathit{Ans.}$

15. Principal $= \$97.57 - \$7.57 = \$90.$

$$90 \times \frac{4}{100} \times x = 7.57.$$

21. Let $x =$ principal.

$$x + \frac{x \times 4 \times 846}{100 \times 360} = 196.92.$$

The interest is then found by subtracting from \$196.92, the value obtained for x.

22. First find the principal (x).

25. See **15.**

934. The recommendation so frequently made, that all written work be preceded by oral exercises of the same character, should not be followed without some modifications. Oral work is necessarily accompanied by a number of devices that tend to simplify the task of handling numbers that are not seen; written work should follow general rules in order to be learned by a majority of the pupils, although later they may adopt some of the short-cuts of their oral exercises. Even the oral addition of two numbers of two figures each, is done in a manner different from the ordinary slate method, the operation in the former case being commenced generally with the tens' figures, and in the

latter case with the units' figures. The reduction to a common denominator recommended in oral division of fractions, is seldom employed in slate work.

The average scholar is able to handle "mental" problems containing small numbers in a way that he cannot always explain, although he may endeavor to stultify himself by repeating a prescribed form of analysis. It is next to impossible, with the average teaching, to get the same pupil to work some varieties of "written" problems containing the same conditions.

In order to furnish a general method of treating some classes of examples, it has been thought best to commence with written work, leaving the mental exercises with their various devices until the former task is accomplished.

The accomparving exercises are so simple as not to need explanation by the teacher; but sufficient time should be given the pupil to work them out in his own way. They differ in this respect from the oral examples of a single operation containing larger figures, but which do not require any effort on the part of the scholar to determine which process is required.

1. Yearly interest is $6; a year and a half will be needed to make the interest $9.

2. The yearly interest is $8, making the rate 4%.

3. Yearly interest is $4, requiring a principal of $100, at the given rate.

5. The pupils may remember (Art. 878, No. 15) that 5% for a year is 1% for 72 da.

6. 4% per year is 1% for 90 da.

11. 2 mo. 12 da. = 72 da. See **5.**

12. 1% for 80 da. is (360 ÷ 80) % for a year.

17. 2% for 6 mo.

18. $3.60 per year is 1 cent per day.

20. See **18.**

935. First payment $=\frac{x}{3}$, leaving $\frac{2x}{3}$ remainder; second payment $=\frac{1}{2}$ of $\frac{2x}{3}=\frac{x}{3}$, leaving $\frac{x}{3}$ remainder; third payment $=\frac{3}{5}$ of $\frac{x}{3}=\frac{x}{5}$; last payment, $=\$2000$. The total cost of the house, $x=$ the sum of the payments, $\frac{x}{3}+\frac{x}{3}+\frac{x}{5}+2000$.

936. The books contain many methods of calculating interest, but it is questionable whether it is not time wasted in giving so much attention to this topic. The average person is required to do comparatively little work in this line; while those called upon to compute interest often, learn short methods of their own or use interest tables.

If a second method is to be taught at all, the one by aliquot parts is the most useful, as modifications of this method may be applied to other operations.

6. See Arithmetic, Art. 384.

937. A modification of the so-called "60-day method."

16. See Art. 901 as to days of grace.

938. **21.** 10% gives 2 years' interest; then 1 yr. ($\frac{1}{2}$ of the foregoing); 6 mo.; 1 mo.; 18 da. ($\frac{1}{10}$ of 6 mo.).

942. **46.** Term, 57 da. (54 da.).

47. Term, 92 da. (89 da.).

48. Term, 16 da. (13 da.).

49. Term, 34 da. (31 da.).

50. Term, 187 da. (184 da.).

943. **9.** See Table, Arithmetic, Art. 1303.

944. **11.** The net price of goods catalogued at x dollars, and sold at a discount of 20 and 10%, will be $\left(x-\frac{20x}{100}, \text{ or } \frac{80x}{100}\right)-\left(\frac{1}{10} \text{ of } \frac{80x}{100}\right)=\frac{80x}{100}-\frac{8x}{100}=\frac{72x}{100}$.

13. If the selling price of the above is \$360, $\frac{72x}{100} = 360$; $72x = 36000$; $x = 500$. Catalogue price $=$ \$500. *Ans.*

14. $750 - (\frac{1}{3} \text{ of } 750) = 500$;

$$500 - \left(\frac{x}{100} \text{ of } 500\right) = 500 - 5x = \text{net price.}$$

$$500 - 5x = 450.$$

Transposing, $\quad -5x = -50.$

Changing signs of both terms, $5x = 50$,

$$x = 10.$$

945. **7.** Let $x =$ selling price of muslin.

$$(84 \times 40) + 105x = (84 \times 55) + (105 \times 20).$$

Another way: He loses 15¢ per yard on 84 yd., which is a loss of $15¢ \times 84$. This he must make up on 105 yd., which is $(15¢ \times 84) \div 105$ on each yard, or 12¢. Selling price of muslin, $20¢ + 12¢$, or 32¢. *Ans.*

8. $\frac{1}{4}$ of them brought \$120; $\frac{1}{3}$ of remainder, or $\frac{1}{4}$ of them, brought \$96; $\frac{1}{2}$ of remainder, or $\frac{1}{4}$ of them, brought \$40; remainder, or $\frac{1}{4}$ of them, brought \$30. Total amount received, \$286.

9. Proceeds of gas stock, $\$25 \times 165 = \$4125$. Cost of lots, $\$4125 - \$27 = \$4098$. Number of square feet in lots, $(32 \times 115) + (30 \times 105) = 3680 + 3150 = 6830$. Value per square foot, $\$4098 \div 6830 = \$0.60$. *Ans.*

10. Two walls, each 16×14, and two others, each 12×14, contain $(32 + 24) \times 14$, or (56×14) sq. ft. $= 784$ sq. ft. The ceiling contains (16×12) sq. ft. $= 192$ sq. ft. Adding this to the walls, makes a total of 976 sq. ft.

The deductions are (8×4) sq. ft. $\times 2$, and (7×3) sq. ft. $\times 3$, or 64 sq. ft. $+$ 63 sq. ft. $= 127$ sq. ft. Number of square feet to be plastered $= 976 - 127 = 849$. Cost at $\frac{18}{9}$¢ per square foot $= 2¢ \times 849 = \$16.98$. *Ans.*

946. 1. A can do $\frac{1}{5}$ of the work in 1 hr., and B can do $\frac{1}{7}$ of it in 1 hr.; together they can do in 1 hr. $(\frac{1}{5}+\frac{1}{7})$ of the work, or $\frac{12}{35}$ of it; and to do the whole work it will take as many hours as $\frac{12}{35}$ is contained times in 1.

$$1 \div \tfrac{12}{35} = 1 \times \tfrac{35}{12} = \tfrac{35}{12} = 2\tfrac{11}{12}. \quad \textit{Ans. } 2\tfrac{11}{12} \text{ hours.}$$

2. Commission of $2\frac{1}{2}\% = \frac{1}{40}$ of amount collected $= \$1.60$. Amount collected $= \$1.60 \times 40 = \$64$. Amount remitted $= \$64 - \$1.60 = \$62.40$. *Ans.*

3. $\frac{1}{4}\%$ of $(\frac{3}{4}$ of $\$12000) = \frac{1}{4}\%$ of $\$9000 = \frac{1}{4}$ of $\$90 = \$22.50$. *Ans.*

NOTE. — It may be advisable to explain to the pupils that property is seldom insured for its full value, because it is not likely that a fire will completely destroy a building, and insurance companies reimburse the person insured, only to the extent of his loss.

4. $32 \times x = 6 \times 4$; $32x = 24$; $x = \frac{24}{32} = \frac{3}{4}$. *Ans.* $\frac{3}{4}$ yd. or 27 in.

5. 5% for 360 days $= 1\%$ for 72 days $= 2\%$ for 144 days. 2% of $\$87 =$ *Ans.*

6. 2% of $\$176$.

7. Let $x =$ commission; $40x =$ amount invested; $x + 40x = 41x = 8200$; $x = 200$. *Ans.* $\$200$.

8. $\$500$ is $\frac{1}{8}$ of cost, $\$4000$.

9. Let $x =$ loss, or 20% of cost; $5x =$ cost; $5x - x = 4x =$ selling price.

x, the loss, is $\frac{1}{4}$ of selling price, $4x$.

10. Let $x =$ gain, which is 20%, or $\frac{1}{5}$, of the cost of the goods; $5x =$ cost; $5x + x$, or $6x$, $=$ selling price.

x, the gain, is $\frac{1}{6}$ of selling price, $6x$.

NOTE. — The amount of money given in these two examples, $1200, does not affect either result. It may be used or not, as the pupil prefers.

11. 3 men earn $\$72 \div 8$ in one day, or $\$3$ per day each. 5 men earn $\$15$ a day, or $\$165$ in 11 days.

12. 3 quarters of the cost, or $\frac{3x}{4}$, $= 225$. Cost $= \$300$. By selling for \$325, there is a gain of \$25, or $\frac{1}{12}$ of the cost. $\frac{1}{12} = 8\frac{1}{3}\%$. *Ans.*

13. $2\frac{2}{3}$ yd., or $\frac{8}{3}$ yd. cost 40¢; 1 yd. costs 40¢ $\div \frac{8}{3}$, or $\frac{3}{8}$ of 40¢ $= 15$¢. 4 yd. 1 ft., or $4\frac{1}{3}$ yd., cost 15¢ $\times 4\frac{1}{3}$.

947. The following is the solution without days of grace:

Let $\qquad x =$ face of the note.

Then, $$x \times \frac{6}{100} \times \frac{1}{12} = \frac{x}{200} = \text{discount};$$

$$x - \frac{x}{200} = \text{proceeds} = 1000.$$

$$200x - x = 200000,$$

$$199x = 200000,$$

$$x = \tfrac{200000}{199} = 1005.03.$$

Face of note $= \$1005.03$. *Ans.*

Proof. Face of note,		\$1005.03 –
Deduct 30 days' discount,	$\frac{1}{2}\%$,	5.03 –
	Proceeds,	\$1000.00

949. **1.** When days of grace are omitted, the term of discount is 90 da.

10. Find the term, and add the number of days to March 15.

950. **2.** (*a*) 1 trillion, 500 billions, etc.

5. The first quarter of 1888 contained $(31 + 29 + 31)$ da., or 91 da. The man was employed 60 da., and unemployed 31 da. His \$3 additional paid the expenses of the working days. Deducting $\$2 \times 31$, or \$62, for the expenses of the other days, his net income $= \$350 - \$62 = \$288$. *Ans.*

6.	Apr. 16, '79 to Mch. 19, '86, $83\frac{1}{10}$ mo.,	@ $8,	$664.80
	Mch. 19, '86 to Mch. 4, '87, $11\frac{1}{2}$ "	" 12,	138.00
	Apr. 16, '79 to Sept. 1, '80, $16\frac{1}{2}$ "	" 2,	33.00
	Apr. 16, '79 to Nov. 22, '82, $43\frac{1}{5}$ "	" 2,	86.40
			Ans. $922.20

10. Last quarter's salary = $287 = 82% of previous quarter's salary $= \frac{82x}{100} = 287$; $x = 350$.

Salary of three quarters @ $350 = $1050; add last quarter's, $287. Total for year, $1337. *Ans.*

951. See Art. 784.

953. To find 19 times 91, subtract 91 from 20 times 91, or 1820 − 91.

$$82 \times 19 = (82 \times 20) - (82 \times 1).$$
$$51 \times 29 = (51 \times 30) - 51.$$
$$27 \times 99 = 2700 - 27.$$

954. See Art. 706. $675 \div 37\frac{1}{2} = 6\frac{3}{4} \div \frac{3}{8} = 54 \div 3$.

955. $136 \times \frac{7}{8} = 136 - (\frac{1}{8} \text{ of } 136) = 136 - 17$.

$$290 \times \frac{9}{10} = 290 - 29.$$
$$64\tfrac{1}{2} \div 5 = 12\tfrac{4\frac{1}{2}}{5} = 12\tfrac{9}{10}.$$
$$22 \times 19\tfrac{1}{2} = (22 \times 20) - (22 \times \tfrac{1}{2}).$$
$$45 \times 9\tfrac{14}{15} = (45 \times 10) - (\tfrac{1}{15} \text{ of } 45).$$
$$160 \div 1\tfrac{1}{2} = 320 \div 3.$$
$$18\tfrac{4}{5} \div 1\tfrac{4}{5} = 94 \div 9 = 10\tfrac{4}{9}.$$

956. 3. At 50¢ each, the cost would be $8; at 1¢ apiece less, the cost is $8.00 − $.16 = *Ans.*

4. ($\frac{3}{4}$ of 100 lb.) × 27 = 100 lb. × ($\frac{3}{4}$ of 27) = 100 lb. × $\frac{81}{4}$ = 100 lb. × $20\frac{1}{4}$.

NOTE. — The 100 should not be used until the end; even then, $20\frac{1}{4}$ is changed to 2025 without thinking of multiplication, $\frac{1}{4}$ being considered 25, and annexed to 20. See Art. 649.

6. $900 \div 75 = (900 \div 100) \div (75 \div 100) = 9 \div \frac{3}{4}$.

7. See Art. 955.

9. $(10\frac{1}{2} \times 4) + \frac{1}{2}$ of $10\frac{1}{2} = 42 + 5\frac{1}{4} = 47\frac{1}{4}$.

12. $\frac{1}{2}$ of $(33 \times 42) = 33 \times 21$.

15. $\$16\frac{1}{4} \div \$1\frac{1}{4} = 65 \div 5$.

20. $16\frac{1}{2} \times 2\frac{1}{4} = (16\frac{1}{2} \times 2) + (\frac{1}{4}$ of $16\frac{1}{2})$.

957. 1. $\frac{85}{100}$ of $(27¢ \times 56 \times 37\frac{1}{2} \times \frac{30}{36})$.

2. $[(\$4.875 \times 17350) \div 196] + [\$4.9375 \times 122.75] + [\$.0825 \times 2240 \times 2\frac{1}{4}]$.

3. $x - \frac{7}{400}x = 49739.55\frac{3}{8}$.

12. Duty $= [\frac{85}{100}$ of $(55¢ \times 45 \times 38)] + [20¢ \times (45 \times 38 \times \frac{42}{36})]$.

958. In multiplying by 427, the first figure of the product by 42 (7×6) is placed under the 2; in multiplying by 832, the first figure of the product by 8 is placed under the 8, and the first figure of the product by 32 (8×4) is placed under the 2.

959. These exercises contain some examples worked by short methods explained in previous chapters. See Arts. 650, 714, 791, 792, and 891.

960. See Arithmetic, Art. 384.

964. 2. It won 17 games out of 30.

3. 1600%.

4. $\frac{2}{3}$ is what per cent of $\frac{16}{5}$? $\frac{1}{3}$ is what per cent of $\frac{2}{5}$? $\frac{5}{15}$ is what per cent of $\frac{6}{15}$? 5 is what per cent of 6? $\frac{5}{6} = 83\frac{1}{3}\%$.

6. The deduction of the first discount leaves 80% of the list price; the deduction from this of 10% of itself leaves 90% of 80%, or 72%.

7. One fills $\frac{1}{6}$ of tank in 1 hr., the other fills $\frac{1}{8}$ in 1 hr.; both together fill $\frac{1}{6} + \frac{1}{8}$ in 1 hr., or $\frac{4}{24} + \frac{3}{24}$, or $\frac{7}{24}$; to fill $\frac{24}{24}$ of tank, it will take 24 hr. $\div 7 = 3\frac{3}{7}$ hr.

8. 6% for 60 da. $= 80¢$; for 12 da., $\frac{1}{5}$ of $80¢$ or $16¢$; for 72 da., $80¢ + 16¢ = 96¢$. Or, \$4.80 for year, and $\frac{1}{5}$ of \$4.80 for 72 da.

9. $16\% = \frac{4}{25}$; $420 = \frac{21x}{25}$; etc.

965. **5.** Selling price $= \frac{3}{4}$ of $\$1.50 = \$1.12\frac{1}{2}$; gain $= 22\frac{1}{2}¢$ $= \frac{1}{4}$ of $90¢$.

6. Selling price $= \$9.60$, a reduction of \$2.40 from marked price, or $\frac{1}{5}$ of \$12, or 20%.

7. The rug is sold for \$24. If this is $\frac{4}{5}$ of marked price, the latter is \$30.

8. See **7.**

NOTE. — It is not to be expected that all the pupils' work will be shortened to this extent, but the majority of the class should be able to give answers at sight to these four examples.

9. Find $\frac{1}{10}$ of £83 2*s.* 6*d.* by compound division; do not reduce to pence.

966. **4.** Let $x =$ profits first year; then $\frac{21x}{20} =$ profits second year; $x + \frac{21x}{20} = 6970$.

5. I wish to gain 15% of \$.96, or $\$.14\frac{2}{5}$, which makes my selling price $\$.96 + \$.14\frac{2}{5} = \$1.10\frac{2}{5}$.

Let $$x = \text{marked price.}$$

$$x - \frac{15x}{100} = 1.104; \text{ etc.}$$

Or, writing all the foregoing in one equation:

$$\frac{85x}{100} = \left(\frac{115}{100} \text{ of } .96\right),$$

$$85x = .96 \times 115,$$

$$x = \frac{.96 \times 115}{85}.$$

7. The average pupil will be able to obtain the meanings of these terms by inquiries of his parents, friends, etc.; and he will remember much longer what he learns in this way, than if he finds the answer in the text-book. As the penalties for taking usurious interest vary in the different states, the teacher should ascertain the law of her own state in this matter. See Art. 1306. A tax bill or a policy of insurance brought in by a pupil and described, will add to the interest. The teacher should not spend too much time upon details that have no relevancy in her section of the country. Poll taxes, for instance, should not be dwelt upon in cities in which they are not collected; etc.

9. The use of the hogshead as a measure of 63 gal. is fast becoming obsolete. The term "barrel," to indicate $31\frac{1}{2}$ gal., is occasionally used in giving the capacity of large tanks, etc. The U.S. authorities require prices to be stated in the currency of the country from which the articles are exported; but as this would make the problem more complicated, the text-books generally give prices in U.S. money. No allowance is now made for leakage, the quantity actually imported being ascertained by measuring.

10. A port of entry is a place in which there is a custom house, established by the government.

967. **4.** Any principal — \$150, \$575, or \$343.75 — will double itself at 5% in $(100 \div 5)$ yr.

12. The pupils will need to obtain a correct idea of the meaning of the word "premium" in this connection, as they will find it used differently when they come to the study of Bonds and Stocks. The premium is the amount paid to the company assuming the risk.

968. **11.** \$3500 is raised on property worth \$1750000; the rate is $\$3500 \div \$1750000 = 2$ mills on \$1. The man's property tax $= 2 \text{ mills} \times 24000 = \$48$; adding to this 1 poll tax, at \$2, gives his total tax of \$50. *Ans.*

969. **2.** Multiply the denominators of the first and the third; divide the numerators of the second and the fourth.

5. While pupils in lower grades may be permitted to reduce both amounts to pence, it is now time to use a shorter method. The sums given may be changed to $38\frac{3}{4}$*s.* and $5\frac{2}{4}$*s.*, or $\frac{155}{4}$*s.* and $\frac{22}{4}$*s.*

970. **1.** See diagram, Arithmetic, Art. 897, Problem 2.

$$18\,x + 15\,x + 18\,x + 15\,x + (18 \times 15) = 63 \times 9,$$
$$66\,x + 270 = 567,$$
$$66\,x = 567 - 270 = 297,$$
$$x = 4\tfrac{1}{2}. \qquad \textit{Ans. } 4\tfrac{1}{2} \text{ ft.}$$

The arithmetical solution is apparent from the foregoing. The sides and the bottom contain 63 sq. yd., or 567 sq. ft. The bottom contains (18×15) sq. ft., or 270 sq. ft. The sides contain 567 sq. ft. $-$ 270 sq. ft. $=$ 297 sq. ft., which is the area of four rectangles, whose bases measure 18 ft., 15 ft., 18 ft., and 15 ft., respectively, the total being 66 ft. $297 \div 66$ gives $4\frac{1}{2}$ as the number of feet in depth.

2. There are 4840 sq. yd. in an acre. $(4840 \times 3) \div (\frac{1}{2} \text{ of } 242)$. Cancel.

3. The area of a trapezoid is found by multiplying one-half the sum of the parallel sides by the perpendicular distance between them. See diagrams, Arithmetic, Art. 929, Problems 6 and 9; and Art. 1265, Figs. 9 and 10.

4. The area $= \left(\dfrac{x + 100}{2}\right) \times 60 = 30\,x + 3000.$ *Ans.*

$30\,x + 3000 = 5400$; $x = 80$. *Ans.* 80 yd.

5. $\left(\dfrac{80 + 120}{2}\right) \times x = 100\,x.$ *Ans.*

$100\,x = 4000$; $x = 40$. *Ans.* 40 yd.

6. $\left(\dfrac{x+x+40}{2}\right)\times 60=(x+20)\times 60=60\,x+1200.$ *Ans.*

$60\,x+1200=6000;\ x=80;\ x+40=120.$

Ans. 80 yd. and 120 yd.

9. The number of square feet in the walls of a room $16\frac{1}{2}$ ft. long, $14\frac{3}{4}$ ft. wide, and $13\frac{1}{3}$ ft. high, may be obtained by adding the bases of the four sides,—$16\frac{1}{2}+14\frac{3}{4}+16\frac{1}{2}+14\frac{3}{4}=62\frac{1}{2}$,—and multiplying this by their common height, $13\frac{1}{3}$. Dividing by 9 gives the number of square yards. The operation of finding the cost may be indicated as follows:

$$\frac{10\cent\times 125\times 40}{9\times 2\times 3}=\frac{25000}{27}\cent=\$9.25\tfrac{25}{27}.\quad \textit{Ans. } \$9.26.$$

10. $(20\times 17\frac{1}{2})\div 2\frac{1}{2}$ gives the number of feet of carpet. Dividing this result by 3 gives the number of yards.

11. $41\frac{1}{4}$ lb. $\times\,(15\frac{3}{4}\times\frac{10}{12}\times\frac{1}{6})$. Cancel.

12. The "development" will be a modification of the one given in problem 20, Arithmetic, Art. 818. In drawing the development, the pupil should be required to approximate the proper proportions, and to place the faces in the proper order. It is not necessary to have the top and bottom faces in the positions shown in Art. 818.

The surface of the four vertical faces should be obtained in one operation, as in **9**; also the surface of the two horizontal faces:

$$[2\times(3\tfrac{3}{4}+2\tfrac{1}{2})\times 1\tfrac{1}{3}]+[2\times(3\tfrac{3}{4}\times 2\tfrac{1}{2})].$$

13. $(135\frac{1}{9}\div 12\frac{2}{3})$ ft.

14. $[(128\times 152\times 105)\div 2150.4]$ bu.

15. $[(77\times 45\times 54)\div 231]$ gal.

16. 40 acres $=(160\times 40)$ sq. rd. $=6400$ sq. rd. The dimensions are 80 rd. by 80 rd., making 320 rd. of fence necessary. There will be 640 posts, at 15¢ each; and 5 times 640 rails, or 3200 rails, at 10¢ each.

17. There will be 16 fields, 4 rows of 4 fields each. Five parallel fences, each a mile long, and five other parallel fences of the same length, and perpendicular to the first, will be required.

19. 1728 cu. in. of water weigh 1000 oz.; 1 cu. in. weighs $(1000 \div 16)$ lb. $\div 1728$; 231 cu. in. weigh $[(1000 \times 231) \div (16 \times 1728)]$ lb.

21. Number of square feet $= (320 + 210 + 320 + 210) \times 6$. Divide by 9 to obtain square yards.

22. Area of outer rectangle in square feet $= 332 \times 222$; of inner rectangle $= (320 \times 210)$ sq. ft. Divide the difference by 9 to obtain square yards.

23. Area of outside plot $= (320 \times 210)$ sq. ft.; of inner plot $= (308 \times 198)$ sq. ft.

972. **1.** Each yard measured with the short yardstick contains $\frac{35}{36}$ yd.; the true length $= 25$ yd. $\times \frac{35}{36}$.

Or, each so-called yard is $\frac{1}{36}$ yd. short, and 25 yd. are $\frac{25}{36}$ yd. short; and the piece contains 25 yd. $- \frac{25}{36}$ yd. $= 24\frac{11}{36}$ yd. *Ans.*

4. 32 boys = 20 men. If 15 men do the work in 12 da., 20 men do it in 12 da. $\times \frac{15}{20}$.

20. Change 6 lb. 14 oz. and 23 lb. 12 oz. to ounces, or to pounds and fractions.

976. **3.** A and B can mow $\frac{1}{7}$ of the field in 1 da., all three can mow $\frac{1}{5}$ of the field in 1 da. C mows $(\frac{1}{5} - \frac{1}{7})$ of the field in 1 da., or $\frac{2}{35}$ of it; in 5 da. he does $\frac{2}{35} \times 5$, or $\frac{2}{7}$ of the work, for which he should receive $\frac{2}{7}$ of \$25.

4.

$$\begin{aligned}
5 \text{ bu. @ } 80¢ &= 400¢. \\
5 \text{ bu. @ } 60¢ &= 300¢. \\
x \text{ bu. @ } 30¢ &= 30x¢. \\
(x+10) \text{ bu. @ } 50¢ &= (30x + 700)¢, \\
50x + 500 &= 30x + 700, \\
20x &= 200, \\
x &= 10. \qquad \textit{Ans. } 10 \text{ bu. of oats.}
\end{aligned}$$

7. Total cost $= (65¢ \times 128) + 80¢$; quantity sold $= 128$ gal. $- 16$ gal. Selling price per gallon $= \frac{6}{5}$ of $[(65¢ \times 128) + 80¢] \div (128 - 16)$.

It is advisable to accustom children to understand that a gain of $\frac{1}{5}$ of cost makes the selling price $\frac{6}{5}$ of cost.

10. On sofas sold for \$1125 there was a loss of $\frac{1}{5}$, making the selling price $\frac{4}{5}$ of cost. On the remaining sofas, the selling price, \$1125, represents $\frac{6}{5}$ of cost.

4 fifths of cost = \$1125, selling price of 25 sofas.
1 fifth of cost = \$281$\frac{1}{4}$, loss on first lot.
6 fifths of cost = \$1125, selling price of remaining sofas.
1 fifth of cost = \$187$\frac{1}{2}$ = gain on second lot.

Loss on the transaction, \$281.25 − \$187.50.

The following is an algebraic solution without fractions:

Let $x =$ loss on first lot;

then $5x =$ cost of first lot.

$$5x - x = 4x = \text{selling price} = 1125.$$

$$x = 281\tfrac{1}{4} = \text{loss in dollars.}$$

Let $x =$ gain on second lot;

$$5x = \text{cost of second lot.}$$

$$5x + x = 6x = \text{selling price} = 1125.$$

$$x = 187\tfrac{1}{2} = \text{gain in dollars.}$$

etc.

11. Let $x =$ cost per egg.

$$18x = \text{cost of 18 eggs.}$$

$$\frac{18x}{11} = \text{selling price per egg.}$$

Gain per egg $= \frac{18x}{11} - x = \frac{7x}{11}$; that is, on x cents I gain $\frac{7}{11}$ of x cents. A gain of $\frac{7}{11}$ of cost $= .63\frac{7}{11}$ of cost $= 63\frac{7}{11}\%$. *Ans.*

12. Let $x =$ marked price.

$$x - \frac{15\,x}{100} = 2.$$

14. $x = \frac{x}{3} + \frac{x}{4} + 5\frac{5}{6}.$

18. Let $x =$ cost of the horse.

$$x + \frac{x}{20} = \text{asking price} = \frac{21\,x}{20}.$$

He sold it at $\frac{19}{20}$ of asking price; *i.e.*, $\frac{19}{20}$ of $\frac{21\,x}{20}$, or $\frac{399\,x}{400}$.

$$\frac{399\,x}{400} = 275.$$

20. Length of field in rods $= (1600 + 146) \div 18.$

21. Let $10\,x$ represent cost, then loss $= x$; and selling price $= 9\,x = \$117$; $x = 13$; cost $= \$130$. A gain of 10% would be \$13, making the price at which he should have sold it to gain 10% $= \$130 + \$13 = \$143$. *Ans.*

22. Wife receives $\frac{2\,x}{3}$; son, $\frac{2}{3}$ of $\frac{x}{3}$, or $\frac{2\,x}{9}$; daughter, \$5000.

$$\frac{2\,x}{3} + \frac{2\,x}{9} + 5000 = x.$$

977. The ten problems of this section will call for no special treatment. An occasional problem of this kind has already been given, although, perhaps, with smaller numbers. After the pupils understand in **1**, that the joint capital, \$700, is the basis upon which the profits are distributed, they will have no difficulty in understanding that B is entitled to $\frac{300}{700}$ of \$182, and that C is entitled to $\frac{400}{700}$ of \$182. Cancellation should be employed. See Art. 1121, No. 5.

There is no need in this connection of discussing the subject of business partnerships that are continued for a year or longer. The division of profits in these cases is the subject of a special

agreement, and is rarely made solely on the basis of the amounts invested. A yearly gain of \$4000 made in regular business by two partners, one of whom invested \$1000, and the other \$3000, might be divided in various equitable ways. The partner investing the larger sum, might first take out \$120 as interest on his excess of capital; and the remaining \$3880 might be equally divided, giving one of them \$1940, and the other \$2060. Another arrangement might permit each partner to withdraw a fixed sum for services, say \$1000, leaving \$2000 to be divided on the basis of 1 to 3. This would make the shares (\$1000 + \$500) and (\$1000 + \$1500), or \$1500 and \$2500.

10. Cases of this kind are found only in the books.

979. The cost of the first item is given. The second item contains 451 sq. ft.; the third contains $(4 \times 42 \times 7\frac{1}{6})$ sq. ft., or 1204 sq. ft.; the fourth contains $(3 \times 43 \times 7\frac{2}{3})$ sq. ft., or 989 sq. ft.; the total of the three items being 2644 sq. ft. The cost @ 10*d.* is

	£110 – 3 – 4
First item,	24 – 7 – 8
	£134 – 11
Less $2\frac{1}{2}\%$ $(\frac{1}{40})$,	3 – 7 – $3\frac{1}{4}$ +
Ans.	£131 – 3 – $8\frac{3}{4}$

The fraction of a penny in the discount is $\frac{3}{10}$, which is nearly 1 farthing, written $\frac{1}{4}d.$

980. German currency being a decimal one, the bill is computed in the ordinary way. The duty is found by reducing the marks to dollars by multiplying by \$.238, and taking 35% of the result.

981. Too much stress cannot be laid upon the importance of requiring children to estimate the probable answer to every "written" problem before placing a figure on paper. The mere drill on the "approximations" found in the text-book is of com-

paratively little value, unless it leads pupils to the employment of this device throughout all of their work. Besides preventing a pupil from making a very serious mistake in an ordinary computation, the habit of careful reading that is necessarily formed by the scholar that is not satisfied with a simple guess, will tend to make his methods simpler and more accurate. He will learn to apply to the apparently more difficult numbers of the written problem the processes employed in solving the comparatively simple "mental" questions.

While all of the pupils should not be expected to give the same answer to each of these examples, they should gradually approach more and more closely to the correct result.

1. 480 is what per cent of 960?
2. 52 bu. is about 3 times (17 bu. 37 lb.).
3. 500 cu. ft. ÷ 128 cu. ft. Less than 4 cords.
4. 120 cu. ft. ÷ about $1\frac{1}{4}$ cu. ft.
5. 1500 sq. rd. ÷ 160 sq. rd. Less than 10 acres.
6. A little less than 70×70.
7. $(6 \times 4 \times 5) \times$ about $7\frac{1}{2}$.
8. $64 \div \frac{1}{10}$, or $64.3 \div \frac{1}{10}$. More than 643.
9. About £200 @ $4.80 to £.
10. About 4 marks to $1. Over 400 marks.

982. The teacher that is allowed any discretion should omit all problems relating to Bonds and Stocks. The average grammar-school pupil cannot be made to understand the subject without the expenditure of more time and energy than should be given to a topic that he will learn by himself when he grows older if he gets the proper foundation.

If, however, the course of study requires that this topic be taken up, the teacher should aim to interest the pupils by making some local corporation the basis of the work. A certificate of stock should be obtained, or at least a copy made, which might be placed upon the blackboard.

The scholars should be led to see that large undertakings, such as the construction of a railroad, the building of water-works, and the like, require more money than any individual might have, or would care to risk. It then becomes necessary to interest a number of persons that will be willing to invest more or less money in the new enterprise. It is found, for instance, that $50000 will be needed to build and equip the street railroad mentioned in **1.** The projectors divide this amount into 500 shares, each of which represents a one-five-hundredth interest in the profits. It may happen sometimes that it is considered advisable to interest people of small means, and who are unwilling to take a $100 share. In these cases the original (par) value of the shares may be fixed as low as $10 each. When the par value is not given, it is understood to be $100.

In the distribution of profits, the owner of 10 shares is entitled to $\frac{10}{500}$ of $2000, or $40.

2. These profits are generally distributed annually, semi-annually, or quarterly. Before the time comes for "declaring" the dividend, the directors of the company meet and determine how much money shall be thus distributed. It may be considered advantageous to reserve a portion of the profits for the purchase of new cars, or for the extension of the road, etc.; so that the amount distributed at any time does not necessarily include all that has been gained. The dividend is generally announced as a per cent of the capital, which in this case is $50000; so that the semi-annual dividend is 4%, equal to 8% per year.

3. The owner of a $100 share, on which he receives $8 per year, is not likely to be willing to sell it for $100 if he can obtain only $4 per year interest on that sum of money deposited in a savings bank. The person desirous of obtaining stock after it is reasonably certain that the railroad is going to prove successful, will have to pay more than $100 per share. Mr. H. pays $150 per share, or 150% of the par value.

4. Mr. H. receives 4% dividend on \$3000, or \$120. From the savings bank he would obtain 2% on \$4500, or \$90.

5. The \$120 semi-annual dividend is $2\frac{2}{3}\%$ on the \$4500 invested, equivalent to $5\frac{1}{3}\%$ per year.

6. The words "per cent" are not used in stating the price of stock.

A \$100 share at $164\frac{1}{2}\%$ is worth $\$164\frac{1}{2}$; 30 shares are worth $\$164\frac{1}{2} \times 30$.

7. Assuming the par value of each to be \$100, the first pays \$6 per year on \$150, or 4%; the second pays \$7 per year on \$175, or 4%.

The par value of \$100 is assumed for convenience; a par value of \$50 would make the cost of a share of gas stock 150% of \$50, or \$75, and its annual dividend would be 6% of \$50, or \$3, the rate on the amount invested being 4%.

8. The buyer of stock at 125 wishes to receive 4% of \$125, or \$5. As the dividend is based on the par value (\$100), the rate must be 5% per year, or $2\frac{1}{2}\%$ semi-annually.

9. $[93\frac{7}{8}\% \text{ of } (\$50 \times 17)] + [102\frac{3}{4}\% \text{ of } (\$10 \times 143)]$.

10. A person that desires to buy stocks is not always likely to know where he can find any for sale; so he goes to a stock-broker, who makes a business of buying and selling stocks on commission. This commission is a small per cent of the par value, the charge of $\frac{1}{8}\%$ for buying or selling a share of the par value of \$100 being $12\frac{1}{2}$¢, whether the actual value of the stock be \$150 or \$50.

This broker receives, therefore, $\frac{1}{8}\%$ of

$$[(\$50 \times 17) + (\$10 \times 143)].$$

11. A bond is a note issued by a corporation, and is generally secured by a mortgage on its property. It is a much larger document than the note of an individual, and frequently contains at the bottom a number of "coupons," one for each half-year's interest, upon which is engraved the date when due and the sum

payable. A 10 years' U. S. 4% coupon bond for \$1000 would have 20 coupons, each worth \$20 when due. At the expiration of each 6 months, the holder of the bond cuts off the proper coupon and presents it for payment. A "registered" bond contains no coupons, a check for the interest being mailed to the owner.

Although the railroad company in this example receives only \$95 for each \$100 bond, it promises to pay \$4 interest per year, and \$100 at the end of 20 years.

In considering the rate of interest received by the owner of such a bond, it is not customary to complicate the example too much by requiring the pupils to take into account the additional \$5 above the cost received when the bond is redeemed at the end of 20 years, although buyers of bonds include it in their calculations. For our purpose, at present, it will be sufficient to assume that the purchaser of one of these \$100 bonds receives \$4 on each \$95 invested, the rate being $400 \div 95$, or $4\frac{4}{19}\%$.

12. Omitting the question of redemption, at which time the purchaser for \$116.50 would receive only \$100, the rate is $400 \div 116\frac{1}{2}$, or $3\frac{101}{233}\%$.

The holder of a U. S. bond knows that the face value of the bond will be paid in full at maturity, and that the interest payments will be made on the dates when due; in the case of the bond of a railroad, or the like, there is always the possibility that something may occur to prevent the company from meeting its obligations.

13. The rate of income from stocks may vary at each dividend period, depending upon the amount of business done, etc.; the rate of income from bonds is fixed as stated on their face.

Bonds are redeemed at the time specified; there is no reason why a successful company should sell out and divide the proceeds among its stockholders. When, however, the property of a corporation is sold, the claims of bondholders and all other obligations must be satisfied before the stockholders receive anything.

14. The stock-broker's fee is called brokerage, or commission.

15. A cotton-mill obtains material through a cotton "factor"; property is purchased through a real estate agent; a grocer may buy butter, eggs, etc., from a commission merchant, the seller in each case remitting the amount received to the owner after deducting his fee, or commission.

16. The *base* in insurance is the sum for which the property is insured; in taxes, it is the assessed value of property; in brokerage, it is the par value of stocks or bonds; in commission, it is the sum for which goods are bought or sold; the principal is the base upon which interest is calculated; the face of the note is the base in bank discount; in commercial discount, the gross price is the base in the first instance, the base for each subsequent discount being the successive remainders left after the deduction of the previous discounts; in stocks and bonds, the base is the par value.

17. The assessed value of property is the value for purposes of taxation, and is fixed annually by officers chosen for this duty, generally called assessors.

18. $2\frac{1}{4}\%$ of assessed value $(x) = \$540$; assessed value $= \$24000$. This is $\frac{2}{3}$ of the actual value, or $66\frac{2}{3}\%$.

For various reasons, the assessed value is placed below the sum that would be realized by the sale of the property under favorable conditions; but care is taken that all property is assessed upon the same basis.

19. If all property were assessed at its actual value, the same amount of taxes would be produced by a lower rate. To obtain \$540 taxes on property assessed at \$36000, the rate would be $1\frac{1}{2}\%$.

21. 1674 ft. $=$ 558 yd.; 558 yd. $\div\ 5\frac{1}{2}$ yd. $=$ 1116 half-yd. $\div$ 11 half-yd., which gives 101 rd. and 5 half-yd. $=$ 101 rd. $2\frac{1}{2}$ yd. $=$ 101 rd. 2 yd. 1 ft. 6 in.

22. $\$8575 \div \$245 = 35 =$ number of shares. Quarterly dividend $= 2\frac{1}{2}\%$ of $(\$100 \times 35)$.

983. 27. Each figure of the product is written two places to the right of the corresponding figure of the multiplicand.

Principal,	\$375.	
3%	11.25	
	\$386.25	Amount $\frac{1}{2}$ yr.
3%	11.5875	
	\$397.8375	Amount 1 yr.
3%	11.9351	
	\$409.7726	Amount $1\frac{1}{2}$ yr.
3%	12.2931	
	\$422.0657	Amount 2 yr.
1%	4.2206	
$\frac{1}{2}$%	2.1103	
	\$428.3966	Amount 2 yr. 3 mo.
Principal,	375.	
	\$53.3966	Interest 2 yr. 3 n

Ans. \$428.40, amount; and \$53.40, interest.

It is not necessary throughout the work to carry the multiplication beyond four places of decimals.

985. 6. After deducting the first 25%, or $\frac{1}{4}$, $\frac{3}{4}$ of list price remains; a second discount of $\frac{1}{4}$ of this remainder leaves $\frac{3}{4}$ of this remainder, or $\frac{3}{4}$ of $\frac{3}{4}$ of list price, or $\frac{9}{16}$ of list price $= 90$¢. List price $= 90$¢ $\div \frac{9}{16} = 90$¢ $\times \frac{16}{9} = \$1.60$.

7. All together can do $(\frac{1}{2} + \frac{1}{3} + \frac{1}{6})$ of the work in 1 hr.

8. Selling price, $\$60 = \frac{3}{4}$ of value of cow; $\frac{1}{4}$ of value $= \$60 \div 3 = \$20$, the loss.

9. Selling price, $\$60 = \frac{5}{4}$ of value of cow; $\frac{1}{4}$ of value $= \$60 \div 5 = \$12$, the gain.

986. 6. The wrong weights are $\frac{15\frac{3}{4}}{16}$, or $\frac{63}{64}$ of correct weights, so that the customer receives for \$352, $\frac{63}{64}$ of this amount, the

gain to the grocer being $\frac{3}{64}$ of $352, or $16.50. By selling $16\frac{1}{2}$ oz. to the pound, the grocer gives $\frac{32}{33}$ of the proper amount, his loss being $\frac{1}{32}$ of $320, or $10. The net gain is $16.50 − $10 = $6.50. *Ans.*

7. Cost of alcohol, $2.50 × 42 = $105; 3 yr. interest on $105 = $18.90. Amount to be realized, $105 + $18.90 = $123.90. Number of gallons to be sold, 42 − 7 = 35. Selling price per gallon, $123.90 ÷ 35 = $3.54.

8. £4500 = $4.85 × 4500 = $21825. Income on consols at 3% = $654.75. Selling price of consols, $21825 × .96; value of U. S. bonds, $21825 × .96 ÷ 108. Canceling, we obtain $19400; 6% of which gives the income on bonds = $1164. Difference = $1164 − $654.75 = $509.25. *Ans.*

9. Number of cubic feet $= 9\frac{3}{8} \times 9\frac{1}{8} \times 6\frac{3}{4} = 609$; weighing 609000 oz.; etc.

988. **11.** Agent collected 80% of $4500 = $3600; on this, his commission at $7\frac{1}{2}$% is $270; making the amount to be given me = $3600 − $270.

13. $x \times \frac{5}{100} \times \frac{3}{2}$, or $\frac{3x}{40} = 15.12.$

Find the discount for 17 da., the time from June 20 to July 7.

15. Ignoring the price — if he could buy 80 lb. with $81\frac{1}{2}$% of his money, he could buy with 100%, (80 lb. ÷ $81\frac{1}{2}$) × 100.

17. Let x = cost of each cow; $6x$ = cost of 6 cows; commission $= \frac{3}{100}$ of $6x = \frac{18x}{100}$; cost and commission $= 6x + \frac{18x}{100} = 525.30.$

18. The note is discounted 35 da. after it is made, so that it has (93 − 35) da. to run, or 58 da. [Without grace, 55 da.] The interest for a year is $36, which is 10 cents a day, or $5.80 for 58 days; etc.

19. If the selling price (regardless of its amount) is six-fifths of the cost, the gain is $\frac{1}{5}$ of cost, or 20%.

989. 6. The walls contain $[(20 + 15 + 20 + 15) \times 10]$ sq. ft.; the ceiling contains (20×15) sq. ft. Dividing by $\frac{19}{12}$, the width in feet of the paper, gives the number of feet of paper required, which is then reduced to yards.

991. 7. Dividend is $3\frac{1}{2}\%$ of $(\$9562.50 \div 1.27\frac{1}{2})$.

993. Mr. Smith wishes to pay Mr. Thompson the exact amount of his bill. A check on a Memphis bank for \$3475.86 would not be sufficient, as Mr. Thompson would have to pay a New York bank for collecting the check in Memphis. As the charge may not always be the same, Mr. Smith cannot know how much to add to the amount of his bill to cover this expense. From a banker that has an account in a New York bank, he can obtain a draft, payable in that city, for the exact amount, by giving the Memphis banker \$3475.86 + \$5.21, or \$3481.07.

Exchange is at a *premium* when the cost of a sight draft is greater than its face; it is at a *discount* when the cost of a sight draft is less than its face.

BANK CHECK.

Quogue, N.Y., Jan. 3, 1896. *No. 1492.*

SHINNECOCK NATIONAL BANK.

Pay to the order of *Lewis K. Thurlow,* *\$68$\frac{35}{100}$*

~~~~~~~~*Sixty-eight*~~~~~~~~ $\frac{35}{100}$ *Dollars*

HOWELL & PENNIMAN.

An examination of the above check will show wherein it differs in form from the draft. A draft may be made payable at a future time, whereas a check is always payable on presentation.
~~~~~~~~

8. $\frac{3}{40}\% = \frac{3}{4000}$; $x - \frac{3x}{4000} = 632.18$. **9.** $x + \frac{3x}{200} = 1000$.

10. $339.66 − (2% of $339.66) = sum remaining for the purchase of dry-goods, etc., and the commission. Dividing this by 1.02 gives the cost of the goods.

Another method of solving the foregoing is to indicate the money remaining as $339.66 × .98. Using 1.02 as a divisor, and canceling, gives the result.

$$\frac{339.66 \times 98}{102}.$$

994. **2.** Noon Monday to 6 P.M. Thursday = 78 hr. The loss in time = 35 sec. × 78 = 45 min. 30 sec. The time shown is 6 hr. − 45 min. 30 sec. = 5 hr. 14 min. 30 sec., or $14\frac{1}{2}$ min. past 5.

4. The number of rows, 2 ft. apart in a space 36 ft. − 4 ft., is $(32 \div 2) + 1 = 17$. The number of plants, 16 in. apart in a row 60 ft. − $2\frac{2}{3}$ ft., or $57\frac{1}{3}$ ft., in length, is $(57\frac{1}{3} \div 1\frac{1}{3}) + 1 = 44$. Total number of plants = 44 × 17 = 748.

If the rows run crosswise there will be 29 of them, each containing 26 plants.

6. Number of revolutions = 14 mi. ÷ 13 ft. 4 in.

995. **8.** Length of a degree on the equator = 25000 mi. ÷ 360. 20° will measure (25000 ÷ 360) × 20. Cancel.

9. The circumference = 18 ft. × 3.1416. Divide by 360. Cancel.

10. The difference is 20°, and the distance will be about one-half that found in **8**.

The teacher should remember that the shortest distance between these two places is not measured on the parallel of 60°. The shortest distance between two points on a sphere is measured by the arc of a *great* circle joining the points, and the 20° are $\frac{1}{18}$ of a *small* circle.

11. $46° 22' 30'' = 46\frac{3}{8}°$. The number of miles = $69\frac{1}{4} \times 46\frac{3}{8}$.

12. The approximate length of the 45th parallel is 25000 mi. × .7071; the length of a degree on this parallel = (25000 mi.

$\times .7071) \div 360$; multiplying by $22\frac{1}{2}$ gives the required distance. Cancel.

996. Time drafts are so little used that it is scarcely worth while to spend much time on their study.

A sight draft being payable on presentation (except in those states allowing days of grace), there is no need of formal acceptance. Acceptance is necessary in the case of time drafts, as they are not payable until the specified time after this acceptance.

The acceptance of a draft makes the person or corporation accepting it liable to its owner for the amount, a draft being transferable by endorsement just as a check or a note.

997. In calculating the cost of a sight draft, days of grace — even when allowed — do not enter into the result, this being included in the rate charged. Time drafts are allowed days of grace, except in the states given in the Appendix, Art. 1305. The number of states in which days of grace are no longer allowed, increases yearly, there being no good reason for promising to pay in 60 da. when the intention of the signer is to take 63 da.

1000. 1. Although days of grace are not allowed in California, the pupils of other states should not be expected to know this. In states that grant days of grace, they should be allowed in every note or time draft, no matter where payable; while in the other states, pupils should be taught not to employ them in any case.

The premium on the draft $= \$1.75 \times .840 = \$1.47$. The interest (with days of grace) $= \$840 \times \frac{6}{100} \times \frac{93}{360} = \$13.02$. The cost of the draft $= \$840 + \$1.47 - \$13.02 = \$828.45$. *Ans.*

Or, without days of grace:

$\$840 \times \frac{6}{100} \times \frac{90}{360} = \$12.60$, the cost being $\$840 + \$1.47 - \$12.60 = \$828.87$. *Ans.*

Some teachers prefer to find the cost of a draft for $1 at the given premium — in this case, $1.00175; from which is deducted the interest on $1 for 93 da., or $.0155; making the cost of a

90-day draft for $1, $1.00175 − $.0155 = $.98625. Multiplying this by 840 gives $825.45, the cost of a draft for $840. This method is not so much shorter as to make it advisable to use it.

2. The discount $= \frac{1}{8}\%$ of $400 $= \frac{1}{800}$ of $400 = 50¢. The interest for 33 da. = $400 $\times \frac{6}{100} \times \frac{33}{360}$ = $2.20. Cost = $400 − $.50 − $2.20 = $397.30. *Ans.*

NOTE.—The word "interest" is used instead of "bank discount," to avoid the confusion arising from the use of "discount" with two meanings in the same example.

5. The six remaining examples should be omitted by pupils that do not use algebraic methods of solution. Scholars that have readily worked the first four examples will find no great difficulty in solving **5–10** by means of the equation.

The premium is $\$\frac{1}{4}$ per $1000, or $\frac{1}{4000}$ of the face of the draft, x. Premium $= \frac{x}{4000}$. The interest on x dollars for 63 da. at 6% $= x \times \frac{6}{100} \times \frac{63}{360} = \frac{21x}{2000}$. Adding the premium to the face of the draft, and deducting the interest, gives $x + \frac{x}{4000} - \frac{21x}{2000}$ as the cost of the draft. This may be changed to

$$\frac{4000x + x - 42x}{4000} = \frac{3959x}{4000}. \quad \textit{Ans.}$$

6. $\frac{1}{8}\%$ of $1200 = $1.50; interest for $(x+3)$ da. $= 1200 \times \frac{6}{100} \times \frac{x+3}{360} = \frac{x+3}{5}$; cost of the draft (with days of grace), $1200 - 1\frac{1}{2} - \frac{x+3}{5} = \frac{12000}{10} - \frac{15}{10} - \frac{2x}{10} - \frac{6}{10} = \frac{11979 - 2x}{10}$. *Ans.*

NOTE.—Unless the pupil has studied algebraic subtraction in Chap. XV., he may make a mistake in the deduction $\frac{x+3}{5}$, by failing to change the sign of the second term. By writing $\frac{x+3}{5}$, $\frac{x}{5} + \frac{3}{5}$, he may see more clearly that the cost of the draft is $1200 - 1\frac{1}{2} - \frac{x}{5} - \frac{3}{5}$, etc. Without days of grace, cost $= 1200 - 1\frac{1}{2} - \frac{x}{5}$; etc.

1001. 1. $15^\circ \times 3\frac{1}{3} = 50^\circ$. *Ans.*

2. $(61 \div 15)$ hr. $= 4\frac{1}{15}$ hr. $= 4$ hr. 4 min. *Ans.*

3. Difference in time $= \frac{75}{15}$ hr. $= 5$ hr. London time $= 1$ P.M. $+ 5$ hr. $= 6$ P.M. *Ans.*

4. 2 P.M. $- 5$ hr. $= 9$ A.M. *Ans.*

5. Vienna is $\frac{25}{15}$ hr. later $= 1\frac{2}{3}$ hr. $= 1$ hr. 40 min. Time at Vienna 40 min. after 1 P.M., or 20 min. to 2 P.M. *Ans.*

6. 3 hr. 40 min. $= 3\frac{2}{3}$ hr. Difference in longitude $= 15^\circ \times 3\frac{2}{3} = 55^\circ$. *Ans.*

7. Difference in longitude $= 75^\circ + 30^\circ = 105^\circ$. *Ans.*

8. Philadelphia time is $\frac{105}{15}$ hr. earlier, or 7 hr. 3 P.M. $-$ 7 hr. $= 8$ A.M. *Ans.*

9. Correct Washington time is $\frac{2}{15}$ hr., or 8 min. earlier than standard time.

10. A town in 84° west longitude is 6° east of 90°, so that its correct time is $\frac{6}{15}$ hr., or 24 min., later. Time, 12:24 P.M. *Ans.*

1002. 1. Longitude difference $= 15^\circ \times 3\frac{44}{60}$. The pupil should see that $15 \times \frac{44}{60} = 44 \div 4 = 11$; so that $15^\circ \times 3\frac{44}{60} = 45^\circ + 11^\circ = 56^\circ$. *Ans.*

2. 15)37 hr. 18 min.
 2 hr. 29 min. 12 sec.

At 1 hr. to a degree, the difference in time would be 37 hr. 18 min.; as it requires 15° to make an hour's difference, dividing 37 hr. 18 min. by 15 gives the result.

Shorter methods should be deferred for the present. Using multiplication to obtain the difference in degrees, and division to obtain the difference in time, is more easily understood by beginners.

3. Time difference $= \frac{1}{15}$ of 87 hr. 35 min. earlier at Chicago, because it is west of Greenwich. Standard Chicago time is the time of 90°, or 90 hr. $\div 15 = 6$ hr. earlier than Greenwich. Standard time $= 1$ P.M. $- 6$ hr. $= 7$ A.M. *Ans.*

4. Vessel's time is $2\frac{1}{2}$ hr. earlier, showing that the **vessel** is $15° \times 2\frac{1}{2}$, or $37\frac{1}{2}°$ west of Greenwich. *Ans.* 37° 30′.

6. Time difference $= 1\frac{1}{2}$ hr. Longitude difference $= 15° \times 1\frac{1}{2} = 22\frac{1}{2}°$. The latter place, having the later time, is the more easterly; so that its longitude is $22\frac{1}{2}°$ east of 11° east, or 33° 30′ east. *Ans.*

7. 3 da. 12 hr. 17 min. $= 84\frac{17}{60}$ hr.; $3313.5 \div 84\frac{17}{60} =$ number of miles per hour.

11. $12\frac{1}{2}$ (ft.) $\times 3\frac{3}{4}$ (ft.) $\times x$ (ft.) $= 730\frac{810}{1728} = 730\frac{15}{32}$ (cu. ft.); $\frac{25}{2} \times \frac{15}{4} \times x = \frac{23375}{32}$; $x = (\frac{23375}{32} \div \frac{25}{2}) \div \frac{15}{4} = \frac{23375}{32} \times \frac{2}{25} \times \frac{4}{15} = \frac{187}{12} = 15\frac{7}{12}$. *Ans.* $15\frac{7}{12}$ ft. = 15 ft. 7 in.

12.

$$\begin{array}{rrll} & 48\% = & 237 \text{ bu. } 3 \text{ pk.} & \\ & +4\% = & \underline{19 \text{ bu. } 3 \text{ pk. } 2 \text{ qt.}} & \tfrac{1}{12} \text{ of } 48\% \\ \text{Remainder,} & 52\% = & 257 \text{ bu. } 2 \text{ pk. } 2 \text{ qt. } \textit{Ans.} & \end{array}$$

14. Number of degrees $= (34 \times 24) \div 48.96$.

1003. **5.** 84 half-dollars − 84 cents = \$42.00 − \$.84.

9. After taking $\frac{1}{3}$, $\frac{2}{3}$ are left; when $\frac{1}{3}$ of the remainder is taken, $\frac{2}{3}$ of remainder are left, or $\frac{2}{3}$ of $\frac{2}{3} = \frac{4}{9} = 4$ gal.; etc.

1004. **5.** 12 men working 8 hr. daily build 90 rd. in 15 da.; 7 men working 10 hr. daily build 70 rd. in ? days.

$$\frac{15 \text{ da.} \times 12 \times 8 \times 70}{90 \times 7 \times 10}.$$

9. 72 (in.) $\times$ 48 (in.) $\times x$ (in.) = 2150.4 (cu. in.) $\times$ 75.

16. $x + (x + 15) + (x + 15 + 27) = 320$.

17. .64 bu. = 4 pk. × .64 = 2.56 pk.; .56 pk. = 8 qt. × .56 = 4.48 qt.; 3.64 bu. = 3 bu. 2 pk. 4.48 qt.; $\frac{9}{16}$ bu. = 4 pk. × $\frac{9}{16}$ = $\frac{9}{4}$ pk. = $2\frac{1}{4}$ pk. = 2 pk. 2 qt.; 3 bu. 2 pk. 4.48 qt. + 2 pk. 2 qt. + 1 bu. 3 pk. 6.52 qt. = 6 bu. 5 qt.; 10 bu. − 6 bu. 5 qt. = 3 bu. 3 pk. 3 qt. *Ans.*

18. Each step takes $7\frac{1}{2}$ in. + 10 in. = $17\frac{1}{2}$ in. = $\frac{17\frac{1}{2}}{36}$ yd. Cost = 90¢ × $\frac{17\frac{1}{2}}{36}$ × 18. Cancel.

24. $x + (x + 1211) = 9891$.

1005. Formerly, bills of exchange were issued to purchasers in sets of three bills, two of which were sent by different steamers to the foreign payee, who presented for payment or acceptance the one that reached him first. The third bill was retained by the purchaser, to be sent in case both of the others failed to reach their destination. At present, only two bills of a set are issued. The second will read as follows:

Exchange for £180 17*s.* 6*d.* NEW YORK, Dec. 14, 1895.

Sixty days after sight of this Second of Exchange (First unpaid) pay to the order of John W. Moran & Bro., One Hundred Eighty pounds sterling, seventeen shillings, six pence.

Value received, and charge the same to account of

To JAMES LENNON & Co., London. No. 39. PETER COMERFORD & SON.

1. No deduction for interest is made for the 60 da., the quotation giving the price per pound for 60-day bills.

The method given in the text-book is a form of the aliquot part method used in calculating interest.

1007. 3. Cost in dollars $= 1000 \div 5.1625$.

4. Cost in dollars $= 1874.35 \times .9525 \div 4$.

5. Number of marks $= 1000 \div (.955 \div 4) = 4000 \div .955$.

6. Number of francs $= 1637.5 \times 5.185$.

8.

	£	s.	d.
	£437	5s.	10d.
Less 4%, or $\frac{1}{25}$,	17	9s.	10d.
	£419	16s.	

Cost $= \$4.885 \times 419\frac{16}{20} = \$4.885 \times 419.8$.

9. 18 pcs., 44 m. each, or 792 m.

@ fr. 25 =	fr. 19800	
Less $7\frac{1}{2}\%$,	1485	fr. 18315
3 pcs. 50 m. each, or 150 m. @ fr. 20,	fr. 3000	
Less 5%,	150	2850
Packing charges,		60.50
		fr. 21225.50

Cost of bill $= 19\frac{1}{2}¢ \times 21225.5 = \$4138.97$. *Ans.*

10.

	M. 3598.60
Less 10%,	359.86
	M. 3238.74
Less 5%,	161.94
	M. 3076.80
Less $2\frac{1}{2}$%,	76.92
	M. 2999.88
Freight, 165 kilos @ M. 4.80,	792.00
	M. 3791.88

$95\frac{7}{8}$¢ $\times$ 3791.88 $\div$ 4 = \$908.87. *Ans.*

XVI

NOTES ON CHAPTER THIRTEEN

1008. The word "endorsement" means something that is written on the back of a document. As applied to notes, checks, drafts, etc., it generally means the signature of the person in whose favor the note, check, etc., is made out, which is written on the back in order to transfer the ownership. If the payee of the following note sells it to William Simms, he writes his name on the back, as shown below.

ACCOTINK, VA., March 4, 1897.

Four months after date, I promise to pay to the order of James McWilliams, Two Hundred Dollars, value received, at the Pohick National Bank.

$\$200\frac{00}{100}$. VICTOR STRUDER.

(Endorsement in blank.)

James McWilliams

(Endorsement in full.)

Pay to the order of
William Simms
James McWilliams

The effect of the "endorsement in blank" is to make it payable to any holder; the "endorsement in full" transfers it to William Simms, who may transfer it to another by either kind of endorsement.

Besides transferring ownership in a note, the effect of an endorsement is generally to bind the signer to pay the note, in case of default by the maker or preceding endorsers. This liability is avoided if the endorser writes after his name the words, "without recourse."

The "endorsements" mentioned in this chapter are a record of the payments received by the holder of the note. This is usually kept on the back of the note, the date and the amount received being written in each instance.

1010. Although the maker of a note is generally supposed to pay the interest at the end of each year, the U. S. Courts, by whom this rule has been formulated, do not permit the collection of interest upon deferred payments of interest.

This rule is followed in all the states except Connecticut (see Art. 1307) for computing the amount due on notes that do not expressly provide for the payment of interest annually (Art. 1172). Connecticut pupils should learn only their own rule; in other states, no reference whatever to the Connecticut rule should be made.

See Art. 1307 for Connecticut-rule answers to the partial payment examples of this chapter.

1013. 6. Let $100\,x =$ cost of coal; $2\,x =$ commission.

$102\,x =$ cost of coal $+$ commission $= 7650$.

1015. 1. The man expended 30% of 50% of $\frac{3}{5}$ of his money, or $\frac{3}{10} \times \frac{1}{2} \times \frac{3}{5}$ of it; which was $\frac{9}{100}$ of his money. $\frac{9}{100}\,x = 1\frac{1}{8} \times 728 = 819$; $x = 9100$ Two-fifths of \$9100 equals the balance in bank.

2. Cost per gallon, \$1.50; selling price, \$1.60; gain per gallon, 10¢ on 150¢, or $\frac{1}{15} = 6\frac{2}{3}\%$.

5. Let $10\,x =$ cost, then x will represent the loss in one case and the gain in another, making the selling prices $11\,x$ and $9\,x$.

$11\,x = 99$; $x = 9$, gain in dollars.

$9\,x = 99$; $x = 11$, loss in dollars, making the net loss \$2.

NOTE.—Some teachers, wishing to avoid fractions as far as possible in equations, assume x for loss or gain, making $10x =$ cost; etc. Solutions of this kind are given occasionally as a suggestion to be followed or not, as may seem most desirable.

8. Arithmetic, Art. 924, 7 and 8.

1016. As there is no such thing as "true discount," it is unprofitable to spend time upon it. Any problem involving finding the "present worth" can be solved by an intelligent pupil, from his previous work in interest.

1019. 2. To $1\frac{1}{2}\%$ of $\frac{3}{4}$ of \$25000, add \$1.

3. Longitude difference $= 5° 59' 18''$. See Art. 1002, 2.

10. Number of yards $= (5616 \div 1.04) \div 1\frac{1}{2}$. Art. 1013, 6.

11. Term of discount 36 (33) days. Yearly interest is \$30, which is \$3 for 36 da., $\frac{1}{10}$ yr. \$500 − \$3 = proceeds, \$497. Without grace, the term is 33 days, the interest for which time is \$2.75. Proceeds, \$497.25.

13. Cost of an acre $= \$21.78 \times 5$. Cost per square foot $=$ $(\$21.78 \times 5) \div (4840 \times 9)$, the divisor being the number of square feet in an acre. To gain 20% or $\frac{1}{5}$ of cost, the selling price must be $\frac{6}{5}$ of cost. Multiplying the foregoing by $\frac{6}{5}$, the selling price per foot will be

$$\frac{\$21.78 \times 5 \times 6}{4840 \times 9 \times 5}.$$

In getting the number of square feet in an acre, the pupil may use $160 \times 30\frac{1}{4} \times 9$, unless he remembers that there are 4840 sq. yd. in an acre.

14. Specific duty (duty by weight) $= \$\frac{1}{2} \times 700 \times 1\frac{1}{4}$. Ad-valorem duty $= 30\%$ of $\$1\frac{1}{4} \times 700$. The sum of the two gives the total duty.

1020. 7–9. See Arithmetic, Art. 384.

1021. 4. (50 ft. + 38 ft) — (7 ft. + 2 ft.).

5. $\frac{9}{8}$ of cost of horse = \$90. Cost = \$80. A selling price of \$100 makes a gain of \$20 = $\frac{1}{4}$ of cost = 25%. *Ans.*

6. Loss $= \frac{1}{5}$ of cost $= 20\%$. *Ans.*

12. 100%. 15–16. See Art. 878. 19. $\frac{1}{4}\%$ of \$700. 20. \$10 for 60 da. $+ \frac{1}{12}$ of \$10 for 5 da. 21. 200%. 22. $\frac{7}{20}$ of \$7. 26. $4\frac{2}{10} \div \frac{3}{10} = 42 \div 3$.

28. $\frac{7x}{8} = \frac{7}{9}$ of $63 = 49$; $\frac{x}{8} = 7$; $x = 56$.

29. $\frac{2x}{3} = 8\frac{1}{3} = \frac{25}{3}$, $2x = 25$; $x = 12\frac{1}{2}$. *Ans.* 12 yr. 6 mo.

30. 5 qt. $= \frac{5}{32}$ pk.; $\frac{5}{8} = .625$; $.625 \div 4 = .156\frac{1}{4} = .15625$.

1022. 1. $\$\frac{3}{4} \times 5\frac{1}{2}$ (yd.) $\times 4$ (yd.) $\div 1\frac{1}{2}$ (yd.).
$\$\frac{3}{4} \times \frac{11}{2} \times \frac{4}{1} \times \frac{2}{3}$. Cancel.

2. What sum in 4 years at 6% will amount to \$105.71? Arithmetic, Art. 1017.

$$x + (x \times \tfrac{6}{100} \times 4) = 105.71;\ x + \frac{24x}{100} = 105.71;$$
$$100x + 24x = 10571;\ 124x = 10571;\ x = 85.25.$$

\$85.25. *Ans.*

3. Term $= 27$ da. $+ 3$ da. $= \frac{1}{12}$ yr.

$$x - (x \times \tfrac{6}{100} \times \tfrac{1}{12}) = 95;\ x - \frac{x}{200} = 95;\ \text{etc.}$$

4. Problems of this kind may be solved without finding the cost; 18¢ per yard represents $\frac{9}{10}$ of cost; what price will represent $\frac{6}{5}$ or $\frac{12}{10}$ of cost? If $\frac{9}{10}$ of cost $= 18$¢, $\frac{1}{10}$ will equal 2¢, and $\frac{12}{10}$ will equal 24¢. *Ans.*

5. $24\frac{1}{2}\% = \frac{49}{200}$; $\frac{49x}{200} = 1372$; $x = 5600$. The whole real estate was worth \$5600; the part remaining after the sale of \$1372 worth $=$ \$5600 $-$ \$1372 $=$ \$4228. \$14000 $+$ \$4228 $=$ \$18228. *Ans.*

6. 6 times (5×5) sq. ft.

7. Mr. Jones paid $\frac{3}{4}\%$ of $\frac{5}{6}$ of $\$48000 = \frac{\$48000}{1} \times \frac{5}{6} \times \frac{3}{400}$ $=$ \$300. The company loses \$40000 less the premium received, \$300.

8. This may be solved without finding the cost, although many pupils will prefer the more tedious way. \$764.40 represents 91% of cost; what per cent does \$894.60 represent?

$$\frac{91\% \times 89460}{76440} = \frac{213\%}{2} = 106\tfrac{1}{2}\%\text{; gain } 6\tfrac{1}{2}\%.$$

10. Cancel. $\$3\tfrac{7}{8} \times 25 \times 8 \times 8 \div 128$.

11. Total cost, \$252.50; loss = \$252.50 − \$141.40 = \$111.10, which is 44% of the total cost.

12. $[14\text{ (yd.)} + 5 + 14 + 5] \times 3$.

14. Profit \$2 per ton, $\frac{1}{3}$ of cost. Cost = $\$198 \times 3$.

15. Art. 546. A bill is receipted by writing the words, "Received payment" at its foot, followed by the date and the name of the seller:

Recd. payment, Oct. 3, 1897,
Abraham & Straus,
per J. S.

If the money is received by a clerk, he writes his initials underneath, preceded by the word "per" or "by."

16. $\frac{8\frac{3}{4}}{x} = 9\tfrac{4}{5}$; $9\tfrac{4}{5}x = 8\tfrac{3}{4}$; etc.

17. Omit $4\tfrac{1}{2}$ bu. I gave away $\frac{1}{3}$ and $\frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$. The remainder $= \frac{4}{15} = .26\tfrac{2}{3} = 26\tfrac{2}{3}\%$.

21. \$3 per yd. = 80% of cost; \$? = 115%.

22. Number of gallons sold = (65 gal. × 60) − 80 gal. = 3820 gal. Selling price per gallon = $\frac{9}{8}$ of $\$1542 \div 3820$. Cancel.

23. \$180 = $\frac{4}{5}$ cost of one horse = $\frac{6}{5}$ cost of other.

24. Number of square yards in walls = $(6 + 4 + 6 + 4) \times 3$; in ceiling, 6×4. Number of cords = $(18 \times 12 \times 9) \div 128$.

25. Let x = less, $x + \frac{4}{5}$ = greater; $x + x + \frac{4}{5} = 4\tfrac{1}{5}$.

26. 16 cu. yd. is $x\%$ of $(10 \times 8 \times 2)$ cu. yd.?

27. 672 yd. @ $\$2\frac{1}{4} = \$1512$. Discount without grace $= \$1512 \times \frac{7}{100} \times \frac{1}{6} = \$17.64$. Profit is \$1 per yard less discount $= \$672 - \$17.64 = \$654.36$. *Ans.*

The discount for 3 days' additional (grace) $= \frac{1}{20}$ of $\$17.64 = \$.88$, making the profit 88¢ less than the above, or \$653.48. *Ans.*

28. $40¢ \times [(55 \times 600 \times 5\frac{1}{2}) \div 27]$. Cancel.

1023. **5.** The circumference of the wheel = distance traveled in 1 revolution = 1 mi. 94 rd. 2 yd. 1 ft. ÷ 526 = 6838 ft. ÷ 526 = 13 ft. = 4 yd. 1 ft.

6. The weight in pounds $= \frac{1000}{16} \times 9\frac{2}{8} \times 9\frac{1}{8} \times 6\frac{3}{4}$.

8. Rate of income received on 6% bonds $= 6 \div 1.18$; rate on $4\frac{1}{2}\%$ bonds $= 4\frac{1}{2} \div \frac{x}{100}$. The income being the same, and the same amount being invested, the rates must be equal; therefore, $\frac{600}{118} = \frac{450}{x}$; $600\,x = 118 \times 450 = 53100$; $x = 88\frac{1}{2}$. Price per \$100 = \$88.50.

9. The shrinkage being 1 lb. in 10, he must sell 9 lb. for the cost of 10 lb. to suffer no loss. 10 lb. cost \$1.80; by charging 20¢ per pound, he receives the cost. To gain 20%, he must sell for $\frac{6}{5}$ of 20¢, or 24¢ per pound. Since he loses 4% of the amount of sales, or $\frac{1}{25}$, he receives only $\frac{24}{25}$ of the price charged per pound. Therefore to receive 24¢, he must charge $24¢ \div \frac{24}{25} = 25¢$ per pound.

$$[(18¢ \div \tfrac{9}{10}) \times \tfrac{6}{5}] \div \tfrac{24}{25}.$$

1024. **1.** $12 \times x = 20 \times \frac{3}{4}$.

2. This may be solved by analysis, or the following method may be employed:

The solid contents of first beam in cubic feet $= 16 \times 2\frac{1}{4} \times \frac{2}{8}$; of the second $= x \times 3\frac{1}{4} \times \frac{15}{24}$. The second weighs $\frac{2028}{1280}$ times the first; its contents, therefore, $= \frac{2028}{1280}$ times the contents of the first.

$$x \times 3\tfrac{1}{4} \times \tfrac{15}{24} = 16 \times 2\tfrac{1}{4} \times \tfrac{2}{8} \times \tfrac{2028}{1280};$$
$$x = (16 \times 2\tfrac{1}{4} \times \tfrac{2}{8} \times \tfrac{2028}{1280}) \div (3\tfrac{1}{4} \times \tfrac{15}{24})$$
$$= 16 \times \tfrac{9}{4} \times \tfrac{2}{8} \times \tfrac{2028}{1280} \times \tfrac{4}{13} \times \tfrac{24}{15}. \quad \text{Cancel.}$$

3. The carpet costs 50¢ per foot. $\$\frac{1}{2} \times 22\frac{1}{2} \times 15\frac{3}{4} \div 2\frac{1}{4} = Ans.$ Or, changing all dimensions to yards: $\$1\frac{1}{2} \times 7\frac{1}{2} \times 5\frac{1}{4} \div \frac{3}{4} = Ans.$

4. As a sight example, some pupils may see that the width of the large box is double that of each small one, and its depth is three times that of each small one, so that with the same length as the small one, it would contain 2×3, or 6, small ones. A length twice as great — $8\frac{1}{8}$ ft. — is required to enable it to hold 12 boxes.

6. $[\frac{1}{2}$ of $(18\frac{1}{3} \times 11\frac{5}{6})]$ sq. ft.

8. $\dfrac{22 \times 14 \times 12 \times 1728}{2150.4}$.

Drop the decimal point in the denominator, and annex a cipher to one of the numbers in the numerator. Cancel.

9. $49 \times 44 \times 27 \div 231 =$ number of gallons. Cancel.

10. Solve at sight. 7 yd., 6 yd., 4 yd.

11. Number of gallons $= 5\frac{1}{2} \times 6 \times 7 \times 1728 \div 231$. Canceling, we obtain 1728 gal. One empties it in $(1728 \div 9)$ min. $= 192$ min.; the other in $(1728 \div 7)$ min. $= 246\frac{6}{7}$ min.; both in $(1728 \div 16)$ min. $= 108$ min.

12. The dimensions of the room are 6 yd. and 5 yd., and the carpet is $\frac{3}{4}$ yd. wide. 6 contains $\frac{3}{4}$ an exact number of times (8), so that if the carpet runs across the room it will take 8 strips each 5 yd. long. As $5 \div \frac{3}{4} = 6\frac{2}{3}$, to lay the carpet lengthwise would require 6 strips, and $\frac{2}{3}$ of a seventh strip, which would have to be cut.

Carpet 30 in. wide, $\frac{5}{6}$ yd., could be laid lengthwise without splitting the breadths, $5 \div \frac{5}{6}$, or 6, strips being needed, each 6 yd. long.

13. 36 yd. are needed to cover the floor; including $4\frac{1}{2}$ yd. cut off in matching the pattern, $40\frac{1}{2}$ yd. must be bought at 95¢. At 10¢ per yard, the sewing and laying should cost $10¢ \times 36 = \$3.60$, but the custom is to charge for the number of yards

purchased, $40\frac{1}{2}$, making \$4.05; (5×6) sq. yd. or 30 sq. yd. @ 5¢ $= \$1.50$ for lining. Total cost, $\$38.47\frac{1}{2} + \$4.05 + \$1.50 = \$44.02\frac{1}{2}$, or \$44.03.

14. A strip $\frac{1}{2}$ of $(18 - 15)$ or $\frac{1}{2}$ of $(21 - 18)$ is left uncovered on each of the four sides, or $1\frac{1}{2}$ ft. The area of the uncovered space in square feet $= (21 \times 18) - (18 \times 15)$.

15. The number of square feet in the walls $= (18 + 24 + 18 + 24) \times 9$. The ceiling contains (18×24) sq. ft. Deduct 60 sq. ft. for two doors, 48 sq. ft. for two windows, 25 sq. ft. for the fireplace. The total number of feet around the four walls $= 18 + 24 + 18 + 24 = 84$ ft. Baseboard will not be required at the doors, 8 ft.; nor at the fireplace, 5 ft. — a deduction of 13 ft., making 71 running feet of baseboard, 1 ft. wide, containing, therefore, 71 sq. ft. The total deduction from the area of walls and ceiling, 1188 sq. ft., are 60 sq. ft. + 48 sq. ft. + 25 sq. ft. + 71 sq. ft. = 204 sq. ft., leaving 1188 sq. ft. − 204 sq. ft., or 984 sq. ft., to be plastered.

16. The first pile contains $(25 \times 20 \times 10)$ cu. ft. and costs \$1400. 1 cu. ft. cost $\$1400 \div (25 \times 20 \times 10)$. Multiplying by $(50 \times 40 \times 20)$, the number of cubic feet in the second pile, gives the cost:

$$\frac{\$1400 \times 50 \times 40 \times 20}{25 \times 20 \times 10}.$$

17. Pupils that endeavor to solve a problem without examining the conditions, will be likely to assume that this example resembles **16**. In the latter, the cost of the second pile is 8 times the cost of the first; in this one, the surface to be painted in the second room is 4 times that of the first room, making the cost \$56. As they may not be familiar enough with similar surfaces to know the ratio, they should find the surface of each.

1025. In dividing decimals, change the divisor to a whole number. See Arithmetic, Art. 663.

1026. 2. $\overline{\text{XXV}} = 25000$.

5. The furlong is seldom used. 3.7082 mi. $\div$ 4 = .92705 mi., the length of one side. Multiplying by 8 to reduce to furlongs, we obtain 7.4164 fur. Change the decimal part, .4164 fur., to rods by multiplying by 40, obtaining 16.656 rd.; .656 rd. = 3.608 yd.; etc.; etc.

6. See Art. 986, **7.** That selling price, \$3.54, must be increased $\frac{1}{6}$, or \$.59, to gain $16\frac{2}{3}\%$. \$3.54 + \$.59 = \$4.13. *Ans.*

7. $\$\frac{5}{8} \times \frac{240 \times 38 \times 9}{27}$. Cancel.

10. The inventors of the expression "true discount" assume that interest is not payable in advance. They claim that a borrower that promises to pay \$380 at the end of 2 yr. 5 mo. should receive as a loan the principal that will amount to \$380 in that time.

$$\text{Let} \quad x = \text{principal.}$$

$$\text{Interest} = x \times \frac{6}{100} \times \frac{29}{12} = \frac{29x}{200}.$$

$$\text{Amount} = x + \frac{29x}{200} = 380;$$

$$200x + 29x = 229x = 76000;$$

$$x = 331.88-.$$

The principal ("present worth") = \$331.88.

The "true discount" = \$380 − \$331.88 = \$48.12.

This "true discount" is the interest at 6% on \$331.88 for 2 yr. 5 mo.

The interest on \$380 at 6% for 2 yr. 5 mo. is \$55.10; the difference between the interest and the "true discount" being \$55.10 − \$48.12 = \$6.98. *Ans.*

1027. **1.** A gain of 25%, or $\frac{1}{4}$ of cost, makes the selling price \$9, equal $\frac{5}{4}$ of cost; $\frac{1}{4}$ of cost, the present gain, is \$9 $\div$ 5, or \$1.80. A gain of 50% would be \$1.80 $\times$ 2 = \$3.60.

N. B. — It is not necessary to find the cost.

2. \$30 in one case represents $\frac{5}{4}$ of cost, making the gain \$6; in the other case, \$30 represents $\frac{3}{4}$ of cost, making the loss \$10. Net loss \$4.

NOTE.—The thoughtful teacher will recollect that every member of the class does not "see through" an example in the same way, nor with equal rapidity. While quick work should be exacted where the question involves but a single arithmetical operation, time should be given, in problem work, to pupils that do not quickly grasp the conditions. Such as can dispense with unnecessary figures should be encouraged to do so as much as possible; but care should be taken not to injure others by requiring them to adopt a short method whose underlying principles they do not thoroughly understand. Each should, to a certain extent, be allowed to use his own mode of "analyzing" oral problems and of setting down his written ones; shorter ways, however, being presented from time to time in the oral and blackboard work of his brighter classmates. The scholar that reaches his results by a circuitous course will, by these models, be led to see the time saved by shorter methods, and he will probably try to master some of them.

3. Together they have $\$300 = x + 2x$.

6. \$40 in $3\frac{1}{3}$ yr. = \$12 per year. This is produced at 6% by \$200. *Ans.*

7. The interest of x dollars for 5 yr. at $6\% = \frac{3x}{10}$; $x + \frac{3x}{10}$, or $\frac{13x}{10} = 52$; $\frac{x}{10} = 4$; $x = 40$.

8. Yearly interest = \$3. To obtain \$18 interest will require 6 yr.

9. The interest is \$2, $\frac{1}{6}$ of principal in $3\frac{1}{3}$ yr.; in 1 yr. it is $\frac{1}{6} \times \frac{3}{10}$ of principal, or $\frac{1}{20} = 5\%$.

Or, the interest on \$12 @ 1% for a year is 12¢, or 40¢ for $3\frac{1}{3}$ yr.; to obtain \$2.00 interest, which is 5 times as much, the rate must be 5%.

10. I lose \$25, or $\frac{1}{3}$ of cost $= 33\frac{1}{3}\%$.

1028. **4.** $23\frac{1}{8} + (23\frac{1}{8} + 3\frac{3}{4}) + (23\frac{1}{8} + 3\frac{3}{4} + 3\frac{1}{8})$.

6. Let $3x$ represent the amount received by one; and $4x$ the amount received by the other. Then, $7x = 21.63$; $x = 3.09$; $4x = 12.36$, and $3x = 9.27$. *Ans.* \$9.27 and \$12.36.

7. $[(26 \times 12) \times (4 \times 12)] \div (8 \times 4)$.

8. $\dfrac{\$10.24 \times 2700 \times 890}{1500 \times 356}$. Cancel.

1044. In addition to the details usually given in a bill, an invoice shows the marks and the numbers placed upon each package shipped. In this invoice, the mark is given in the first column, and is the same on each case. The number of each case is written in the second column. Besides informing the receiver in what case a particular article is packed, it is required by the U. S. custom authorities. A certain percentage of cases in each invoice is examined, and their contents must agree in description, quantity, value, etc., with the invoice.

1500 yd. @ $1\frac{7}{8}d.$,	£11 – 14 – $4\frac{1}{2}$
1500 " " 2 "	12 – 10
3000 " " $1\frac{7}{8}$ "	23 – 8 – 9
2889 " " $2\frac{5\frac{1}{2}}{12}$ "	29 – 11 – $10\frac{1}{4}$
	£77 – 4 – $11\frac{3}{4}$
Less $\frac{1}{40}$,	1 – 18 – $7\frac{1}{2}$
	£75 – 6 – $4\frac{1}{4}$

Value in U. S. money, $366.53. Duty @ 50% = $\frac{1}{2}$ value = $183.27, nearly.

English accountants employ to a great extent a method by aliquot parts called "practice." The cost of the fourth item is obtained as here shown. The cost at $1d.$ per yard, $2889d.$, is easily reduced to pounds, etc. It is repeated to obtain the cost @ $2d.$ $\frac{4}{12}d.$ is $\frac{1}{3}$ of $1d.$, $\frac{1}{12}d.$ is $\frac{1}{4}$ of $\frac{4}{12}d.$, and $\frac{\frac{1}{2}}{12}$ is $\frac{1}{2}$ of $\frac{1}{12}d.$ As the smallest denomination is the farthing, or $\frac{1}{4}$ of a penny, $\frac{1}{2}$ of $\frac{3}{4}d.$ is called 2 farthings, or $\frac{1}{2}d.$, instead of $\frac{3}{8}d.$

2889 @ 1 $d.$ =	£12 – 0 – 9
" 1 "	12 – 0 – 9
" $\frac{4}{12}$ "	4 – 0 – 3
" $\frac{1}{12}$ "	1 – 0 – $0\frac{3}{4}$
" $\frac{\frac{1}{2}}{12}$ "	10 – $0\frac{1}{2}$
	£29 – 11 – $10\frac{1}{4}$

A similar method may be used in reducing the above result to U. S. money. See Arithmetic, Art. 1005. The value of £50 is ascertained by taking $\frac{1}{2}$ of $486.65, or $243.325. £25 = $\frac{1}{2}$ of £50. 5s. = £$\frac{1}{4}$; etc.

In assessing duties, the government ignores fractions of a dollar in the cost less than $\frac{1}{2}$; over 50¢ is considered another dollar. The duty that

would be collected upon the foregoing would be 50% of \$367, or \$183.50. Children should not, of course, be burdened with such details; their answer should be \$183.27.

Some zealous teachers fall into the mistake of endeavoring to make their pupils familiar with the methods of calculation peculiar to some callings. The time assigned to the study of arithmetic can be employed more profitably to the scholar by giving him the ability to handle ordinary problems with reasonable readiness, than by dissipating his energies in trying to make him understand a multitude of small matters that are entirely outside of his present experience. The average boy would make a better accountant if he did not hear of taxes, partnership, insurance, bonds, stock, brokerage, commissions, etc., during his school life, provided the time thus misspent were given to elementary algebra and constructive geometry, as well as to better work in what are considered the more elementary portions of arithmetic.

1046. The ratio of 3 to 6 is generally expressed as 1 to 2.

1049. 3. The ratio of the price of coffee to that of sugar is $\frac{5}{1}$, or 5 to 1.

4. A goes $1\frac{1}{4}$ times as fast as B. The ratio of A to B is $\frac{5}{4}$, or 5 to 4.

5. E's earnings are to D's as 4 to 3.

6. 3 to 4.

10. 30 to 3 = 10 to 1.

1050. 6. One wheel makes 70 revolutions per second; the other makes 90 per second.

7. The diameter = twice radius = 4 ft.

8. One goes 48 mi. per hour; the other goes the same distance in the same time.

9. Area of first $= 6\frac{2}{3} \times 4\frac{1}{8}$; of second $= 4\frac{2}{3} \times 2\frac{1}{12}$. Ratio $= \dfrac{4\frac{2}{3} \times 2\frac{1}{12}}{6\frac{2}{3} \times 4\frac{1}{8}}$. Reduce.

1051. 2. Cost of farm = \$75.50 × $156\frac{1}{4}$ × 124.6 ÷ 160 = \$9201.52. Interest for 1 yr. on one-half of the cost = \$230.04; \$4600.76 + \$230.04 = \$4830.80, amount of mortgage. Deducting \$500 then paid, gives \$4330.80, balance due.

4. Number of cubic feet of excavation $= 41\frac{1}{4} \times 8 \times 33 = 10890$. The inner dimensions of the cellar are $38\frac{1}{4} \times 30 \times 8$, deducting from the length and the breadth 3 ft., which is the thickness of two walls. The number of cubic feet in the cellar $= 9180$, leaving 10890 cu. ft. $-$ 9180 cu. ft. $=$ 1710 cu. ft. as the contents of the walls. The cost of the excavation is $\$\frac{1}{2} \times 10890 \div 27 = \$201\frac{2}{3}$; to lay the wall, costs $\$15 \times 17.1 = \$256.50$. Total cost, $\$201.66\frac{2}{3} + \$256.50 = \$458.16\frac{2}{3}$, or \$458.17. *Ans.*

NOTE. — One of the practices of builders in some sections is to take only the outside measure of walls in ascertaining the contents. The number of cubic feet in this case, according to their calculations, would be $(41\frac{1}{4} + 33 + 41\frac{1}{4} + 33) \times 8 \times 1\frac{1}{2} = 1782$ cu. ft., or 72 cu. ft. too many. By this method the four corners, measuring each $1\frac{1}{2} \times 1\frac{1}{2} \times 8$, or 72 cu. ft. in all, are included twice. As has already been said, children should be expected to obtain only the correct results, leaving later experience to furnish information as to local usages.

5. For \$107.25 there can be obtained, at $3\frac{1}{4}\%$, insurance amounting to $\$107.25 \div .0325 = \$3300$. If this is 80% of the value, the flour must have cost him $\$3300 \div \frac{4}{5} = \$4125$, which is $\$4125 \div 500$, per barrel, or \$8.25.

The algebraic method would be:

Let $\quad x =$ the cost per barrel.

$$x \times 500 \times 80\% \times 3\tfrac{1}{4}\% = 107.25;$$

or, $$500x \times \tfrac{4}{5} \times \tfrac{13}{400} = 13x = 107.25;$$

$$x = 8.25.$$

6. The bank discount is calculated on the sum due at maturity, which is \$1250 + interest from June 12 to Dec. 15 (12), 186 da. (183 da.), at 5% $= \$1250 + \$32.29$ (\$31.77) $=$ \$1282.29 (\$1281.77). The discount for 30 da. (including grace) on \$1282.29 at 6% is \$6.41, making the proceeds \$1275.88. The discount on \$1281.77 for 27 da. (no grace) at 6% $=$ \$5.77, making the proceeds \$1276.

8. A furnished 5 men for 20 da. and 6 men for 15 da., which is the same as 100 men and 90 men for 1 da. B furnished

the equivalent of 120 men + 180 men for 1 da. The money received should be divided on the basis of 190 men for A and 300 for B, 490 in all; and A should receive $\frac{190}{490}$ of the sum, and B $\frac{300}{490}$ of it.

$$\tfrac{190}{490} \text{ of } \$857.50 = \$332.50, \text{ A's share;}$$

$$\tfrac{300}{490} \text{ of } \$857.50 = \$525.00, \text{ B's share.}$$

9. Arrange the work so as to have the required term in the last place:

A ditch $(403 \times 3 \times 3)$ cu. ft. is dug in (62×13) hr. by 27 men.
A ditch $(750 \times 4 \times 3)$ cu. ft. is dug in (250×12) hr. by ? men.

If $(403 \times 3 \times 3)$ cu. ft. are dug in (62×13) hr. by 27 men,

1 cu. ft. will be dug in (62×13) hr. by $\dfrac{27 \text{ men}}{403 \times 3 \times 3}$;

1 cu. ft. will be dug in 1 hr. by $\dfrac{27 \text{ men} \times 62 \times 13}{403 \times 3 \times 3}$;

etc. See Arithmetic, Arts. 973, 974.

$$\textit{Ans.} = \frac{27 \text{ men} \times 62 \times 13 \times 750 \times 4 \times 3}{403 \times 3 \times 3 \times 250 \times 12}. \quad \text{Cancel.}$$

10. Let $x =$ price per barrel; $\dfrac{23x}{4} =$ cost of flour. $2\frac{1}{2}\%$ of $\dfrac{23x}{4}$, or $\dfrac{1}{40}$ of $\dfrac{23x}{4}$, $= \dfrac{23x}{160} =$ commission.

Then $$\frac{23x}{4} + \frac{23x}{160} = 1508.80.$$

NOTE. — The words "after deducting his commission" mislead some pupils, who think that this requires them to begin work by deducting a commission of $\frac{1}{40}$ of the amount sent. If the purchasing agent did not receive money in advance, he would render his employer the following account:

To 256 bbl. flour @ $5.75,	$1472.00
Commission at $2\frac{1}{2}\%$,	36.80
	$1508.80

$2\frac{1}{2}\%$ on the amount sent, $1508.80, would be $2\frac{1}{2}\%$ of the cost of the flour, $1472, and $2\frac{1}{2}\%$ of the $36.80 commission.

A pupil that wishes to deduct the commission first, may work in this way:

Let $\quad x = \text{commission, then } 40x = \text{cost of flour};$

$$41x = 1508.80;\ x = 36.80.$$

\$1508.80, amount sent, $-$ \$36.80, commission, $=$ \$1472 left for the purchase of the flour.

The commission to be deducted is $\frac{1}{41}$ of the sum sent.

1052. See Arithmetic, Arts. 936, 937.

1053. Pupils generally find it most convenient to change the time to days, leaving the reduction to lowest terms for the subsequent cancellation.

11. $2 \text{ yr. } 11 \text{ mo. } 18 \text{ da.} = 720 \text{ da.} + 330 \text{ da.} + 18 \text{ da.} = \frac{1068}{360} \text{ yr.}$

$$\overset{2}{\cancel{240}} \times \frac{x}{100} \times \frac{\overset{356}{\cancel{1068}}}{\underset{3}{\cancel{360}}} = \frac{712x}{100} = \text{interest} = 32.04;$$

$$712x = 3204;\ x = 4\tfrac{1}{2}.$$

NOTE. — The teacher will notice that canceling 100 and 356 by dividing each by 4, and reducing the equation to $\frac{178x}{25} = 32.04$, is of no advantage.

The French method of solving an example like the foregoing, is to indicate all the work, and then to cancel:

(a) $$\frac{240 \times x \times 1068}{100 \times 360} = \frac{3204}{100};$$

(b) $$240 \times x \times 1068 = \frac{3204}{100} \times 100 \times 360;$$

(c) $$x = \frac{\overset{3}{\cancel{3204}} \times \cancel{100} \times \overset{3}{\cancel{360}}}{\cancel{100} \times \underset{2}{\cancel{240}} \times \cancel{1068}} = \frac{9}{2} = 4\tfrac{1}{2}.$$

The first member of equation (a) is "cleared of fractions" by multiplying the second number by the compound denominator, 100×360; see (b). The value of x is then found by dividing the second member by the compound coefficient of x, 240×1068; see (c). The result is then obtained by cancellation. In practice, the second equation (b) is not employed, the multiplication by 100×360, and the division by 240×1068, being indicated at the same time.

1054. See Art. 948.

In states that do not allow days of grace, there will be no difference in these examples between the words "term" and "time." The notes in **19**, **21**, and **25** are assumed to be discounted on the day they are drawn, which will make the "term" in states allowing days of grace, 3 days longer than the specified time. Teachers in other states are advised not to use "days of grace" in discount examples, no matter what is the date of the note or where it is made. Answers in which days of grace are not included, are enclosed in parentheses.

1055. The short methods previously studied should be employed. For **1**, **2**, **4**, see Arithmetic, Art. 891; **6** and **8**, see Art. 792; **10**, Art. 717; **12**, Art. 716; **14** and **16**, Art. 791; **17** and **18**, Art. 958; **19**, Art. 758; **20**, Art. 910.

3. $1648 \times 87\frac{1}{2} = \frac{7}{8}$ of 164800. See Arithmetic, Art. 891.

$$\begin{array}{l} 1648 \times 87\frac{1}{2} \\ \underline{206} \text{ deduct } \frac{1}{8} \\ 144200 \textit{ Ans.} \end{array}$$

5. $2416 \times 875 = \frac{7}{8}$ of 2416000. **7.** $848 \times 125 = \frac{1}{8}$ of 848000. **9.** $= \frac{1}{4}$ of 79200. **13.** $= \frac{5}{8}$ of 157600. **15.** $= \frac{3}{8}$ of 1128000.

The pupils should be encouraged to employ these methods, where practicable. When too many figures are used in blackboard work, the teacher should call the attention of the class to the saving of time that is rendered possible by the employment of a shorter way.

1056. The use of diagrams will tend to simplify these problems for many pupils. The average scholar finds no difficulty in ascertaining the time difference when the difference in longitude is given, and *vice versa;* but the introduction of the other elements tends to confuse him.

First mark the meridian of 0°; then locate the two places, writing under each the longitude, as far as given; and above each, its time, as far as given. The next step is to find the time difference or longitude difference, writing it in the place designated; and from it to calculate the other, writing it in its place. The last step is to calculate the required time or longitude.

1. The difference in time is 1 hr. 24 min. The time at A is later; 1 hr. 24 min. is, therefore, deducted from 1:30 P.M., giving 12:6 P.M. as the time at B.

2. The longitude difference is 36°, which gives a time difference of 36 hr. ÷ 15 = 2 hr. 24 min., both of which should be written on the diagram. As the right-hand, or more easterly, place is later, the time at B is 12 M. – 2 hr. 24 min. = 9:36 A.M.

3. A look at the diagram shows the pupil which difference must be found, from which the other is to be calculated. The time difference in this problem is 55 min. 30 sec., making the longitude difference 55′ 30″ × 15 = 13° 52′ 30″. As B has the later time, it is the more easterly. A glance at the diagram, if correctly made, will show the pupil that he must deduct from the longitude of A, 156° 48′, the above difference, 13° 52′ 30″, to obtain the longitude of B, which is 142° 55′ 30″ west. Other rules than those already learned should not be given.

4. B is 52° 36′ east of 0°, or east longitude.

5. The time difference = 101 hr. ÷ 15 = 6 hr. 44 min. The time at A = 12 M. – 6 hr. 44 min. = 5:16 A.M.

1057. **6.** Long. diff. = 9°; time diff. = 36 min.; 9 A.M – 36 min. = 8:24 A.M.

7. Long. diff. = 25° 55′; time diff. = 1 hr. 43 min. 40 sec.; 1:45 P.M. – 1 hr. 43 min. 40 sec. = 12:1:20 P.M.

8. Time diff. = 55 min.; long. diff. = 13° 45′; 156° 48′ – 13° 45′ = 143° 3′ W.

1058. Find the square root of each term. After extracting the roots of **13–21**, reduce the resulting improper fractions to mixed numbers.

1059. **6.** Find the amount of $1030.05 at 4% for 2 yr. 9 mo. 12 da.

7. See Manual, Art. 1026, **10**.

1060. See notes on Special Drills of previous chapters.

1062. $36 \times 31 = (36 \times 30) + 36$. $36 \times 29 = (36 \times 30) - 36$. Art. 953.

1063. Use chiefly as "sight" work. $675 \div 75 = 6\frac{3}{4} \div \frac{3}{4} = 27 \div 3$; $225 \div 12\frac{1}{2} = 2\frac{1}{4} \times 8$; $150 \div 6\frac{1}{4} = 1\frac{1}{2} \times 16$; $825 \div 37\frac{1}{2} = 8\frac{1}{4} \div \frac{3}{8} = 66 \div 3$; $750 \div 62\frac{1}{2} = 7\frac{1}{2} \div \frac{5}{8} = 60 \div 5$.

1064. $315 \times 1\frac{14}{15} = (315 \times 2) - (\frac{1}{15}$ of $315)$; $32 \times 39\frac{7}{8} = (32 \times 40) - (\frac{1}{8}$ of $32)$; etc.

The square of $7\frac{1}{2}$ (Art. 1032) $= 7$ times $7 + (2 \times 7 \times \frac{1}{2})$, or 1 time $7 + \frac{1}{4} = 8$ times $7 + \frac{1}{4}$; $8\frac{1}{2} \times 8\frac{1}{2} = 9$ times $8 + \frac{1}{4}$. The square of 75 is found by affixing 25 to the product of 7 by 8, 5625; $85^2 = 7225$.

$18\frac{2}{5} \times 5\frac{1}{3} = 18\frac{2}{5} \times 5\ (92) + \frac{1}{3}$ of $18\frac{2}{5}\ (6\frac{2}{15}) = 98\frac{2}{15}$. See Arithmetic, Art. 758.

When the divisor is a whole number, $97\frac{1}{2} \div 3$, etc., do not reduce the dividend to an improper fraction; $\frac{1}{3}$ of $97\frac{1}{2} = 32$ with a remainder of $1\frac{1}{2}$, or $\frac{3}{2}$; $\frac{1}{3}$ of $\frac{3}{2} = \frac{1}{2}$. $19\frac{1}{5} \div 2\frac{2}{5} = \frac{96}{5} \div \frac{12}{5} = 96 \div 12$; etc.

1065. These problems are applications of the drills upon the preceding page. Their solution involves simply the ability to handle large numbers without a pencil, and does not require any mental effort in determining the nature of the operations required. It is of more advantage to the pupil to be able to obtain the results in examples of this kind than in the more complicated ones usually given in the higher grades, for which reason teachers should be careful not to omit them.

6. $[(30 \times 16) - (\frac{1}{2}$ of $16)]$ oz.

8. 5700 tenths $\div$ 19 tenths.

10. $[(12$ yr. $\times 9) + (10$ yr. $\times 6)] \div (9 + 6)$.

16. $77 \div 5\frac{1}{2} = 77 \div \frac{11}{2} = 77 \times \frac{2}{11} = 7 \times 2$.

17. $[(10\frac{2}{3} \times 6) + (\frac{1}{2}$ of $10\frac{2}{3})]$ sq. yd.

18. 31 doz. @ 15¢.

21. 100 marks = \$23.80; 50 marks = $\frac{1}{2}$ of \$23.80.

24. $1\frac{1}{4}$ thousand $\times 16 = 20000$.

25. $5600 \div 87\frac{1}{2} = 56 \div \frac{7}{8} = 56 \times \frac{8}{7} = 8 \times 8$.

1066. The value of "All others" should be written directly in its place. See Arithmetic, Art. 384. To obtain results with two decimal places that will give a total of exactly 100, it will be necessary to extend the division to three places, as here shown, and to increase by 1 the hundredths' figure of the four having the largest figures in the thousandths' place, rejecting the thousandths' figures of the others. The usual method of calling 5 thousandths or over 1 hundredth, does not always make the total correct. See **8**, in which .715 – is made .72, although the thousandths' figure is not quite 5.

1.	63.301 +	63.30
2.	13.143 –	13.14
3.	8.576 –	8.58
4.	3.826 +	3.83
5.	3.414 +	3.41
6.	2.094 –	2.09
7.	1.113 –	1.11
8.	.715 –	.72
9.	3.818 –	3.82
	100.000	100.00

1067. **7.** (*a*) $28.128 × 39½ = $1104.02. A payment on Dec. 21 is entitled to a rebate of 11 days' interest on the foregoing, at $7\frac{3}{10}$%, or $2.46, making the amount actually paid = $1104.02 – $2.46 = $1101.56. If paid Jan. 15, the interest on $1104.02 at 9% for 45 da., $12.42, would be added, making a total of $1116.44.

1070. Permit the pupils to work out in their own way these preliminary examples. The sign : : is another form of the sign of equality. The rule for proportion is given in Arithmetic, Art. 1073.

1074. In oral problems, pupils should generally be permitted to use their own method. Nothing will be gained by requiring them to use proportion in the solution of these.

1076. **1.** 80% of 70¢. **2.** 90% of 60¢. **3.** 60% of 75¢. **8.** 90% of 66⅔¢.

1077. **11.** The first discount is the given one, 30%; the second is 30% of the remaining 70%, or 21%; the total single discount $= 30\% + 21\% = 51\%$. **12.** $20\% + \frac{1}{4}$ of $(100 - 20)\% = 20\% + 20\% = 40\%$. **13.** $25\% + \frac{1}{5}$ of $(100 - 25)\% = 25\% + 15\% = 40\%$. The results are the same in **12** and **13**, it making no difference which discount is taken first.

Another method of obtaining the single discount is to ascertain the net per cent, and to subtract this from 100%. Thus, in **11**, $(100 - 30)\%$ of $(100 - 30)$ per cent $= 70\%$ of 70 per cent $= 49$ per cent. The single discount $= (100 - 49)\% = 51\%$.

1078. **21.** 30 and $20\% = (70\%$ of $80\%)$ net $= 56\%$ net. 40 and $10\% = (60\%$ of $90\%)$ net $= 54\%$ net. The latter, being the smaller price, is better for the buyer.

1079. **1.** 60% of 90% of $\$250 = \$135$. *Ans.* **2.** 95% of $(100 - x)\%$ of 800, or $\frac{95}{100}$ of $\frac{100 - x}{100}$ of $\frac{800}{1} = 684$; canceling, $\frac{3800 - 38x}{5} = 684$; $3800 - 38x = 3420$; $-38x = -380$; $38x = 380$; $x = 10$. *Ans.* 10%.

3. $66\frac{2}{3}\%$ of 90% of x, or $\frac{2}{3}$ of $\frac{9}{10}$ of $x = \frac{3x}{5} = 90$; $3x = 450$; $x = 150$. *Ans.* \$150.

4. 70% of $(100 - x)\%$ of $600 = 378$; $\frac{7}{10} \times \frac{100 - x}{100} \times \frac{600}{1} = \frac{2100 - 21x}{5} = 378$; $2100 - 21x = 1890$; $-21x = -210$; $21x = 210$; $x = 10$. *Ans.* 10%.

$$\begin{array}{lr} & 800 \\ 5\%, & 40 \\ \hline & 760 \\ x\%, & \frac{38x}{5} \\ \hline & 760 - \frac{38x}{5} \end{array}$$

NOTE. — Some pupils may prefer to use the longer method, deducting each discount in turn to obtain the net price, and making this equal to the given net price. Thus in **2** the operations would be shown as here indicated, and the equation would be $760 - \frac{38x}{5} = 684$, which, after clearing of fractions, reduces to the form given above, $3800 - 38x = 3420$. The first method is shorter and less likely to give rise to mistakes.

5. $x = 70\%$ of 20% of 16.

6. $\frac{1}{2}$ of $\frac{9}{10}$ of $x = 27$.

7. $\dfrac{4}{5}$ of $\dfrac{100 - x}{100}$ of $5 = 3.20$.

8. $\frac{7}{10}$ of $\frac{7}{10}$ of $x = 73.50$.

9. $x = \frac{3}{4}$ of $\frac{4}{5}$ of 200.

10. $\dfrac{1}{2}$ of $\dfrac{100 - x}{100}$ of $1.50 = .60$.

1084. 2. There are 5280 ft. in a mile. $5280 \times 176 \div 3520$ = number of minutes. Cancel.

3. $\frac{875}{10000}$ A. = \$9; 1 A. = $\$9 \div \frac{875}{10000} = \$9 \times \frac{10000}{875}$; $\frac{3}{32}$ A. $= \$9 \times \frac{10000}{875} \times \frac{3}{32}$. Cancel.

4. Reduce both to pence.

5. A receives $\frac{2000}{7500}$ of \$576; etc.

6. (580 tiles $\times 6 \times 6) \div (4 \times 3)$. Cancel.

7. $\dfrac{x}{400} = 1500$.

10. 108 (in.) $\times 80 \times 77 \div 231$.

11. $.625 + .4375 + .75 + .09375 + 2.46 = 312.5\,x$.

12. $142.50 \div (5.25 - 4.95)$ = number of tons, 475. Total cost = $\$4.95 \times 475$.

14. The distance traveled in 1 revolution is 13 ft., the circumference of the wheel.

15. Three hundred forty-nine thousandths; three hundred (units) and forty-nine thousandths; three hundred forty nine-thousandths; three hundred (units) and forty nine-thousandths.

In dictating such numbers, it would be necessary to be still more explicit.

16. 4 lb. 6 oz. 12 pwt. = 1092 pwt.; 7 lb. 9 oz. 12 pwt. = 1872 pwt. *Ans.* (£13 8*s.* 4*d.*) $\times 1872 \div 1092$ = (canceling) £13 8*s.* 4*d.* $\times \frac{12}{7}$. If preferred, £13 8*s.* 4*d.* may be reduced to pence.

17. Total amount realized = \$20000 + \$1500 + \$1000 = \$22500. A furnishes one-third of the \$15000 capital, and is entitled to $\frac{1}{3}$ of \$22500, or \$7500. Having drawn \$1500 already, he should receive \$6000 additional. B's share = \$15000 − \$1000 = \$14000.

18. The gain on an article bought for 80% of its value and sold for 120% of its value, is 40% of its value. 40% is $\frac{1}{2}$ of the cost, 80%, so that the gain is 50%.

The words "per cent" occur so frequently in the foregoing as to confuse some children. Calling the value $100x$, the cost is $80x$ and the selling price is $120x$, a gain of $40x$, which is $\frac{1}{2}$ of the cost, $80x$, or 50%.

19. \$9 half-yearly, or \$18 yearly, is the interest on the difference between \$1200 and \$750, or \$450. The rate is 4%.

21. M does $\frac{1}{4}$ in a day; N, $\frac{1}{5}$; O, $\frac{1}{6}$; together, $\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ in 1 da., or $\frac{15+12+10}{60}=\frac{37}{60}$ in 1 da. To do the whole work will require 1 da. $\div \frac{37}{60} = \frac{60}{37}$ da. $= 1\frac{23}{37}$ da.

The teacher should not neglect to have her pupils make an estimate of the answer of each problem before beginning work. They will find in doing this, that they can solve many of the problems without using the pencil. It will be found useful as an occasional class exercise to require each pupil to write on paper at a signal, his estimate of, say, ten problems similar to some of the foregoing. Anything that will cause a pupil to read carefully and understandingly before setting his pencil to paper, will by of great value to him in problem work. See Art. 981.

1086. **1.** Let $x=$ first cost of the goods. Then, $\frac{24x}{100}=122.16$; $24x=12216$; $x=509$; cost $=$ \$509. The number of yards purchased $=32\times 32=1024$; invoice price per yard $=$ \$509 $\div 1024 = 49\frac{181}{256}$¢. Cost of the goods, including duties, etc., $=$ \$509 $+$ \$122.16 $+$ \$40.96 $=$ \$672.12; total cost per yard, $=$ \$672.12 $\div 1024 = 65\frac{163}{256}$¢.

2. The amount paid $=$ \$12500 $+$ 9 months' interest on \$3500 $+$ 18 months' interest on \$2600 $+$ 28 months' interest on \$2400.

3. At the end of 30 da., the provisions will last 1200 men 70 da. If half the men are then withdrawn, the remaining 600 will consume in 30 da. what 1200 men would use in 15 da., leaving 55 days' provisions for 1200 men. If the force is increased then by 900 men, there will be 1500 in the garrison. 55 days' provisions for 1200 men will last 1500 men $\frac{1200}{1500}$ of 55 da. $= \frac{4}{5}$ of 55 da. $= 44$ da. Total time, 30 da. + 30 da. + 44 da. = 104 da.

4. The ad valorem duty = 20% of $7¢ \times 267 \times 37$. The specific duty $= 5\frac{1}{2}¢ \times 267 \times 37 \times \frac{30}{36}$. The sum of both will be the entire duty.

5. $\frac{1}{5}$ of $\frac{3}{4}$ of $\frac{25}{8} = \frac{15}{32}$; $\frac{15}{32} \times \frac{1}{2}$ of $\frac{15}{32} \div \frac{1}{2}$.

6. $97\frac{1}{2}\%$ of amount = \$762742.50.

9. The tea cost 25¢ per lb. A gain of 15¢ = 60%.

1087. These exercises should be omitted if not required by the course of study.

	Face of Draft		+ Premium, or − Discount		− Interest	=	Cost of Draft
1.	100	+	.02			=	x
2.	x			−	$\frac{x}{1000}$	=	499.50
3.	1800	+	$\frac{9x}{5}$	−	18	=	1778.85
4.	x	+	$\frac{x}{400}$			=	701.75
5.	200	−	.20	−	1	=	x
6.	600	+	2.25	−	$36x$	=	598.95
7.	1000	−	.75	−	$60x$	=	999.25
8.	1200			−	$\frac{31x}{10}$	=	1178.30
9.	800	+	$\frac{4x}{5}$	−	3.20	=	796.80
10.	400	+	.80	−	$\frac{x}{10}$	=	400.30

2. Interest $= x \times \frac{6}{100} \times \frac{6}{360} = \frac{x}{1000}$.

3. The rate is x dollars per \$1000 premium; on \$1800, the premium $= \frac{1800}{1000} \times x = \frac{9x}{5}$. The interest on \$1800 for 60 days @ $6\% = 1\%$ of $\$1800 = \$18$. The equation becomes $\frac{9x}{5} + 1782 = 1778.85$; $\frac{9x}{5} = 1778.85 - 1782 = -3.15$; $9x = -15.75$; $x = -1.75$. The minus sign indicates that it is a discount. *Ans.* \$1.75 discount per \$1000.

6. Let $x =$ time in years. The equation becomes $602.25 - 36x = 598.95$; $-36x = 598.95 - 602.25 = -3.30$; $36x = 3.30$; $x = \frac{3.3}{36} = \frac{33}{360}$. *Ans.* $\frac{33}{360}$ year $= 33$ days.

7. $-.60x = 0$; $x = 0$ years; *i.e.* sight.

8. The interest $= 1200 \times \frac{x}{100} \times \frac{93}{360} = \frac{31x}{10}$. The equation becomes $\frac{31x}{10} = 21.70$; $31x = 217$; $x = 7$. *Ans.* 7%.

9. Rate $= x$ per M. Premium $= \frac{800}{1000} \times x = \frac{4x}{5}$; $x = 0$. *Ans.* Par.

1088. 4. 10 sq. ch. = 1 A. = 160 sq. rd. 1 sq. ch. = 16 sq. rd. 1 chain $= \sqrt{16}$ rods = 4 rods = $16\frac{1}{2}$ ft. × 4.

5. Each face contains 1350 sq. in. ÷ 6 = 225 sq. in. Length $= \sqrt{225}$ in. = 15 in.

6. Hypotenuse = 5 in.

7. Hypotenuse = $3\frac{1}{4}$ in.

1089. Another method of subdividing square C is here shown, Fig. 3. From each side of C, a right-angled triangle is cut, of the same dimensions as the original triangle m, leaving a small square X. In Fig. 4, is shown a rearrangement of these

Fig. 1.

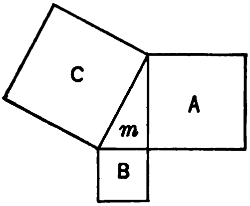


Fig. 2.

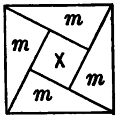


Fig. 3.

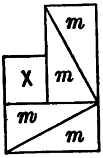


Fig. 4.

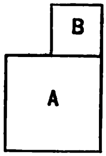


Fig. 5.

triangles, making a polygon equal in surface to the sum of the squares *A* and *B*, Fig. 5.

1091. 1. To find $\frac{1}{40}$, divide by 4, placing each quotient figure one place to the right. Nothing should be written beyond what is shown in the Arithmetic. To find the interest, see Art. 983, **27**.

2. 1% each quarter. In dividing by 100, the dividend is repeated, with each figure two places to the right.

3. 3% each half year. Multiply by 3, placing each figure of the product two places to the right.

$$
\begin{array}{rl}
\$1500. & \\
45. & \\
\hline
\$1545. & \tfrac{1}{2}\text{ year} \\
46.35 & \\
\hline
\$1591.35 & 1\text{ year.}
\end{array}
$$

1092. In some cities, the quotations of stocks give the price per share. A share whose par value is $50 and which sells for $48, is quoted in the New York papers as worth 96; *i.e.* 96%

of \$50. In a few places, the price is quoted as 48, meaning \$48 per share. In the examples given in the Arithmetic, the rate always means the per cent of the par value.

1. $(87\frac{3}{4} + \frac{1}{8})\%$ of $(\$10 \times 240)$.

2. Brokerage $= \frac{1}{4}\%$ of $(\$100 \times 120) = \$30$. Value of stock $= \$11460 - \$30 = \$11430$; value per share $= \$11430 \div 120 = \$95.25$.

3. Let $x =$ par value per share. Then $(87\frac{1}{2} + \frac{1}{4})\%$ of $(x \times 150) = 5265$; $\dfrac{x \times 150 \times 87\frac{3}{4}}{100} = \dfrac{x \times 150 \times 351}{100 \times 4} = 5265.$

Using the cancellation method, $x = \dfrac{5265 \times 100 \times 4}{150 \times 351}$.

4. $\frac{1}{8}\%$ of \$27500.

5. $102\frac{1}{4}\%$ of $(\$25 \times 200) =$ selling price. Deduct therefrom $\frac{1}{8}\%$ of $(\$25 \times 100)$.

6. His income is 18% of the par value $(\$100 \times 60) = \$1080$. His investment is 450% of $(\$100 \times 60) = \$27000$.

\$1080 is 4% (*Ans.*) of \$27000.

Or, irrespective of number of shares, $18\% \div 4\frac{1}{2} = 4\%$. *Ans.*

7. The stock $= \$50 \times 4000 = \$200000$. The rate of dividend $= \$20000 \div \$200000 = \frac{1}{10} = 10\%$. *Ans.* $10\% \div 1.75 = 5\frac{5}{7}\%$. *Ans.*

8. \$10000 interest must be paid to the bondholders, leaving (\$47500 − \$10000) \$37500 to be paid as dividends on \$1000000 of stock. $37500 \div 1000000 = .0375 = 3\frac{3}{4}\%$.

9. Let $x =$ brokerage. $\dfrac{168\frac{1}{2} + x}{100}$ of $25 \times 360 = 15176.25$; $9000\,(168\frac{1}{2} + x) = 1517625$; etc.

10. Let $x =$ amount of brokerage.

$$107\tfrac{3}{4}\% \text{ of } (100 \times 250) - x = 26875;$$
$$26937.50 - x = 26875;$$
$$x = 62.50;\ \text{brokerage} = \$62.50.\ \textit{Ans.}\quad \text{Etc.}$$

11. $\$35050 \div 1.75\frac{1}{4} = \$20000$ = par value of stock; $7\frac{1}{2}\%$ of $\$20000 = \$1500$. *Ans.*

12. Mr. Tower receives \$30 interest, and \$100, the face value of the bond, in six years, \$130 in all. His investment was \$104, on which he has received \$130 − \$104, or \$26 interest in 6 yr., or $\$4.33\frac{1}{3}$ per year. $4\frac{1}{3} \div 104 = .04\frac{1}{6} = 4\frac{1}{6}\%$. *Ans.*

1093. **1.** A corporation is an association of a number of persons legally empowered to transact business as a single individual. The charter specifies the name of the corporation, the amount of capital, the business it is authorized to carry on, the powers and privileges conferred, etc. The stock is the money invested in the business of the corporation; a share is one of the equal parts into which the stock is divided; a shareholder is the owner of one or more shares; a stockbroker is a person engaged in the business of buying and selling stocks on commission; a dividend is a *pro rata* division of profits among the stockholders; an assessment is a sum levied upon stockholders to meet some unexpected expenses, losses, etc.

2. Income from bonds $= 6\%$ of $\$125 \times 109 \div 1.09 = 6\%$ of $\$12500 = \$750$, an increase of \$68.75 over the previous income of \$681.25.

3. Sum loaned $= \$59.57 \div \frac{1}{6} = \$59.57 \times 6$.

4. In 3 yr. at 7%, \$1 will amount to \$1.21; *i.e.* a payment of \$1.21 in 3 yr. is considered equal to a cash payment of \$1. The "present worth" of the delayed payment $= \$4235 \div 1.21 = \$3500$, which is a loss of $\$3675 - \$3500 = \$175$. *Ans.*

5. The rate on one $= 5\% \div (.98\frac{1}{4} + .00\frac{1}{4}) = 5 \div .98\frac{1}{2}$; on the other $= 6\% \div 1.09$.

The cost of a \$100 5% bond is $\$98.25 + 25¢$ brokerage $= \$98.50$; the interest on the bond is 5% of $\$100 = \$5$; the rate is, therefore, $5\% \div .98\frac{1}{2}$; etc.

6. The time of the place being 1 hr. later than the time of the starting-point, the traveler is 15° east of the latter place.

8. As the note is stated to be due in 3 mo., which may be assumed to include days of grace where allowed, the date of its maturity is Sept. 20. The amount of the note for 92 da., at 9%, = \$2455.20. The discount on this sum at 6% from Aug. 8 to Sept. 20, 43 da. = \$17.60. Proceeds = \$2455.20 − \$17.60 = \$2437.60. *Ans.*

10. \$4800 less 63 days' interest, \$50.40, = \$4749.60.

(\$4797.58 less 60 days' interest, \$47.98, = \$4749.60.)

1097. **4.** Having found that $\sqrt{.10} = .316+$, the pupils should see that $\sqrt{.1} = .316+$.

In pointing off, begin at the units' place, pointing off two places to the right or the left.

7. 1.6 is made 1.60. See **10.**

1098. **7.** Find the amount of \$467.50 at 6% from July 5, 1881, to Dec. 19, 1885.

8. Amount stolen = \$650. The first sender is entitled to $\frac{150}{650}$ of \$523.25; the second, to $\frac{202}{650}$ of \$523.25; etc.

9. WASHINGTON, D.C., Dec. 1, 1896.

THE UNITED STATES

To JAMES RYAN, *Dr.*

	To Services, Nov. 19–30, 12 da.	45	65		
	" Traveling Expenses, 12 da., \$4.00	48	00		
	" Stage Fare (\$12.50) St'mb't Fare (\$17.40) Telegraph (\$3.25)	33	15		
				\$126	80

10. The account for the first quarter, exclusive of box rents, is \$124.96. On \$50, the commission allowed = \$50. On the remaining \$74.96, the commission is 60% = \$44.98. Total = \$77 + \$50 + \$44.98 = \$171.98.

Salary for the year = \$171.98 + \$194.01 + \$174.08 + \$167.87 = \$707.94.

1099. Base^2 + perpendicular^2 = hypotenuse^2; $H=\sqrt{B^2+P^2}$.

1. $225+64=x^2$; $x^2=289$; $x=17$, hypotenuse.

2. $1225+x^2=1369$; $x^2=1369-1225=144$; $x=12$, perpendicular.

3. $x^2+225=1521$; $x^2=1521-225=1296$; $x=36$, base.

1100. 11. Let x = perpendicular. Area $=\frac{1}{2}(200+160)\times x=180x=32400$; $x=180$.

12. $\frac{1}{2}(20+x)\times 15=225$; $\dfrac{300+15x}{2}=225$; $300+15x=450$; $15x=450-300=150$; $x=10$.

13. $\frac{1}{2}(x+x+6)\times 10=(x+3)\times 10=10x+30=150$; $10x=150-30=120$; $x=12$, $x+6=18$. 12 rd. and 18 rd. *Ans.*

14. The base = 7 rd.; the altitude, or perpendicular, = 24 rd. Area $=\frac{1}{2}(24\times 7)$ sq. rd.

15. Number of stones $=(84\times 36)\div(6\times 3)$. Cost $=\$1\frac{1}{4}\times(28\times 12)$.

16. The angle of 90° indicates a right-angled triangle. Hypotenuse = 20 chains. Number of chains of fence $=12+16+20=48$. 1 chain = 66 ft. = 4 rd. Number of rods $=48\times 4=192$.

17. A perpendicular let fall from the upper right corner would form a right-angled triangle, whose perpendicular is 40 ft., base 30 ft., making the hypotenuse $=\sqrt{1600+900}$ ft. = 50 ft., the fourth side.

The number of yards of fence $=(40+70+50+100)\times 5\frac{1}{2}$.

Area in acres $=[\frac{1}{2}$ of $(70+100)\times 40]\div 160$.

18. The diagonal is the hypotenuse of a right-angled triangle whose other sides measure, respectively, 90 yd. and 120 yd.

19. The diagonal $=\sqrt{1296+729}$ chains = 45 chains. The distance by the road = 27 chains + 36 chains = 63 chains. The saving is 18 chains, of 22 yd. each.

20. A 40-acre field contains 6400 sq. rd. Each side measures $\sqrt{6400}$ rd. $= 80$ rd. The diagonal $= \sqrt{80^2 + 80^2}$.

1101. 1. The loss is $\frac{1}{6}$ of the cost, \$300.

2. If 3 boys solve 3 problems in 3 min., 1 boy will solve 1 problem in 3 min., and 6 boys will solve 6 problems in 3 min.

Or, 3 boys will solve 6 problems in 6 min., therefore 6 boys will solve 6 problems in 3 min.

3. The 3 boys eat 12 cakes, 4 each; for which the third boy pays 12¢, or 3¢ apiece. The boy that brought 7 cakes supplies 3; for which he should receive 9¢. The other boy furnishes 1, and is entitled to 3¢.

4. The cost of the article is 50¢; a sale for \$2.00 is a gain of \$1.50, or 300%.

6. A 6-ft. fence needs 2 posts, one at each end; a 12-ft. fence, 3 posts; a 30-ft. fence, 6 posts.

7. One half is profit, the other half is the cost. The profit equals the cost, and is 100%.

8. $\frac{1}{2}x \times \frac{3}{10}x = \frac{3}{20}x^2 = 60$; $3x^2 = 1200$; $x^2 = 400$; $x = 20$.

9. $\dfrac{3x}{400} = 90.$

10. The difference between 28 and 75, inclusive, $= (75 - 28) + 1$.

1102. 2. The quantity of provisions for each man $= 2\frac{1}{4}$ lb. $\times 20 = 45$ lb. To last 24 da., the allowance should be 45 lb. $\div$ 24 $= 1\frac{7}{8}$ lb. $= 1$ lb. 14 oz.

5. $80\frac{1}{2}$ min. $\times 112 \div 46$. Cancel.

6. See Arithmetic, Arts. 821, 822.

7. First beam contains $(66 \times 10 \times 8)$ cu. in. The second contains $(x \times 12 \times 12)$ cu. in. The contents of the latter $= \frac{3024}{924}$ times the contents of the former.

$$x \times 12 \times 12 = \frac{3024 \times 66 \times 10 \times 8}{924},$$

$$x = \frac{3024 \times 66 \times 10 \times 8}{924 \times 12 \times 12} = 120.$$

Ans. 120 in. = 10 ft.

9. The weight of the provisions = 3 lb. $\times 32 \times 45$. Dividing by 40 gives the daily allowance for the increased crew; dividing this by their number, $32 + 16$, gives the allowance of each.

$$\frac{3 \text{ lb.} \times 32 \times 45}{40 \times 48}.$$

12. The difference in deposits = \$450, which sum produces \$18 interest. The rate is 4%.

14. \$7500 at $x\%$ for $3\frac{1}{3}$ years produces \$1125 interest.

15. $\left(\frac{5}{100} \text{ of } \frac{x}{2}\right) - \left(\frac{4}{100} \text{ of } \frac{x}{2}\right) = 40;\ \frac{5x}{200} - \frac{4x}{200} = \frac{x}{200} = 40;$ $x = 8000$.

16. $[\frac{5}{100} \text{ of } (x + 400)] - (\frac{4}{100} \text{ of } x) = 30;$

$$\frac{5x + 2000}{100} - \frac{4x}{100} = 30,$$

$$5x + 2000 - 4x = 3000,$$

$$x = 1000,$$

$$x + 400 = 1400.$$

Ans. \$1000 at 4%, \$1400 at 5%.

17. $\left(\frac{4}{100} \text{ of } \frac{4x}{5}\right) + \left(\frac{5}{100} \text{ of } \frac{x}{5}\right) = 2940.$

19. A receives $\frac{320}{800}$ of selling price; he invested, therefore, $\frac{320}{800}$ of \$600; etc.

20. F receives $\frac{7}{15}$. E's share is $\frac{2}{15}$ more than D's. If $\frac{2}{15}$ = \$90, $\frac{1}{15}$ = \$45; F's share, $\frac{7}{15}$, = \$315.

21. The bank receives at the end of 63 days, the sum loaned, \$593.70, + \$6.30 interest. The problem is: At what rate will \$593.70 produce, in 63 days, \$6.30 interest?

$$593.70 \times \frac{x}{100} \times \frac{63}{360} = 6.30,$$

$$x = \frac{630 \times 100 \times 360}{59370 \times 63} = 6\tfrac{126}{1979}.$$

22. The two supply pipes fill $\frac{1}{2} + \frac{1}{3}$, or $\frac{5}{6}$, of tank in 1 hour. If all the pipes are set to work when the tank is full, the exhaust pipe takes off each hour $\frac{1}{6}$ of the tank more than the others supply. To empty the tank would, therefore, require 6 hours.

1103. **7.** The diagonal $= \sqrt{137\frac{1}{2}^2 + 137\frac{1}{2}^2}$.

9. Let $100x =$ value. Buying price $= 90x$; selling price $= 110x$; gain $= 20x$, which is $\frac{2}{9}$ of the cost, $90x$, or $22\frac{2}{9}\%$.

10. $420¢ \div (\frac{3}{4}¢ - \frac{2}{5}¢) =$ number.

1106. **1.** $\$500 \div \frac{5}{4} =$ cost of one portion; $500 \div \frac{5}{6} =$ cost of other portion.

4. If $\frac{2}{3}$ of farm is sold for $\frac{3}{4}$ cost, the selling price of the whole farm at the same rate would be $\frac{3}{4}$ cost $\div \frac{2}{3} = \frac{9}{8}$ cost, making the gain $\frac{1}{8}$ cost $= 12\frac{1}{2}\%$.

7. $10¢$ per bu. of 60 lb. $= 16\frac{2}{3}¢$ per 100 lb., or $\frac{2}{3}¢$ higher than $16¢$ per 100 lb., or $\frac{1}{24}$ of $16¢$ higher, or $4\frac{1}{6}\%$.

8. See Art. 1084, **15**.

10. If $\frac{2}{5}$ of the selling price is profit, the cost must be $\frac{3}{5}$ of the selling price; the latter is therefore $\frac{5}{3}$ of the cost. This is a profit of $\frac{2}{3}$ of the cost, or $66\frac{2}{3}\%$. *Ans.*

Or, a profit of 2-fifths on a cost of 3-fifths is $\frac{2}{3}$ of the cost.

11. $x + 4\frac{1}{2}x = 60\frac{1}{2}$.

14. $1\frac{1}{2}\%$ of $\frac{2x}{3} = 150$.

17. See Arithmetic, Art. 915, **6–8**.

22. $(100 \div 5\frac{1}{2})$ years. The $\$200$ is unnecessary.

23. $(96 \times 48 \times 45) \div (231 \times 31\frac{1}{2})$. Cancel.

24. 20 acres $= 3200$ sq. rd. Each side measures $\sqrt{3200}$ rd. The diagonal = the square root of the sum of the squares of two equal sides. The square of each $= 3200$.

The diagonal $= \sqrt{3200 + 3200}$ rd. $= \sqrt{6400}$ rd. $= 80$ rd.

NOTE. — By constructing a square on the diagonal of a square, the pupils will see that the former will be twice as large as the latter; that is, that a square on the diagonal of the above will contain 6400 sq. rd., making each side 80 rd.

In the above example, the square root of 3200 should not be extracted.

1107. **6.** Find the proceeds of \$1572.50 for 81 da. (Omitting days of grace, the proceeds for 78 da. = \$1552.06.)

9. One costs $85\frac{1}{4}\%$ of \$1500; the other costs $102\frac{1}{4}\%$ of \$1300.

10. (\$2562.50 ÷ 1.025) ÷ \$62.50 = number of acres.

1. It is frequently difficult, for various reasons, to measure the altitude of a triangular field. On this account, a method of determining the area when the lengths of the sides are given, is useful, even though the underlying principles be not understood. A pupil can satisfy himself as to its accuracy, by calculating the area of the right-angled triangle in **3**, and of the isosceles triangle in **5**.

2. The half sum $= \frac{1}{2}$ of $(35 + 84 + 91) = 105$. The remainders are: $(105 - 35)$ 70, $(105 - 84)$ 21, and $(105 - 91)$ 14. Area $= \sqrt{105 \times 70 \times 21 \times 14}$ sq. ft. $= 1470$ sq. ft.

3. This is a right-angled triangle, since $21^2 + 28^2 = 35^2$; its area, therefore, is $\frac{1}{2}$ of (21×28) sq. rd., or 294 sq. rd.

By the other method, the area $= \sqrt{42 \times 21 \times 14 \times 7}$ sq. rd., or 294 sq. rd.

4. The sides of one triangle measure 39, 52, and 65 rd. respectively. Its area $= \sqrt{78 \times 39 \times 26 \times 13}$ sq. rd. $= 1014$ sq. rd. The sides of the other are 25, 60, and 65 rd., respectively; and its area $= \sqrt{75 \times 50 \times 15 \times 10}$ sq. rd. $= 750$ sq. rd. The area of the quadrilateral $= 1014$ sq. rd. $+ 750$ sq. rd. $= 1764$ sq. rd.

Each of these triangles is right-angled, AC being their common hypotenuse. Their areas are $\frac{1}{2}$ of (39×52) sq. rd., and $\frac{1}{2}$ of (25×60) sq. rd., respectively.

5. Since the altitude AC, Fig. 2, Arithmetic, Art. 1263, of an isosceles triangle divides the base into two equal parts, $BC = 15$ yd. $AC^2 = AB^2 - BC^2 = 625 - 225 = 400$; $AC = 20$ yd. The area $= \frac{1}{2}$ of 600 sq. rd. $= 300$ sq. rd.

The area by the other method $= \sqrt{40 \times 10 \times 15 \times 15}$ sq. yd.

6. In the right-angled triangle ACB (Art. 1263, Fig. 2), $BC = \frac{1}{2}$ of 96 ft. $= 48$ ft; $AB = 64$ ft.; $AC = \sqrt{64^2 - 48^2}$ ft. $= \sqrt{4096 - 2304 = 1792}$ ft. The area $= \frac{1}{2}$ of (96×42.332) sq. ft.

7. $\frac{1}{2}$ sum $= 9$ ft. Area $= \sqrt{9 \times 3 \times 3 \times 3}$ sq. ft.

8. The base $= \sqrt{70^2 - 42^2}$ ft. $= 56$ ft. Area $= \frac{1}{2}$ (42×56) sq. ft.

9. $\sqrt{50^2 - 48^2} = 14$, the number of feet in one-half the base. See Fig. 2, Arithmetic, Art. 1263.

10. The common base of the two triangles will be one diagonal, 2 in. The altitude of a triangle will be one-half the other diagonal $= \sqrt{2^2 - 1^2}$ in. $= \sqrt{3}$ in. $= 1.732$ in. The diagonal $= 3.464$ in. The area of each triangle $= \frac{1}{2}$ (2×1.732) sq. in. $= 1.732$ sq. in. The area of the rhombus $= 3.464$ sq. in.

11. The scholars should be led to ascertain for themselves the approximate relation between the diameter of a circle and its circumference. Place a point marked on the circumference of a spool, or something similar, on a given point on the surface of a sheet of paper. Roll the spool until the point on the circumference again touches the surface of the paper. The distance between the two points on the paper will be equal to the circumference of the circle. Measure this distance very carefully, also the diameter of the circle, and ascertain the ratio.

12. See Art. 1274, **6–14**, for the reason for the rule for determining the area of a circle.

13. The $\frac{1}{2}$ circumference $= \frac{1}{2}$ $(2x \times 3.1416) = 3.1416x$; the $\frac{1}{2}$ diameter $= x$; the area $= 3.1416x \times x = 3.1416x^2$.

14. $3.1416x^2 = \text{area} = 314.16.$

$$x^2 = 100; \; x = 10.$$

15. Diameter $= 15.708$ ft. $\div 3.1416 = 5$ ft.

$$\text{Area} = [(\tfrac{1}{2} \text{ of } 15.708) \times (\tfrac{1}{2} \text{ of } 5)] \text{ sq. ft.}$$

1108. The balance, \$851.72, is found by adding the credits, and subtracting from \$2535.35 in one operation. See Art. 384.

1110. 1. Amount of \$500, July 25 to

April 1, 250 da.,		\$520.83
Amount of \$100, Sept. 18 to April 1, 195 da.,	\$103.25	
" " \$200, Feb. 5, " " " 55 "	201.83	305.08
Due April 1, 1894,		\$215.75

3. Amount of \$600 for 354 da.,		\$635.40
" " \$300 " 152 "	\$307.60	
" " \$200 " 57 "	201.90	509.50
Due at settlement,		\$125.90

5. In the debit column, place the interest for 329 da., \$46.06. The total of this column is written on the same line as the total of the credit column, and is \$886.06. The amount is also written as the total of credits. The total interest (\$28.38) on \$500 for 297 da., \$24.75, and on \$200 for 109 da., \$3.63, is written among the credits; and the cash payment is ascertained by writing in its place the sum necessary (\$157.68) to make the total, \$886.06. See Art. 384.

6. From the amount of \$725 for 234 da. + the amount of \$603 for 174 da., take the amount of \$600 for 183 da. + the amount of \$300 for 37 da.

1113. 3. The field contains 160 sq. rd. $\times 7\frac{1}{2}$. Its length is $(160 \times 7\frac{1}{2} \div 30)$ rd. $= 40$ rd. The diagonal $= \sqrt{40^2 + 30^2}$ rd. $= 50$ rd.

4. The rate per cent that will produce \$36 interest in $1\frac{2}{3}$ yr. is $7\frac{1}{5}$. The interest on \$212.50 at $7\frac{1}{5}\%$ for the given time $=$ \$52.02. *Ans.*

Or, the problem may be solved as follows (Arithmetic, Art. 974):

Interest on \$300 for 20 mo. is \$36,
" " \$212.50 " $40\frac{4}{5}$ " " ?

$$\frac{\$36 \times 212.50 \times 40\frac{4}{5}}{300 \times 20} = \frac{\$36 \times 212.50 \times 204}{300 \times 20 \times 5}.$$

5. The date not being given, the number of days is taken as $120 + 3$. The proceeds for 120 da. $= \$490$.

8. The interest on \$635 for 205 da. at 5% $= \$18.08$.

Amount $= \$635 + \$18.08 = \$653.08$.

XVII

NOTES ON CHAPTER FOURTEEN

1115. 1. The total interest on the given sums of money is equal to the interest of \$3000 for 1 mo. As the total sum is \$1000, the interest of \$3000 for 1 mo. equals the interest on \$1000 for 3 mo.

1116. 3. Since the time is required, the products may be expressed in years (or months or days), and their total divided by the total of the multipliers.

$$\begin{array}{lcrcl}
2 & \text{yr.} \times & 600 & = & 1200 \text{ yr.} \\
1\frac{1}{2} & \text{"} \times & 500 & = & 750 \text{ "} \\
1 & \text{"} \times & 300 & = & 300 \text{ "} \\
\frac{3}{4} & \text{"} \times & 400 & = & 300 \text{ "} \\
\hline
? & \times & 1800 & = & 2550
\end{array}$$

Ans. 2550 yr. ÷ 1800 = 1 yr. 5 mo.

5. [(15 da. × 200) + (30 da. × 300) + (45 da. × 400)] ÷ (200 + 300 + 400).

6. [(1 mo. × 210) + (2 mo. × 210) + (3 mo. × 210) + (4 mo. × 210)] ÷ 840.

Since the sum due at each period is the same, the 210 may be omitted; (1 mo. + 2 mo. + 3 mo. + 4 mo.) ÷ 4.

7. [(2 mo. × 320) + (4 mo. × 160) + (5 mo. × 240) + (6 mo. × 240)] ÷ 960.

Or, (2 mo. × $\frac{1}{3}$) + (4 mo. × $\frac{1}{6}$) + (5 mo. × $\frac{1}{4}$) + (6 mo. × $\frac{1}{4}$) = $4\frac{1}{12}$ mo. *Ans.*

8. (2 mo. × $\frac{1}{10}$) + (3 mo. × $\frac{1}{5}$) + (4 mo. × $\frac{1}{6}$) + (12 mo. × $\frac{8}{15}$) = $7\frac{13}{15}$ mo. = 7 mo. 26 da. *Ans.*

9. (0 mo. $\times \frac{1}{3}$) + (3 mo. $\times \frac{1}{6}$) + (6 mo. $\times \frac{1}{6}$) + (9 mo. $\times \frac{1}{3}$) = $4\frac{1}{2}$ mo. *Ans.*

10. [(0 da. × 300) + (30 da. × 800) + (60 da. × 1000)] ÷ (300 + 800 + 1000) = 40 da. July 1 + 40 da. = Aug. 10. *Ans.*

1117. **11.** The total amount received ÷ number of bushels sold = average price.

The arrangement of the work may follow that given in Art. 1116, Problem 3.

12. The first puts in the equivalent of 180 cows for a week; the second, 120 cows for a week; the third, 180 cows for a week; a total of 480 cows for a week. The first should pay $\frac{180}{480}$ of \$84; the second, $\frac{120}{480}$ of \$84; the third, $\frac{180}{480}$ of \$84.

$$
\begin{aligned}
15 \times 12 &= 180 \\
20 \times \ \ 6 &= 120 \\
18 \times 10 &= \underline{180} \\
480 : 180 &:: \$84 : x \\
480 : 120 &:: \$84 : y \\
480 : 120 &:: \$84 : z
\end{aligned}
$$

Proportion is commonly employed in working examples of this kind. As the whole number for a week (480) is to the first man's number for a week (180) so is the whole rent (\$84) to the first man's share (x). The second proportion is used to ascertain the second man's share (y); and the third, to ascertain the third man's share (z).

13. A furnishes \$2000 for 2 yr. and \$1000 for 1 yr., which is the equivalent of \$5000 for a year. B furnishes the equivalent of \$6000 for a year. The total is \$11000 for a year, and the profits of \$1100 are distributed in the ratios of $\frac{5000}{11000}$ and $\frac{6000}{11000}$, as indicated by the proportions here given. A receives \$500 of the profits and his capital of \$3000, or \$3500 in all. B receives \$3600.

$$
\begin{aligned}
2000 \times 2 &= 4000 \\
1000 \times 1 &= \underline{1000} \quad 5000 \\
3000 \times 2 &= \qquad\quad \underline{6000} \\
11000 : 5000 &:: \$1100 : \text{A} \\
11000 : 6000 &:: \$1100 : \text{B}
\end{aligned}
$$

14.
$$40x + 30(100 - x) = 36 \times 100,$$
$$40x + 3000 - 30x = 3600,$$
$$40x - 30x = 3600 - 3000,$$
$$10x = 600,$$
$$x = 60 = \text{number of bushels at } 40¢,$$
$$100 - x = 40 = \text{number of bushels at } 30¢.$$

15. $(60 \times x) + (50 \times 80) = 52(x + 80).$
$$60x + 4000 = 52x + 4160;\ \text{etc.};$$
that is, x bu. @ 60¢ + 80 bu. @ 50¢ = $(x + 80)$ bu. @ 52¢.

16. A does $\frac{1}{20}$ in 1 da.; B does $\frac{1}{30}$ in 1 da.; both do $\frac{1}{20} + \frac{1}{30} = \frac{1}{12}$ in 1 da. They finish the work, therefore, in 12 da., and receive \$60. If A does $\frac{1}{20}$ in 1 da., in 12 da. he does $\frac{12}{20}$, and should receive $\frac{12}{20}$ of \$60 = \$36. B should receive \$24.

17. C's capital of \$4000 for $\frac{1}{2}$ yr. may be considered as \$2000 for a year. This, with \$4000 furnished by A and B, makes the capital \$6000. A takes $\frac{1500}{6000}$, or $\frac{1}{4}$ of the profits; etc. See **13.**

18.
600 : 180 :: 180 tons : A's share.
600 : 105 :: 180 tons : B's share.
600 : 315 :: 180 tons : C's share.

19. Let x = number of quarts of water, then $40 - x$ = number of quarts of milk. 40 quarts of the adulterated article at 5¢ per quart = 200¢. $(40 - x)$ quarts of milk at 6¢ per quart = $(240 - 6x)$¢. x quarts of water cost nothing.
$$200 = 240 - 6x,$$
$$6x = 40,$$
$$x = 6\tfrac{2}{3}.$$
The can contains $6\frac{2}{3}$ qt. water and $33\frac{1}{3}$ qt. milk.

1120. **1.** \$999 = rent for 1 yr. 10. mo. 6 da., or $22\frac{1}{5}$ mo. The rent for 1 mo. = $\$999 \div 22\frac{1}{5} = \$999 \times \frac{5}{111}$; for 12 mo. = $\$999 \times \frac{5}{111} \times 12$.

2. What per cent of 55 oz. is 121 oz.?
$$\frac{x}{100} \text{ of } 55 = 121;\ 55x = 12100;\ x = 220.$$

3. The field contains 1600 sq. rd.; each side measures 40 rd., or 220 yd.; etc.

4. A solves 20 per hour; B solves $15 \times \frac{12}{11}$, or $16\frac{4}{11}$, per hour; both solve $36\frac{4}{11}$ per hour. To solve 100 will require $(100 \div 36\frac{4}{11})$ hours.

6. Six faces, each containing (15×15) sq. in.

7. Cost 90¢. See Arithmetic, Art. 924, **7**. Selling price $= 90¢ \times 1.43\frac{1}{3}$.

8. March 4, 1861, + 4 yr. 1 mo. 11 da. = Apr. 15, 1865; Apr. 15, 1865, − 56 yr. 2 mo. 3 da. = Feb. 12, 1809. *Ans.*

9. The selling price, $\$4.50 = 90\%$ of cost; the latter is, therefore, $5 per barrel. Loss per barrel, 50¢; on 50 bbl., $25.

A profit of $6\% = 30¢$ per barrel; the gain on 100 bbl. = $30. Net gain = $5. *Ans.*

10. 60% of $66\frac{2}{3}\% = 40\%$. If $\frac{40}{100}$ of a number $= 810$, the number $= 810 \times \frac{100}{40} = 2025$.

NOTE. — Observe the difference between this example and **8** of Art. 1101.

1121. **1.** Having used $\frac{7}{8}$ of $\frac{4}{7}$ box, the remainder $= \frac{1}{8}$ of $\frac{4}{7}$ box $= \frac{1}{14}$ box. If $\frac{1}{14}$ box $= \$\frac{17}{25}$, a box $= \$\frac{17}{25} \times 14 = 68¢ \times 14 =$ $9.52. *Ans.*

3. The 40-ft. ladder forms the hypotenuse of two right-angled triangles; CE and DE, Arithmetic, Art. 1250, Problem **9**. CA, one perpendicular, measures 21 ft.; DB, the other, measures 33 ft. AB is the width of the street.

$$AE = \sqrt{40^2 - 21^2}\,;\ EB = \sqrt{40^2 - 33^2}\,;\ AE + EB = AB.$$

4. 16 oz. : 12 oz. :: $28 : x.

5. A partnership is the association of two or more persons for the transaction of business on joint account. One advantage of a partnership is the employment of a larger capital, with a smaller percentage of expenses than would be the case were each member to establish a separate business. The firm obtains the combined business experience of its several members, and can

utilize the services of each in the department in which he can best serve the interests of the firm; etc.

NOTE. — In the absence of an agreement to the contrary, each partner is entitled to an equal share of the profits, and is liable for an equal portion of the losses; in the examples given, however, the gains and the losses are distributed according to the amount invested by each and the length of time each one's capital remains in the business. See also Art. 977.

6. A, $\frac{16}{104}$ of $13; B, $\frac{24}{104}$; C, $\frac{28}{104}$; D, $\frac{36}{104}$; etc.

7. The analysis of a mathematical problem or operation should be occasionally used as an exercise in composition, the same attention being given to penmanship, spelling, language, and arrangement as in other such exercises.

1122. 1. Make *men* the last term; Art. 974, 8.

If 1 be eaten in 35 da. at 24 oz. daily by 3600 men,
2 will be eaten in 45 da. at 14 oz. daily by ? men.

$$\frac{3600 \text{ men} \times 35 \times 24 \times 2}{45 \times 14} = 9600 \text{ men. } \textit{Ans.}$$

4. $\$3700 \div (1 - .075)$.

8. See Arithmetic Art. 1005, 2. £500 = ½ of $4866.50. Take £250, £25, £5, 10*s*., 5*s*., 2*s*. 6*d*., 1*s*. 3*d*., 2*d*.

Or,

$$\begin{array}{r} 20)\underline{\$4.8665} \times 780 = \\ 12)\underline{\$\ \ .2433} \times \ \ 18 = \\ \$\ \ .0203 \times \ \ 11 = \end{array}$$

9. See Art. 1250, 8. Calling the part remaining x, the part broken off will be $100 - x$. The latter is the hypotenuse of a right-angled triangle; x is the perpendicular; 40 is the base.

$$\begin{aligned} 40^2 + x^2 &= (100 - x)^2, \\ 1600 + x^2 &= 10000 - 200x + x^2, \\ 200x &= 10000 - 1600 = 8400, \\ x &= 42, \\ 100 - x &= 58. \end{aligned}$$

The length of the part broken off = 58 ft. *Ans.*

By assuming x as the length of the part broken off, the hypotenuse $= x$, and the perpendicular $= 100 - x$.

$$x^2 = 40^2 + (100 - x^2),$$
$$x^2 = 1600 + 10000 - 200x + x^2,$$
$$200x = 11600,$$
$$x = 58.$$

10. Number of feet in length $= 10 \div (\frac{15}{12} \times \frac{11}{12})$.

13. At $1\frac{1}{2}d.$ per pound, 2240 lb. are worth $3360d. =$ £14. Cost of 120 T. = £1680; the duty in English money is $\frac{1}{5}$ of £1680 = £336; freight = £30; etc.

15.

Dr. U.S. TREASURY DEPARTMENT. *Cr.*

1883.					1883.					
Jan.	3	To 2575 lb. Twine .12	309	—	Feb.	4	By Cash	175	—
Apr.	4	" 25 doz. Pens 25.—	625	—	Apr.	30	" Cash	350	—
May	7	" 645 reams Paper 2.—	1290	—	July	15	" Cash	700	—
July	9	" 45 doz. Ink 3.—	135	—	Nov.	5	" Cash	2300	—
Oct.	30	" 1000 M Envelopes 2.—	2000	—	Dec.	31	" Breakage	75	—
Dec.	5	" 8 doz. Inkstands 1.97	15	76	Dec.	31	" Shortage	60	—
Dec.	31	" Cartage	45	—	Dec.	31	" *Balance*	759	76
			4419	76				4419	76
1884.									
Jan.	1	To Balance	759	76					

The above represents the account as it stands upon the books of Samuel Adams. The debit column contains the amounts due from the Treasury Department, and the credit column contains the sums received, etc. The account is balanced by placing the footing of the debit column under each, and by writing in the credit side the words "By Balance" in red ink, followed by the sum necessary to make the total of the credit column equal to the total of the debit column. Red ink is used to show that the money has not been paid. The account is reopened by writing "To Balance" on the debit side, followed by the sum remaining due.

The statement rendered by Samuel Adams to the Treasury Department would be made out as follows:

WASHINGTON, D.C., Jan. 2, 1884.

U.S. TREASURY DEPARTMENT,

In Account with SAMUEL ADAMS.

1883.		—— *Dr.* ——				
Jan.	3	To Mdse., as per bill rendered	309	—		
Apr.	4	" " " " " "	625	—		
May	7	" " " " " "	1290	—		
July	9	" " " " " "	135	—		
Oct.	30	" " " " " "	2000	—		
Dec.	5	" " " " " "	15	76		
Dec.	31	" Allowance for Cartage	45	—	4419	76
1883.		—— *Cr.* ——				
Feb.	4	By Cash	175	—		
Apr.	30	" "	350	—		
July	15	" "	700	—		
Nov.	5	" "	2300	—		
Dec.	31	" Breakage 75.— Shortage 60.—	135	—	3660	—
		Balance due			759	76

The quantities and prices are omitted, as they have been given in the bills rendered at the time the articles were supplied.

For the form of this account as it would appear on the books of the Treasury Department, see Art. 1146, **24**.

1123. **1.** $5280 \div 15.7$.

The owner of a cyclometer should calculate the number of revolutions of a wheel necessary to move the index of the cyclometer over one of its smallest divisions. The circumference of a wheel, 26 in. in diameter, measures nearly $2\frac{1}{4}$ yd. Nine revolutions of the wheel should indicate a trifle over 20 yd. on the cyclometer; 8 revolutions should indicate a trifle over $\frac{1}{100}$ mile, 17.6 yd.

3. The premium $= \frac{3}{400}$ of $\frac{3}{4}$ of $\$6000 = \$33.75$. The loss will be the value of the uninsured one-quarter, \$1500, and the above premium.

4. $\$1.40 \times (5\frac{1}{3} \times 5\frac{1}{2}) \times (8\frac{1}{2} \div 3)$.

$$\frac{\$1.40 \times 16 \times 11 \times 17}{3 \times 2 \times 2 \times 3}$$

The rods are reduced to yards by multiplying by $5\frac{1}{2}$; the feet, by dividing by 3.

5. Let $x =$ the smaller number; $x + 1$ bu. 2 pk. 5 qt. $=$ the larger one.

$$x + x + 1 \text{ bu. } 2 \text{ pk. } 5 \text{ qt.} = 12 \text{ bu. } 1 \text{ pk. } 3 \text{ qt.},$$
$$2x = 12 \text{ bu. } 1 \text{ pk. } 3 \text{ qt.} - 1 \text{ bu. } 2 \text{ pk. } 5 \text{ qt} = 10 \text{ bu. } 2 \text{ pk. } 6 \text{ qt.},$$
$$x = 5 \text{ bu. } 1 \text{ pk. } 3 \text{ qt} = \text{the smaller number},$$
$$x + 1 \text{ bu. } 2 \text{ pk. } 5 \text{ qt.} = 7 \text{ bu.} = \text{the larger number.}$$

8. Each side $= 40$ rd. Area $= 1600$ sq. rd. $= 10$ A.

9. See Art. 1097. $\sqrt{.44'\,10}$.

10. 7000 gr. $\times\, 17\frac{11}{16} =$ the number of Troy grains. Reduce to pounds, ounces, pennyweights, etc.

11. At what rate (x) will \$324.61 in 2 yr. 7 mo. 13 da. produce (\$384.13 − \$324.61) interest.

$$324.61 \times \frac{x}{100} \times \frac{943}{360} = 59.52;$$

Canceling decimals, $30610723\,x = 214272000$;

$$x = 7 \text{ nearly.}$$

12. $\frac{7}{800}x = 140$.

13. The field contains 6400 sq. rd.; each side measures 80 rd.; the diagonal $= \sqrt{6400 + 6400}$ rd.

14. See Arithmetic, Art. 591.

15. One edge measures $\sqrt{256}$ ft. $= 16$ ft. Solid contents $= (16 \times 16 \times 16)$ cu. ft., or (256×16) cu. ft.

16. Cost and selling price.
Cost and profit (or loss).
Selling price and profit (or loss).

17. $92\text{¢} \times 21643 \div 60$.

18. $\frac{1}{2}$ of $(22\frac{7}{8} \times 19\frac{3}{4})$ sq. ft.

19. See Arithmetic, Art. 924, **7**.

21. 14 da. in Oct. $+ 30 + 31 + 31 + 28 + 31 + 24$ in April exclusive of April $25 = 189$ da. 9 tons will be needed.

22. 112 A. 96 sq. rd. $= 112.6$ A. Remainder $= 112.6$ A. $- 48.64$ A. $= 63.96$ A.; $63.96 = \frac{x}{100}$ of 112.6; $x = 6396 \div 112.6 = 63960 \div 1126 = 56\frac{452}{563}$. *Ans.* $56\frac{452}{563}\%$.

23. By 4-ft. wood is meant that the sticks are 4 ft. long. This makes the pile 4 ft. wide. Cancel.

25. See Art. 1117, **12**.

$1 = 18 \times 4 =$	72	$= 1008$ fourteenths.	Share	$= \frac{1008}{4905} = \frac{112}{545}$,
$2 = 12\frac{4}{7} \times 5 =$	$62\frac{6}{7}$	$= 880$	"	"	$= \frac{880}{4905} = \frac{176}{981}$,
$3 = 6\frac{1}{2} \times 11 =$	$71\frac{1}{2}$	$= 1001$	"	"	$= \frac{1001}{4905}$,
$4 = 16 \times 9 =$	144	$= 2016$	"	"	$= \frac{2016}{4905} = \frac{224}{545}$.
Total,	$349\frac{19}{14}$	$= 4905$	"		

26.

$$\tfrac{1}{2}(48 \times \text{perpendicular}) = 160 \text{ rd.} \times 13\tfrac{1}{2},$$

$$24 \times \text{perpendicular} = 160 \text{ rd.} \times 13\tfrac{1}{2},$$

$$\text{perpendicular} = \frac{160 \text{ rd.} \times 27}{24 \times 2} = 90 \text{ rd.}$$

Hypotenuse $= \sqrt{90^2 + 48^2}$ rd. $= 102$ rd. Length of fence $= 48$ rd. $+ 90$ rd. $+ 102$ rd.

1124. **2.** $\frac{1}{2}$ circumference $= 3.1416x$; $\frac{1}{2}$ diameter $= x$; area $= 3.1416x^2$.

NOTE. — The pupil should memorize the ratio between the circumference and the diameter, 3.1416. After learning from Art. 1274, **6–14**, that the area of a circle is equal to the product of the semi-circumference by the semi-diameter, and ascertaining from **2** that this is equal to the square of the radius multiplied by 3.1416, the latter rule can occasionally be employed. See **6**.

4. When the circumference is x, the diameter $= \frac{x}{3.1416}$. Then $\frac{x}{2} \times \frac{x}{3.1416 \times 2} = \frac{x^2}{12.5664} = .07958x^2$.

5. $18^2 \times 3.1416$.

6. $R^2 \times 3.1416 = 153.9384$; $R^2 = 153.9384 \div 3.1416 = 49$; $R = 7$. *Ans.* 7 yd.

7. Let x = circumference; then, from **4**, $\frac{x^2}{12.5664}$ = area = 198.95; $x^2 = 198.95 \times 12.5664 = 2500.08528$; $x = 50.008$. *Ans.* 50 rd.

8. The square of the diagonal, x^2, = twice the square of a side. The square of a side is, therefore, $\frac{x^2}{2}$, which is the area of the square.

9. Let the pupil draw a square. On its diagonal, which may be called 150 rd., draw another square. Produce two sides of the smaller square so as to make diagonals of the larger one. An examination of the small square will show that its area is one-half that of the other, or $\frac{1}{2}$ of (150×150) sq. rd.

10–12. See **6**, **7**, and **1**, of Measurements, Art. 1107.

13. See **4**. 100 sq. ft. $\div$ 12.5664 = 7.958 sq. ft. *Ans.*

Area = circumference^2 $\times$ .07958.

14. The altitude $= \sqrt{625 - 49} = 24$. Area $= (40 \times 24)$ sq. rd.

15. Find the perpendicular, $\sqrt{100^2 - 80^2}$; etc.

16. See **10** of Measurements, Art. 1107.

17. Calculate the altitude. Area $= \frac{1}{2}(60 + 130) \times$ altitude.

18. See **4**, Measurements, Art. 1107.

19. The altitude $= \sqrt{30^2 - 15^2}$.

20. Find the area as in **1**, Measurements, Art. 1107. Divide the area by one-half the base to obtain the altitude.

21. Area = 800 sq. rd.; area of the square constructed on its diagonal = 800 sq. rd. $\times$ 2 = 1600 sq. rd.; length of the diagonal $= \sqrt{1600}$ rd. = 40 rd. *Ans.*

22. See Measurements, Art. 1107, **7**, for the area of one of the six equal triangles.

23. $(6^2 \times 3.1416)$ sq. in.

24. Its area is one-half the area of the square constructed on the diameter; that is, $\frac{1}{2}$ of 100 sq. ft.

25. The area of the sector is $\frac{1}{5}$ that of the circle. Area of circle $= 100 \times 3.1416$.

1125. **2.** See Supplement for the definitions.

3. \$6.75 is what per cent of \$2700?

6. The note is due April 13 (10), 1891; the term of discount is 102 (99) da.

$$\begin{array}{rr} & \$261. \\ 6\% & 15.66 \\ \hline & \$276.66 \end{array}$$

7. Do not place the rate under the principal. See Arithmetic, Art. 983, **27**.

8. Make the divisor a whole number, $68702050000 \div 48665$. See Art. 1007, **7**.

9. To yield $4\frac{1}{2}\%$, the bonds must have a face value of $\$225 \div .045 = \$5000$. Their cost will be $\$5000 \times 1.0525$.

10. See Art. 1051, **10**.

1126. **10.** $73\frac{1}{40}$ mi. $\div 2\frac{1}{6}$.

11. See Art. 1056.

12. $\$3 \times 4 \times 4 \div \frac{3}{8}$. Teachers should not require pupils to use pencils unnecessarily. See page 5.

16. $\dfrac{x}{25} + 68 + \dfrac{4x}{7} = x.$

18. See Arts. 546 and 1022, **15**.

19. $(\frac{7}{8}$ of $[(14 + 16 + 14 + 16) \times 8] \div 1\frac{1}{2}) \div 24$. Cancel. The perimeter of the room multiplied by the height gives the surface of the walls; $\frac{7}{8}$ of this gives the number of square feet remaining after the openings are deducted; dividing by $1\frac{1}{2}$, gives the number of feet of paper needed; dividing by 24, which is the number of feet in a roll, gives the number of rolls, or $11\frac{2}{3}$. As a part of a roll is not obtainable, 12 rolls must be purchased.

1127. 5. Cellar contains $(10 \times 8 \times 2)$ cu. yd.

6. $(48 \times 32) \div (16 \times \frac{1}{8})$.

9. See Art. 1100, **19**.

1128. 7. $16 : 20 :: x : 25$. See Arts. 1068–1073.

13. $\frac{4}{500}$ of $\frac{2}{3}$ of \$18000. Cancel.

14. $\$120 \div \frac{4}{500}$.

15. Rate, \$8 per \$1000.

1129. See notes on previous Special Drills.

1131. $44 \times 22 = (44 \times 20) + (44 \times 2)$; $44 \times 18 = (44 \times 20) - (44 \times 2)$. $26 \times 62 = (26 \times 60) + (26 \times 2)$; $26 \times 58 = (26 \times 60) - (26 \times 2)$.

1133. See Art. 1064. 49×49, Art. 1032.

1134. See Art. 1065.

5. 9 times 16 ft.

12. 175×12 hundredths $= 1\frac{3}{4} \times 12$.

14. $143 \div \frac{11}{5} = \frac{143}{11} \times 5 = 13 \times 5 = 65$.

15. 70% of 69 = 70% of 70 − 70% of 1.

17. $(29 \times 16) + 26 = 10$ more than (30×16).

20. $2 \times (87 + 49) = [(87 + 50) - 1] \times 2$.

21. Each brick contains $(\frac{2}{3} \times \frac{1}{3} \times \frac{1}{6})$ cu. ft. $= \frac{1}{27}$ cu. ft.

27. 30 da. after April 6 = May 6; 30 da. thereafter = June 5; 30 da. thereafter = July 5; adding days of grace = July 8.

28. Each side measures 2 yd. The surface of 1 face = 4 sq. yd.; of 6 faces = 24 sq. yd.

29. Without grace, the interest for 60 da. is 1% of \$100 = \$1; the proceeds = \$99.

For 3 additional days, the interest is $\frac{1}{20}$ of \$1, or 5¢; the proceeds = \$98.95.

1135. See Art. 1044. The mark on each case is H. B. The numbers of the cases are 5453 and 5454. The goods are sent (consigned) to Messrs. Hamburger Bros., to be sold on commission.

$94\frac{1}{2}$ yd. at 2*s*. 3*d*.	£10	12*s*.	$7\frac{1}{2}$*d*.
$140\frac{3}{4}$ yd. at 1*s*. 9*d*.	12	6	$3\frac{3}{4}$
61 yd. at 3*s*. 0*d*.	9	3	0
348 yd. at 1*s*. 9*d*.	30	9	0
	£62	10	$11\frac{1}{4}$
Less $\frac{1}{40}$,	1	11	$3\frac{1}{4}$
	£60	19	8

The value in U.S. money = \$296.78. Ad valorem duty at 50% = \$148.39. Specific duty (295 lb. + 351 lb.) = 646 lb. @ 44¢ = \$284.24. The entire duty = \$432.63. *Ans.*

1137. Divide area by $\frac{1}{2}$ base. See Art. 1107, Measurements, 1 and 4.

1139. 1. A's equivalent = 72, (6×12); B's = 70, $[(5 \times 11) + (3 \times 5)]$. Total, 142. A pays $\frac{72}{142}$ of \$175; etc.

2. Total debts (\$750 + \$1125 + \$1245) = \$3120.

3120 : 750 :: \$1287 : A
3120 : 1125 :: \$1287 : B
3120 : 1245 :: \$1287 : C

3. K, 50×26; L, 60×26; M, 70×20; N, 90×22; etc.

4. Some persons prefer to employ smaller figures for the capital invested, by dividing each by the same number. A's can be taken as \$25; B's as \$40; and C's as \$50; or \$5, \$8, and \$10 may be used.

$$\begin{aligned} 5 \times 12 &= 60 \\ 8 \times 9 &= 72 \\ 10 \times 5 &= \underline{50} \end{aligned}$$

182 : 60 :: \$15000 : A's share.
182 : 72 :: \$15000 : B's share.
182 : 50 :: \$15000 : C's share.

5. Counting each ox as 3 sheep,

$$\begin{array}{lr}
60 \times 10 = 600 & \\
50 \times \ \ 8 = \underline{400} & 1000 \\
75 \times \ \ 8 = 600 & \\
30 \times \ \ 7 = \underline{210} & 810 \\
54 \times 10 = 540 & \\
10 \times 12 = \underline{120} & 660 \\
90 \times 12 = & \underline{1080}
\end{array}$$

$$\begin{array}{l}
3550 : 1000 :: \$152.50 : W \\
3550 : \ \ 810 :: \$152.50 : X \\
3550 : \ \ 660 :: \$152.50 : Y \\
3550 : 1080 :: \$152.50 : Z
\end{array}$$

6. 220 yd. were built in 11 da. by 18 men. 480 yd. will be built in 18 days by ? men.

$$\frac{18 \text{ men} \times 11 \times 480}{220 \times 18} = 24 \text{ men.} \quad 6 \text{ extra men. } \textit{Ans.}$$

7. See Art. 1122, **1**.

8. 14 men in $(8\frac{1}{4} \times 12)$ hr. mow 168 acres. 20 men in $(7\frac{4}{5} \times 11)$ hr. mow ? acres.

NOTE.—A thoughtless scholar will sometimes fail to see that the 15 min. should be joined to 8 hr.; he will, therefore, compare 8 hr. with 7 hr., and 15 min. with 48 min.

9. To do 4 times the work in $\frac{1}{5}$ of the time will take 20 times 12 men, or 240 men. (Omit 20 da.)

10. At the time they meet the sinking vessel, 60 men have provisions for 24 da.; these will last 72 persons 20 da.

$$72 : 60 :: 24 \text{ da.} : x$$

11. Omit the dimensions of the boards. If 76 are worth \$19.76, 50 are worth \$19.76 × 50 ÷ 76.

1140. This table will furnish some practice in addition and division, and should not be passed over.

1141. The "developed" entire surface of a square prism is shown in Arithmetic, Art. 818, **20.** In drawing the development of the convex surface, the upper and lower squares, denoting the bases, will be omitted. The drawing should be done with reasonable care, to a scale of, say, $\frac{1}{2}$ inch to the inch.

In making a model of a solid, narrow strips for pasting should be added, as shown in Fig. 1. In the development of a triangular prism, the bases are usually drawn above and below the middle rectangle, but the pupils should learn that they may be placed in other positions, one of which is here shown. It will be noticed that the pasting flaps do not form rectangles, the sides being inclined at an acute angle to make neater work in the completed model.

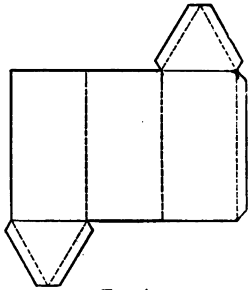


FIG. 1.

The shape and the arrangement of the gumming flaps for the bases of a cylinder are shown in Fig. 2. Interested pupils may be safely left to themselves to ascertain the length of the rectangle that is needed for the model of a given cylinder.

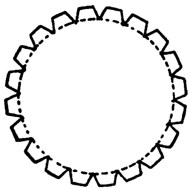


FIG. 2.

The scholars will learn more geometrical facts while constructing these models than they will obtain by memorizing many pages of definitions or listening to numerous "explanations."

5. The entire surface includes the convex surface and the surface of the bases.

8. If x represents one side of a cube, $x^2 =$ the surface of one face, and $6x^2 =$ the entire surface $= 216$ sq. in. *Ans.*

9. The convex surface $= 4x^2 = 144$ sq. in. *Ans.*

11. The perimeter $= (600 \div 15)$ ft.

Or, let $x =$ one side of base; the perimeter $= 4x$; the convex surface $= 4x \times 15 = 60x = 600$; $x = 10$; etc.

12. Let $x =$ the altitude. $(15+15+15+15)\times x + 15^2 + 15^2 = 1650$; $60x + 225 + 225 = 1650$; $60x = 1650 - 225 - 225 = 1200$; $x = 20$; the convex surface $= 60x = 1200$. 20 in.; 1200 sq. in. *Ans.*

13. Let $x =$ one side of base; $4x =$ the perimeter; $4x \times 15 = 60x =$ the convex surface $= 540$; $x = 9$. The entire surface $= 540$ sq. in. $+ 81$ sq. in $+ 81$ sq. in.

14. Circumference of base $= 3.1416$ ft. Convex surface $= (3.1416 \times 1)$ sq. ft. $= 3.1416$ sq. ft. Radius of base $= \frac{1}{2}$ ft.; area $= (\frac{1}{2} \times \frac{1}{2} \times 3.1416)$ sq. ft. $= .7854$ sq. ft. Entire surface $=$ 3.1416 sq. ft. $+$ .7854 sq. ft. $+$ .7854 sq. ft. $= 4.7124$ sq. ft. *Ans.*

15. See Arithmetic, Art. 1290. While pupils should be permitted to "develop" these solids in their own way, provided it be a correct one, they should be advised in making drawings for models to use a pattern that will require a minimum of pasting. While Fig. 3 would serve for the development of the entire surface, it would not answer as a pattern from which to construct a hollow pyramid.

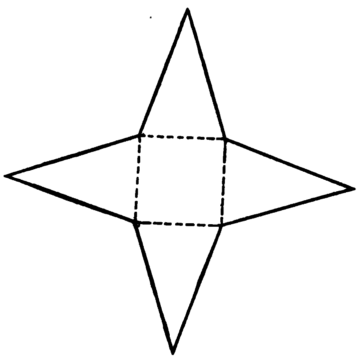


FIG. 3.

16. The convex surface of a pyramid is equal to the perimeter of the base $\times \frac{1}{2}$ the slant height. The slant height of a regular pyramid is the altitude of one of the equal triangles that constitute its convex surface.

18. One side of base $= \sqrt{144}$ in.

19. Either calculate the slant height, which is $\sqrt{2^2 - 1^2} = \sqrt{3}$; or employ the method given in **1**, of Measurements, Arithmetic, Art. 1107.

20. The developed convex surface of a cone is a sector, whose radius is the slant height of the cone and whose arc is equal in length to the circumference of the base of the cone.

The circumference of the base of the given cone = (4 times 3.1416) in. The circumference of the circle of which the sector forms a part, is (2×6 times 3.1416) in., or (12 times 3.1416) in.; the sector is, therefore, $\frac{1}{3}$ of the circle, and its arc measures $\frac{1}{3}$ of 360°, or 120°.

Any sector of 120° will form a hollow cone of the proper proportions.

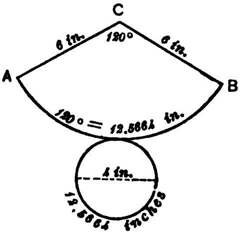


FIG. 4.

The base shown in Fig. 4 is not required by the terms of this problem; it is merely introduced here to show the development of the entire surface. As it is difficult to lay off the required number of inches for the arc AB, the pupil will appreciate the foregoing method of determining the number of degrees it should contain. The compasses or the protractor may be employed to construct an angle of 120° at C.

The convex surface of a cone is equal to the circumference of the base $\times \frac{1}{2}$ slant height.

An examination of Fig. 4 will show the resemblance between the methods of calculating the surface of a sector and of a triangle. The area of a triangle = $\frac{1}{2}$ (base $\times$ altitude); that of a sector = $\frac{1}{2}$ (base $\times$ radius).

22. The altitude, one-half the base, and the slant height, form a right-angled triangle; and the lengths of the two first being 12 in. and 5 in., respectively, the length of the latter is $\sqrt{144+25}$ in., or 13 in. The convex surface = $\frac{1}{2}(10 \times 3.1416 \times 13)$.

23. The entire surface = $[\frac{1}{2}$ of $(6 \times 3.1416 \times 10)] + (3^2 \times 3.1416) = (30 \times 3.1416) + (9 \times 3.1416) = 39 \times 3.1416$.

Using π (pī) instead of 3.1416, the circumference of the base = 6π inches. The radius of the sector representing the development, is 10 in., and the circumference of the whole circle = 20π inches. As the arc of the sector must be 6π inches, it measures in degrees $\frac{6}{20}$ of 360°, or 108°.

24. The slant height will be $\frac{1}{2}$ of 6 in. The circumference of the base will equal the arc of the semicircle, 3π inches; its diameter will, therefore, be 3 in.

1142. **3.** Due Sept. (21) 24. Term of discount from July $21 = 65$ (62) da.

4. \$600 yearly interest represents a principal of \$10000.

5. Length of one fence, $(20 + 20 + 20 + 20)$ rd.; of the other, $(40 + 10 + 40 + 10)$ rd.

6. The distance between the center of the first and of the last post = 10 ft. $\times (11 - 1) = 100$ ft. Adding $\frac{1}{2}$ of the diameter of each post, gives 100 ft. 6 in.; and an additional 3 in. at each end to fasten the wire, makes a total of 101 ft. of wire required for each length, or 303 ft. in all. *Ans.*

1143. **7.** Troy weight.

8. $43\frac{2}{5}$ mo. @ 25¢ per mo. 43 quarter dollars = \$10.75; $\frac{2}{5}$ of 25¢ = 10¢. Total \$10.85. *Ans.*

11. Without grace, 1% of \$400, or \$4. *Ans.* With grace, \$4 + $\frac{1}{20}$ of \$4, or \$4.20. *Ans.*

12. $\frac{1}{4}$ of cost = \$2; etc.

13. The principal is unimportant. Time = $(100 \div 8)$ yr.

17. $(100 \div 6)$ yr. **23.** $\frac{1}{2}$% of \$1234.

24. 2% of \$1234.

25. The number of rings = 60 pwt. $\div 2\frac{1}{2}$ pwt.

26. $2^\circ\ 3' \times 15$. **27.** [Twice $(4 + 3) \times 10$] + [twice (4×3)].

28. \$$1.12\frac{1}{2} + \frac{1}{4}$ of \$$1.12\frac{1}{2}$ = \$$1.12\frac{1}{2}$ + \$$.28\frac{1}{8}$.

30. Selling price = $\frac{3}{2}$ of cost = $\frac{1}{2}$¢; cost = $\frac{1}{2}$¢ $\div \frac{3}{2} = \frac{1}{3}$¢. *Ans.*

1144. **4.** 14% profit = 7¢; cost = 7¢ $\div .14$ = 50¢; selling price = 57¢. *Ans.*

7. $x + \frac{x}{40} + \frac{x}{200} = 7828$; etc.

17. Cost of 350 tons @ $\$3.50 = \$1225$. Selling price $= \$4.25 \times 350 \times \frac{2240}{2000}$.

NOTE.— The scholars should not use pencils to obtain answers to problems that can be solved at sight.

1145. See Art. 1290.

3. Area of base $= 6$ sq. in.; etc.

5. See Art. 1107, Measurements, **7**.

8. Changing given dimensions to inches, the number of gallons will be $36^2 \times 3.1416 \times 66 \div 231$.

9. 1 cu. ft. $= \frac{1000}{16}$ lb. Cubical contents $= (3^2 \times 3.1416 \times 5\frac{1}{2})$ cu. ft.

10, 11. Careful pupils will be much interested in ascertaining how closely their calculations as to the contents of the measure, agree with the number of cubic inches it is supposed to contain. There should be 231 cu. in. $\div$ 8, in a quart. The cup used must be cylindrical. Tapering measures should be left until the frustum of a cone has been studied, Art. 1295. The paper box used for ice cream, a frustum of a pyramid, can also be employed at that time. Some of these measurements should be made out of school, and comparisons made as to the results obtained by different pupils and the methods employed by them to secure accuracy. A random measurement will not obtain the correct diameter of a quart measure.

After calculating the altitude of an equilateral triangle or the diagonal of a square, the pupil should draw the figure to a scale, measure the altitude or the diagonal, and compare the measured length with the length obtained by calculation.

Pupils should ascertain the weight of a cubic foot of water by weighing a quart of water, for instance, etc.

12. Measure the height to which the water rises in the box, etc.

If a solid heavier than water, is placed in a rectangular or a cylindrical vessel containing sufficient water to cover it, and the difference in the depth of the water before and after immersion is noted, the volume of the solid

can be calculated. It will be equal to that of a solid whose base is the base of the vessel, and whose altitude is the difference in depth above mentioned.

If the water in a rectangular box, whose base measures $5\frac{1}{2}$ by 3 in., is raised $1\frac{7}{8}$ in. by the introduction of a piece of marble, the volume of the latter $= 5\frac{1}{2} \times 3 \times 1\frac{7}{8}$ cu. in.

This method is useful in determining the contents of a solid of irregular shape.

13. The radius of the base $= \frac{1}{2}$ of $(25.1328$ yd. $\div 3.1416)$ $= 4$ yd. Volume of cone $= (4^2 \times 3.1416 \times \frac{1}{3}$ of $18)$ cu. yd.

14. See Art. 1141, **22**.

15. The slant height of the pyramid $= \sqrt{24^2 + (\frac{1}{2} \text{ of } 14)^2}$. See Art. 1283, **13**.

1146. **6.** $[(10\frac{3}{8}¢ \times 1\frac{1}{4}) + (3\frac{7}{8}¢ \times 1\frac{3}{8})] \times 10840)$.

7. 1000 grams $\div$ 279 = weight in grams of a 10-mark piece. Weight in Troy grains $= (1000 \div 279) \times 15.432349$. Dividing this result by $23\frac{22}{100}$ gives the number of U.S. gold dollars.

$$\frac{1000 \times 15 \odot 43.2349}{279 \times 23 \odot 22.}$$

NOTE. — 23.22 is changed to a whole number by removing the decimal point two places to the right, and a corresponding change is made in one of the numbers in the dividend.

11. Interest on

$$\begin{array}{rll} \$237453250 & @\ 3\ \% = & \$\ 7123597.50 \\ 250000000 & @\ 4\frac{1}{2}\% = & 11250000. \\ 737954700 & @\ 4\ \% = & 29518188. \\ \hline \$1225407950 & @\ x\ \% = & \$47891785.50 \end{array}$$

$$4789178550 \div 1225407950 = 3.9083+$$

The interest on the entire amount at $2\frac{1}{2}\%$ would be \$30635198.75, the saving being \$17256586.75. *Ans.*

12. \$100 worth of stock costs $\$85\frac{3}{4}$. The annual dividend is 5% of \$100, or \$5. This is $(5 \div .8575)$ per cent on the money invested.

13. $\$8930 \div 1.11\frac{5}{8}$.

14. See Art. 1122, **15**. The following shows the account as it stands on the books of the Interior Department. The items that appear as debits on Mr. Well's books, here appear as credits, and *vice versa.*

Dr. RICHARD WELLS. *Cr.*

1882					1882				
Jan.	31	To Cash	885	—	Jan.	1	By 645 bbl. Flour 9.45	6095	25
Feb.	5	" "	450	—	"	16	" 1912 bu. Oats .57	1089	84
Apr.	11	" "	615	85	Apr.	4	" 9231 lb. Bacon .09	830	79
May	30	" "	4162	15	May	3	" 8264 bu. Corn .74	6115	36
June	25	" 345 lb. Bacon .09	31	05	June	20	" 325 bbl. Pork 12.65	4111	25
"	25	" 35 bbl. Pork 12.65	442	75	"	30	" Cartage	65	—
"	30	" Penalty	75	—					
"	30	" Cash in full	11646	19					
			18307	49				18307	49

1148. Multiply the length in feet by the width in feet by the thickness in inches.

16. $3¢$ (per ft.) $\times 15 \times 16 \times \frac{3}{4} \times 3$.

17. The floor contains $(36 \times 17\frac{1}{2})$ sq. ft., or 630 sq. ft. If 1-inch boards were used, 630 board feet would be required. The number of feet of $2\frac{1}{2}$-inch planks required $= 630$ ft. $\times 2\frac{1}{2}$.

18.
$$\begin{aligned} (150 \times 13 \times \tfrac{2}{3} \times 1) \text{ board ft.} &= 1300 \text{ board ft.} \\ (60 \times 14 \times \tfrac{3}{4} \times 2) \text{ `` `` } &= 1260 \text{ `` ``} \\ (40 \times 15 \times \tfrac{5}{12} \times 4) \text{ `` `` } &= \underline{1000} \text{ `` ``} \\ \text{Total, } & 3560 \text{ `` ``} \end{aligned}$$

the duty on which, at \$1 per M, is \$3.56. *Ans.*

19. The length of the fence $= 480$ ft. $+ 360$ ft. $+ 480$ ft. $+ 360$ ft. $= 1680$ ft. For a fence 4 boards high, (1680×4) running feet of boards will be needed, or 6720 running feet. If the boards are $\frac{1}{2}$ ft. wide and 1 in. thick, the number of board feet $= 6720 \times \frac{1}{2} \times 1 = 3360$ ft. Cost $= \$18 \times 3.36$.

20. The length of the fence $= (25 + 100 + 25 + 100)$ ft. Surface $= (250 \times 6)$ sq. ft. $= 1500$ sq. ft. As the boards are 1 in. thick, the number of board feet $= 1500$. Cost $= \$25 \times 1.5$

$= \$37.50$. The number of posts $= 250 \div 6\frac{1}{4} = 40$; cost, at 25¢ each, \$10. The number of running feet of scantling, two strips, $= 250 \times 2 = 500$; the number of board feet $= 500 \times \frac{1}{4} \times 2 = 250$; cost, $\$18 \times .25 = \$4.50$. Total cost, $\$37.50 + \$10 + \$4.50 = \$52$. *Ans.*

1149. 1. (*a*) A note made payable to the *order* of a certain person or to *bearer* is negotiable; in the former case, an endorsement is necessary to transfer its ownership. A note payable to *bearer* does not require endorsement. A note payable to Charles Naumann (without the words, "or order," or the like) is not transferable by endorsement. If Charles Naumann wishes to sell the note, he must *assign* his interest in it by another document.

NOTE.—The above is the general rule; in some states there are special laws bearing on the subject:

In Alabama and Kentucky, a note to be negotiable must be payable at a fixed place; in Indiana and Virginia, at a bank; in West Virginia, at a bank or public office. In Pennsylvania, it should contain the words "without defalcation"; in New Jersey, "without defalcation or discount"; in Missouri, "negotiable and payable without defalcation or discount."

(*b*) A person unable to write his name, makes his mark, as shown below, in the presence of a witness:

Witness:	WILLIAM $\underset{\text{mark}}{\overset{\text{his}}{\mathbf{X}}}$ DEVERS.
THEODORE H. FICKLIN.	

3. In old deeds, the contents of a farm are given in acres (A.), roods (R.), and poles (P.), the rood being $\frac{1}{4}$ acre, and containing 40 poles, or square rods. In long measure, the word *pole* is occasionally employed instead of *rod.*

6. $2x = x + \frac{x}{2} + \frac{x}{3} + 18$; etc.

7. The distance between the ships is the hypotenuse of a right-angled triangle, whose other sides are 72 mi. and 128 mi., respectively.

8. The first capital $= \$3500 \div 1.40 = \$2500$; C put in $\frac{2100}{3500}$, or $\frac{3}{5}$ of $\$2500 = \$1500$; etc.

9. See Art. 1026, **10.**

10. See Supplement.

1150. **3.** $x^2 + x^2 = \text{hypotenuse}^2 = 100$. $x^2 = 50$, the area of the inscribed square. *Ans.*

4. Area of circle $= (5 \times 5 \times 3.1416)$ sq. in. $= 78.54$ sq. in.

5. Arc of $90° = \frac{1}{4}(10 \times 3.1416)$ in. $= 7.854$ in.; the chord $= \sqrt{50}$ in.

6. Arc of 90° in a circle whose radius is 10 in. $= 15.708$ in. Area of sector $= \frac{1}{2}$ of (15.708×10) sq. in. $= 78.54$ sq. in. *Ans.* Area of triangle $= \frac{1}{2}$ of (10×10) sq. in. $= 50$ sq. in.; area of segment $= 78.54$ sq. in. $- 50$ sq. in. $= 28.54$ sq. in. *Ans.*

8. $R^2 \times 3.1416$. *Ans.*

9. Area of outer circle in square yards $= (15^2 \times 3.1416) \div 9$; of inner circle $= (10^2 \times 3.1416) \div 9$.

10. (125×3.1416) sq. ft.

11. $[(6^2 - 3^2) \times 3.1416]$ sq. in. The radius of the outer circle is 6 in.; of the inner circle, 3 in.

12. $36 \times 3.1416 : 9 \times 3.1416 = 4:1$.

13. $[(30 \times 30) - (20 \times 20)]$ sq. ft.

14. Dividing the walk into four trapezoids, as in Fig. 5, the area of each will be $[\frac{1}{2}$ of $(30 + 20) \times 5]$ sq. ft. $= 125$ sq. ft. The broken line, XY, drawn midway between the parallel sides measures 25 ft.; the whole length of the center lines is 100 ft. The area of the walk is (100×5) sq. ft.

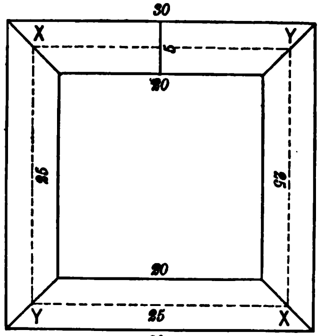


FIG. 5.

The area of the circular frame in **11** can be ascertained in the same way. The center of the frame is $4\frac{1}{2}$ in. from the center of the glass. The middle line of the frame $= (3.1416 \times 4\frac{1}{2} \times 2)$ in. $= 28.2744$ in. The area $= (28.2744 \times 3)$ sq. in.

15. The area of the first $= 240$ sq. ft.; of the second, 960 sq. ft.

1153. **18.** The surface of the sphere $= (4\pi \times \frac{1}{4})$ sq. ft. $= 3.1416$ sq. ft.; the convex surface of the cylinder $= 3.1416$ sq. ft.

19. The entire surface $= 3.1416$ sq. ft. $+ (2 \times \frac{1}{4} \times 3.1416)$ sq. ft. $= 1\frac{1}{2}$ times 3.1416 sq. ft.

20. 4 times ($\frac{1}{2}$ circumference) $10 \times$ ($\frac{1}{2}$ diameter) $\frac{10}{3.1416} = \frac{400}{3.1416} =$ surface in square inches.

1154. **1.** Rate per cent $= 18750 \div 12500 = 1\frac{1}{2}$. $1\frac{1}{2}\%$ of $\$6000 = \$90$.

3. The price of silver is now given by the ounce.

4. The interest on \$200 at $4\frac{1}{2}\% = \$9$. $\$9 \div .08 = \$112.50$. *Ans.*

5. $\$2100 + \$4400 + \$13000 + \$7200$ (90% of \$8000) $= \$26700 =$ total assets. Total liabilities $= \$1625 + \$5625 = \$7250$. $\$26700 - \$7250 = \$19450$. To this, add the amounts withdrawn, $\$850 + \$1075 = \$1925$, making the sum of the capital and profits \$21375. Of this, H is entitled to $\frac{1}{3}$, \$7125, less the amount withdrawn by him, \$1075, or \$6050.

1157. **9–11.** Find the cube root of the numerator and of the denominator separately.

12–15. Reduce to an improper fraction before extracting the cube root; then reduce the root to a mixed number.

1159. **2.** $[\frac{4}{3} \times 3.1416 \times (\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2})]$ cu. in. Cancel.

3. The volume of the first in cubic inches $= \frac{4}{3}\pi \times \frac{1}{8} = \frac{1}{6}\pi$. The volume of the second $= \frac{4}{3}\pi \times 1 = \frac{4}{3}\pi$.

4. The volume of the sphere $= .5236$ cu. in.; the volume of the cube $= 1$ cu. in.

6. The ball contains .5236 cu. ft. Its weight $= (.5236 \times 7.5 \times 1000 \div 16)$ lb.

1160. 1. 40 tons were to be moved; there remain 22 tons to be moved in 3 da. In 6 da. 18 men moved 18 tons, so it requires 22 men to remove 22 tons in 6 da., or 44 men to remove 22 tons in 3 da. 44 men. *Ans.*

3. A diagonal on the floor measures $\sqrt{40^2 + 30^2}$ ft. $= 50$ ft. A diagonal on one wall measures $\sqrt{40^2 + 14^2}$ ft. $= 42.38 -$ ft.; on the other wall it measures $\sqrt{30^2 + 14^2}$ ft. $= 33.11 -$ ft.

4. Selling price, $\$6500 + 15\%$ of $\$6500 = \$6500 + \$975 = \$7475$.

As no date is given, the note may be assumed to be for 120 da., or 123 da. including grace. Proceeds of \$7475 for 123 da. = \$7321.76. Profits = \$7321.76 − \$6500 = \$821.76. *Ans.* Without grace, the proceeds would be \$3.74 more, making the profits \$825.50. *Ans.*

5. Let $x =$ face; $\frac{13x}{400} =$ premium; bank discount $= x \times \frac{33}{360} \times \frac{8}{100} = \frac{11x}{1500}$ (including grace).

$$x + \frac{13x}{400} - \frac{11x}{1500} = 4265,$$
$$6000x + 195x - 44x = 25590000,$$
$$6151x = 25590000,$$
$$x = 4160.30-. \quad \$4160.30. \ \textit{Ans.}$$

Without grace, $x + \frac{13x}{400} - \frac{x}{150} = 4265$; etc.

7. Longitude difference $= 48° \ 24'$; time difference $= 48$ hr. 24 min. $\div 15 = 3$ hr. 13 min. 36 sec. As the more easterly place has the later time, the watch is fast. *Ans.*

The above result is based upon the assumption that the "sun time" of each place is used. The difference in the "standard" time of the two cities is 3 hr.

9. $\frac{1}{4}$ of $\frac{1}{8}$ of $\frac{8}{8}$ of $x=6$; $\frac{x}{32}=6$; $x=192$.

10. $x+\frac{x}{5}=x-\frac{x}{8}+1040$;

$$40x+8x=40x-5x+41600;$$
$$13x=41600;\ x=3200. \qquad \textit{Ans. } \$3200.$$

11. $2050:x::41:69$. Cancel.

14. \$1450 − $\frac{1}{4}\%$ of \$1450 − 63 (60) days' interest on \$1450 at 5% = cost of draft.

17. A can do $\frac{1}{12}$ in 1 da.; B can do $\frac{1}{20}$; both can do $\frac{2}{15}$ in 1 da., and can do the whole work in $7\frac{1}{2}$ da. *Ans.*

19. See Art. 1150, **4**.

1161. The definitions and principles called for throughout this chapter should be formulated, as far as possible, by the pupils, the latter being led through the teacher's questioning to see their mistakes, and to make the necessary corrections. If this preliminary work is done as it should be, the scholars will be ready to understand the definition finally given by the teacher. Too much time, however, should not be wasted on formal definitions, as they are of next to no help to a pupil in his mathematical work, and it is very unlikely that he will ever be called upon to use them in after life. See Supplement.

1. (*d*) A decimal fraction is frequently defined as one whose denominator is 10 or some power of 10. In this place, however, the expression is used as synonymous with "decimal." The rule asked for, refers to the method of "pointing off" the product.

7. If all three are opened, they will fill $(\frac{1}{10}+\frac{1}{6}-\frac{1}{5})$ or $\frac{1}{15}$ of the cistern in 1 hr. To fill the whole cistern will require 15 hr. *Ans.*

9. As they meet in 15 hr., they must approach each other at the rate of (105 ÷ 15) miles per hour, or 7 mi. As one goes 3 mi. per hour, the other must travel 7 mi. — 3 mi. = 4 mi. per hour.

18. A's capital at end of year is \$8000, B's is \$10000, C's, \$3000: total, \$21000. Profits = \$18000 + \$12000 − \$21000 = \$9000.

$$
\begin{array}{llrr}
\text{A,} & 10000 \times \tfrac{1}{2}\ (\text{yr.}) & = 5000 & \\
 & 8000 \times \tfrac{1}{2} & = \underline{4000} & 9000 \\
\text{B,} & 6000 \times \tfrac{1}{2} & = 3000 & \\
 & 10000 \times \tfrac{1}{2} & = \underline{5000} & 8000 \\
\text{C,} & & & \underline{3000}
\end{array}
$$

$$20000 : 9000 :: 9000 : 4050$$
$$20000 : 9000 :: 8000 : 3600$$
$$20000 : 9000 :: 3000 : 1350$$

A is entitled to his capital at the end of the year, \$8000, and \$4050 profits, or \$12050. As he receives \$12000 worth of goods, his cash receipts are \$50. B receives \$10000 + \$3600 = \$13600. C receives \$3000 + \$1350 = \$4350.

19. $\frac{x}{5} + \frac{1}{4}$ of $\frac{4x}{5} + 20 + \frac{2x}{5} = x;$

$$\frac{x}{5} + \frac{x}{5} + \frac{2x}{5} + 20 = x; \text{ etc.}$$

20. He sold 800 bbl. at \$7.50, which amounted to \$6000. The cost was \$7200 + \$312 + \$350 = \$7862. His profits being \$138, he must have received \$7862 + \$138 = \$8000. As the flour realized only \$6000, he must have received \$2000 from the railroad company.

28. 16% of the person's money = 20% of \$160000; etc.

29. 4 men do $\frac{1}{3}$ of the work in 60 hr.; to do the remainder, they would need 120 hr.; and 1 man would require 480 hr. There are 8 da. of 10 hr., or 80 hr., in which to finish it; 6 men, therefore, will be needed to complete it, or 2 men additional.

30. Cost = \$100 + \$5 + \$50 + \$20 = \$175. Selling price, \$300 less 10% (\$270) − \$20 = \$250. Profit, \$75.

If the commission merchant had refunded \$20 before making returns, his commission would have been 10% of \$280, or \$28; and the gain would have been \$77.

36. (*b*) See Arithmetic, Art. 668.

38.

A,	$5000 \times \frac{1}{2} = 2500$	
	$3000 \times \frac{1}{2} = 1500$	4000
B,		6000
C,	$4000 \times \frac{1}{2} = 2000$	
	$12000 \times \frac{1}{2} = 6000$	8000
		18

18 : 4 :: \$6000 : A, etc.

39. Proceeds of \$12000 for 93 da. = \$11814. Amount of \$10000 for 6 mo. at 6% = \$10300. Sum remaining = \$11814 − \$10300 = \$1514.

Without grace, the proceeds of 90-days note = \$11820; sum remaining = \$1520.

40. Cost, \$40 each. 10 were sold @ \$44 each = \$440; 10 @ \$46 each = \$460; 15 were sold for \$100: total \$1000. To obtain \$900 for remaining 15, he must charge \$60 each.

1164. **1.** (30 + 30 + 31 + 30 + 31 + 31 + 30 + 31 + 30 + 17) da. = 291 da. Exact interest = $\$400 \times \frac{1}{20} \times \frac{291}{365}$.

$$\begin{array}{rrrl} 0 \text{ da.} \times & 630 = & 0 & \text{da.} \\ 243 \text{ ''} \times & 820 = & 199260 & \text{''} \\ 274 \text{ ''} \times & 950 = & 260300 & \text{''} \\ & 2400 &)459560 & \text{da.} \\ & & 191+ & \text{da.} \end{array}$$

4. Business men generally find the number of days' credit to which each item is entitled. \$820 is due March 5, or 243 da. after July 5; \$920 is due April 5, or 274 da. after July 5. The equated time is 191 da. after July 5, or Jan. 12. *Ans.*

Using months instead of days, the average term of credit is found to be 6 mo. 9 da., nearly, making the equated time Jan. 14.

5. 7500 : A :: 1200 : 250

7500 : B :: 1200 : 950.

6. Bank stock pays $(25 \times 7\frac{1}{2} \div 85)$ per cent interest semi-annually; the railroad stock pays $(25 \times 3 \div 31)$ per cent interest semi-annually.

9. The bases are equilateral triangles, each side of which measures 5 in. Art. 1107, **7**, Measurements.

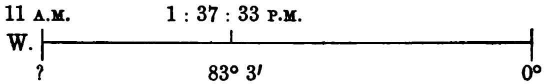
1165. **1.** See Supplement.

Quantity is anything that can be measured.

See Arithmetic, Art. 1072.

2. $\frac{2}{3}$ of $\frac{11}{12} \times \frac{81}{1} \times \frac{2}{11} \times \frac{3}{2} \times \frac{3}{11} \times \frac{1}{6} \times \frac{11}{2}$. Cancel.

3. Time difference $= 2$ hr. 37 min. 33 sec.



Longitude difference = ?

The longitude difference $= 2° \; 37' \; 33'' \times 15 = 39° \; 23' \; 15''$. Longitude of San Francisco $= 83° \; 3' + 39° \; 23' \; 15'' = 122° \; 26' \; 15''$ west.

5. $x + 2x + 5x = 11480$.

6. The equated time for the payment of \$600 is $[(200 \times 1) + (400 \times 2)]$ yr. $\div 600 = 1\frac{2}{3}$ yr. The present worth of \$600 due in $1\frac{2}{3}$ yr. $=$ \$600 $\div 1.1 =$ \$545.45+.

Another way is to calculate the present worth of each, and to add the results: (\$200 $\div$ 1.06) + (\$400 $\div$ 1.12) = \$188.68 + \$357.14 = \$545.82. *Ans.*

The latter is the more consistent way inasmuch as it employs the "present worth" method throughout. The first solution uses the "present worth" method to calculate the value at date, of \$600 whose equated time has been found by the "interest" (bank discount) method.

7. Selling price = \$120 + 15% of \$120 = \$138. Asking price = \$138 + \$12 = \$150. I threw off \$12 from \$150, or 8%.

8. Arithmetic, Art. 1250, **8**. $AD = 39$, $AC = 52$, $DC = \sqrt{52^2 + 39^2} = 65 = BC$. Height of tree $= 52$ ft. $+ 65$ ft. $= 117$ ft.

9. $\sqrt[3]{10.125000} = Ans.$

10. If $\frac{4}{5}$ gain $= \frac{2}{15}$ cost, the gain $= \frac{2}{15}$ cost $\times \frac{5}{4} = \frac{1}{6}$ cost $= 16\frac{2}{3}$ per cent.

1166. **4.** $[1 \div (\frac{1}{20} + \frac{1}{15})]$ weeks.

5. $x + x + 1\frac{3}{4} = 7\frac{1}{2}$.

15. B's gain of \$1400 is $\frac{7}{12}$ of total gain; $\frac{1}{12}$ of total $= \$200$; A's gain, $\frac{5}{12}$ of total $= \$1000$.

1167. **4.** Let $x =$ cost per barrel.

$$75\% \text{ of } 500\,x \times .02\tfrac{1}{5} = 80.85.$$

5. A does $\frac{1}{27}$ in 1 da.; B does $\frac{1}{15}$ in 1 da. A does $\frac{12}{27}$ of the work, or $\frac{4}{9}$, and B does $\frac{5}{15}$ of it, or $\frac{3}{9}$, leaving $\frac{2}{9}$ to be done by C in 4 da. To do the whole work C would require 4 da. $\div \frac{2}{9}$ $= 18$ da.

6. A ditch 20 yd. $\times$ 18 in. $\times$ 4 ft. is dug in (3×10) hours by 72 men. A ditch 30 yd. $\times$ 27 in. $\times$ 5 ft. is dug in (9×15) hours by ? men.

$$\frac{72 \text{ men} \times 3 \times 10 \times 30 \times 27 \times 5}{20 \times 18 \times 4 \times 9 \times 15} = Ans.$$

7. £2400 income is produced at 3% by bonds whose face value is £80000. Their cost $=$ £80000 $\times .94\frac{3}{8} =$ £75500 $= \$367433\frac{1}{3}$, 12% of which $= \$44092.00$.

$$\$4.86\tfrac{2}{3} \times (2400 \div .03) \times 94\tfrac{3}{8} \times .12.$$

8. $x - \dfrac{40x}{100} = 30 + 30\%$ of $30 = 39$.

$$100\,x - 40\,x = 3900;\ \text{etc.}$$

10. $[(15 + 10 + 15 + 10) \times 9\frac{3}{4}] + (15 \times 10) =$ number of square feet in the walls and ceiling $= 637\frac{1}{2}$ sq. ft. $= 70\frac{5}{6}$ sq. yd. The cost $= 21d. \times \frac{425}{6} = 1487\frac{1}{2}d.$; etc.

1168. **1.** $\frac{2}{3} + \frac{3}{4} = 1\frac{5}{12} = 1.41666 +$; $\frac{1}{2} + \frac{1}{5} + \frac{5}{7} = 1\frac{29}{70} =$ $1.41429 -$.

$.7409375 \div 237100 = .007409375 \div 2371$. See Arithmetic, Art. 668.

2. $\dfrac{1\frac{1}{4}}{1\frac{1}{12}}$ of $\dfrac{2}{3} = \left(\dfrac{5}{4} \div \dfrac{13}{12}\right) \times \dfrac{2}{3} = \dfrac{5}{4} \times \dfrac{12}{13} \times \dfrac{2}{3} = \dfrac{10}{13}$.

$$\frac{2\frac{1}{2} - 1\frac{5}{6}}{\frac{1}{4} + 1\frac{5}{6}} = \frac{2}{3} \div 2\tfrac{1}{12} = \frac{2}{3} \div \frac{25}{12} = \frac{2}{3} \times \frac{12}{25} = \frac{8}{25}.$$

$$8\tfrac{1}{5} \div 7\tfrac{4}{5} = \tfrac{41}{5} \div \tfrac{39}{5} = 41 \div 39 = \tfrac{41}{39}.$$

$$\frac{10}{13} + \frac{8}{25} - \frac{41}{39} = \frac{750 + 312 - 1025}{975} = \frac{37}{975};$$

$$\frac{37}{975} \div x = \frac{2}{3};\ \frac{37}{975x} = \frac{2}{3}.$$

Clearing of fractions, $37 = 650x$; $x = \frac{37}{650}$. *Ans.*

$$\tfrac{82}{9} \text{ of } \tfrac{8}{21} \text{ of } \tfrac{5}{8} \text{ of } \tfrac{7}{12} \times \tfrac{17}{6} \times \tfrac{11}{14} \times \tfrac{21}{4}.$$

$x =$ smaller number; $x + \frac{7}{18} =$ larger.

$x + x + \frac{7}{18} = \frac{113}{126}$; $126x + 126x + 49 = 113$; $252x = 113 - 49 = 64$; $x = \frac{64}{252} = \frac{16}{63}$; $x + \frac{7}{18} = \frac{16}{63} + \frac{7}{18} = \frac{32}{126} + \frac{49}{126} = \frac{81}{126} = \frac{9}{14}$. *Ans.* $\frac{16}{63}$ and $\frac{9}{14}$.

3. 16*s.* $4\frac{1}{8}$*d.* $= 196\frac{1}{8}$*d.* *Ans.* $= £(\frac{5}{9}$ of $196\frac{1}{8}) \div 240$.

$\frac{5}{8}$ mile $= 1000$ meters; 1 mi. $= 1000$ m. $\div \frac{5}{8} = 1600$ m. 17 mi. $= 1600$ m. $\times 17 = 27200$ m.; 6 furlongs $= 200$ m. $\times 6 = 1200$ m.; $82\frac{1}{2}$ yd. $= \dfrac{1600 \text{ m.}}{1760} \times \dfrac{165}{2} = 75$ m.

$$27200 \text{ m.} + 1200 \text{ m.} + 75 \text{ m.} = 28475 \text{ m.} \ \textit{Ans.}$$

27 yd. 2 ft. 9 in. $= 1005$ in.; 17 yd. 1 ft. 11 in. $= 635$ in.

(635×1) sq. in. cost \$25.40; 1 sq. in. $=$ \$2540 $\div 635$; $(1005 \times \frac{7}{8})$ sq. in. $= (\$25.40 \div 635) \times 1005 \times \frac{7}{8} = \$35.17\frac{1}{2}$.

4. Let $x =$ B's money; $x + 17.50 =$ A's.

$$\frac{2x}{5} = \frac{x + 17.50}{3}.$$

A's rate is $\frac{17}{18}$ of B's; B's time is $\frac{17}{18}$ of A's. To run the whole distance, A needs 34 min. $\div \frac{17}{18} = 36$ min. If he runs $2\frac{1}{8}$ miles

in $16\frac{4}{5}$ min., in 1 min. he runs $2\frac{1}{3}$ mi. $\div 16\frac{4}{5}$, and in 36 min. he runs $(2\frac{1}{3}$ min. $\div 16\frac{4}{5}) \times 36 = \frac{7}{3}$ mi. $\times \frac{5}{84} \times \frac{36}{1} = 5$ mi. *Ans.*

5. $x - 1\frac{7}{8}\%$ of $x = 96084$; $x - \dfrac{15x}{800} = 96084$; $800x - 15x = 76867200$; $785x = 76867200$; $x = 97920$.

6. For the information of the teacher, the following method is given:

$$\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{2-2\sqrt{2}+1}{2-1} = \frac{3-2\sqrt{2}}{1} = 3-2\sqrt{2}.$$

We learn in algebra that the sum of two numbers $(x+y)$ multiplied by their difference $(x-y)$ gives the difference of their squares (x^2-y^2). If the sum of $\sqrt{2}$ and 1 be multiplied by their difference $(\sqrt{2}-1)$, we obtain the difference of their squares $(2-1)$. Multiplying the numerator also by $\sqrt{2}-1$, we retain the equality and obtain a divisor that has no decimals. See Art. 1169, **3**.

It is not expected that this method should be given to the pupils.

7. For 42 da., 50 men were at work. To do the same work, 30 men would have required 70 da., or $(70+40)$ da. to do the whole work. 110 da. $-$ 84 da. $=$ number of days the contractor would have been behindhand.

8. The number of square feet in the wall $= (23\frac{2}{3} + 15\frac{5}{6} + 23\frac{2}{3} + 15\frac{5}{6}) \times 11\frac{3}{4} = 928\frac{1}{4}$ sq. ft. The two windows contain (19×5) sq. ft. $= 95$ sq. ft.; the fireplace contains $(4\frac{1}{2} \times 6)$ sq. ft. $= 27$ sq. ft.; the door contains $(7\frac{1}{2} \times 3\frac{1}{2})$ sq. ft. $= 26\frac{1}{4}$ sq. ft.; a total of 95 sq. ft. $+$ 27 sq. ft. $+ 26\frac{1}{4}$ sq. ft. $= 148\frac{1}{4}$ sq. ft. There remain to be papered $928\frac{1}{4}$ sq. ft. $- 148\frac{1}{4}$ sq. ft. $=$ 780 sq. ft. $= \frac{780}{9}$ sq. yd. A roll of paper contains $12 \times \frac{26}{36}$ sq. yd.; the number of rolls will be, therefore, $\dfrac{780}{9} \div \left(12 \times \dfrac{26}{36}\right) = \dfrac{780 \times 1 \times 36}{9 \times 12 \times 26}$; and its cost, $\dfrac{\$4.08 \times 780 \times 1 \times 36}{9 \times 12 \times 26} = \$40.80$. *Ans.*

9. $x+\frac{7x}{9}=336.$

10. The "present worth" of \$365 due in 30 da. $=$ \$365 $\div$ 1.005 $=$ \$363.18 $+$ $=$ the cost of the horse in cash. The "present worth" of the selling price $=$ \$435 $\div$ 1.02 $=$ \$426.47 $+$. Gain $=$ \$426.47 $-$ \$363.18 $=$ \$63.29, which is 17.43% of the cost.

If the seller has the note for \$435 discounted at a bank, he will receive in cash \$435 $-$ \$8.70 $=$ \$426.30. If he uses this money to buy the note he has given, he should pay, at bank rates, \$365 $-$ \$1.82½ $=$ \$363.17½. The profit would be \$426.30 $-$ \$363.17½ $=$ \$63.12½, which is 17.38% of the cost.

11. £57 1*s.* 8*d.* $=$ 13700*d.*; £2 11*s.* 4½*d.* $=$ 616½*d.*

$$13700 \times \frac{15}{200} \times x = \text{interest} = \frac{2055x}{2} = 616\tfrac{1}{2},$$

$$2055\,x = 1233,$$

$$x = \tfrac{1233}{2055};\ \tfrac{1233}{2055}\text{ yr.} = 7\text{ mo. }6\text{ da. } \textit{Ans.}$$

13. A man that does only $\frac{5}{6}$ of a day's work, does 14 da. less work in 84 da. than the average. The contractor therefore loses, in 84 da. on three men, 14 da. $+$ 12 da. $+$ $9\frac{1}{3}$ da. $=$ $35\frac{1}{3}$ da. He gains on two others $10\frac{1}{2}$ da. $+$ $8\frac{2}{5}$ da. $=$ $18\frac{9}{10}$ da. The net loss $=$ $35\frac{1}{3}$ da. $-$ $18\frac{9}{10}$ da. $=$ $16\frac{13}{30}$ da. The extra 17 men have to do the equivalent of $16\frac{13}{30}$ days' work; each has, therefore, to do $16\frac{13}{30}$ days' work $\div$ 17 $=$ $\frac{29}{30}$ of a day's work, or $\frac{1}{30}$ less than the average.

14. Making no allowance for waste, etc., two strips, each 260 ft. long, will be needed for two sides; two strips, each $(93-3-3)$ ft., or 87 ft. long, will be needed for the other two, or 520 ft. $+$ 174 ft. $=$ 694 ft. $=$ $231\frac{1}{3}$ running yards, 1 yd. wide, making $231\frac{1}{3}$ sq. yd. Cost at 90¢ $=$ \$208.20.

The surface to be carpeted $=(260-5)$ ft. by $(93-5)$ ft. $=$ 85 yd. $\times$ $29\frac{1}{3}$ yd. Cost $=$ \$2.09 $\times 85 \times 29\frac{1}{3} \div \frac{38}{36}$ $=$ \$4936.80. Total, \$4936.80 $+$ \$208.20 $=$ \$5145. *Ans.*

15. Since the meeting-place is twice as far from A as from B, the first man goes twice as fast as the other; the latter, there-

fore, walks $2\frac{1}{2}$ mi. per hour. If x is the distance between A and B, the first will require $\frac{x}{5}$ hr., and the second $\frac{x}{2\frac{1}{2}}$ hr. $= \frac{2x}{5}$ hr.

$$\frac{2x}{5} - \frac{x}{5} = 1;\ x = 5.\quad \textit{Ans.}\ 5\text{ mi.}$$

16. Let $x =$ number of miles between A and C. Then $x - 15$ $=$ distance between B and C.

$\frac{x-15}{15} =$ time required for first train to run from B to C; $\frac{x}{25} =$ time required for second train to run from A to C. As the latter train leaves 3 hr. later and arrives one-half hour later, the running time of the first is $2\frac{1}{2}$ hr. longer.

$$\frac{x-15}{15} = \frac{x}{25} + \frac{5}{2};$$

$$10x - 150 = 6x + 375;$$

$$4x = 525;\ x = 131\tfrac{1}{4}.\quad \textit{Ans.}\ 131\tfrac{1}{4}\text{ mi.}$$

1169. **2.** The width of the road $= (60 \div 16\frac{1}{2})$ rd.; its area $= (104 \times 60 \div 16\frac{1}{2})$ sq. rd. $= [104 \times (60 \div 16\frac{1}{2}) \div 160]$ acres.

$$\text{Its cost} = \frac{\$154 \times 104 \times 60 \times 2}{33 \times 160} = \$364.$$

$$\text{The cost of grading} = \frac{\$200 \times 104}{320} = \$65.$$

$$\text{The cost of fencing} = \$\tfrac{1}{2} \times 104 \times 5\tfrac{1}{2} = \$286.$$

3. See Art. 1168, **6**.

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{5+2\sqrt{15}+3}{5-3}$$

$$= \frac{8+2\sqrt{15}}{2} = 4 + \sqrt{15}.$$

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{5-2\sqrt{15}+3}{5-3}$$

$$=\frac{8-2\sqrt{15}}{2}=4-\sqrt{15}.$$

$$4+\sqrt{15}-(4-\sqrt{15})=2\sqrt{15}=\sqrt{60}.$$

4. A cubic centimeter $= .3937^3$ cu. in. $= (.3937^3 \div 1728)$ cu. ft. Its weight = weight of 1 gram $= (.3937^3 \times 1000 \div 1728)$ oz. $= [(.3937^3 \times 1000) \div (1728 \times 16)]$ lb. A kilogram $= [(1000 \times .3937^3 \times 1000) \div (1728 \times 16)]$ lb. The weight of the anchor in kilograms = 6500 lb. ÷ the number of pounds in a kilogram; or,

$$\frac{6500 \times 1728 \times 16}{1000 \times .3937 \times .3937 \times .3937 \times 1000}.$$

5. $x-(x \times \frac{8}{100} \times \frac{63}{360})=1500;$

$$x-\frac{7x}{500}=1500;\ 500x-7x=750000;$$

$$493x=750000;\ x=1521.30-. \quad \textit{Ans.}\ \$1521.30.$$

The proceeds of the new note + \$200 must pay the note of \$2000; the proceeds must therefore be \$1800.

$$x-(x \times \tfrac{8}{100} \times \tfrac{93}{360})=1800.$$

1170. **2.** $(80.005-.013) \div 88.$

5. Let x = number of cents received by Thomas. Then $\frac{3x}{5}$ = number received by Henry, and $\frac{6x}{25}$ = number received by Richard.

$$x+\frac{3x}{5}+\frac{6x}{25}=4.14;$$

$$25x+15x+6x=103.50;$$

$$46x=103.50;\ x=2.25.$$

6. 272 liquid quarts = 231 cu. in. × 272 ÷ 4. A dry quart = 2150.4 cu. in. ÷ 32.

$$\text{Number of dry quarts}=\frac{231 \times 272 \times 32}{4 \times 2150.4}=\frac{231 \times 272 \times 320}{4 \times 21504}.$$

7. The tub holds $12\frac{1}{2}$ qt. $\times 4\frac{1}{3} = 54\frac{1}{6}$ qt. Both pipes discharge $12\frac{1}{2}$ qt. $+ 83$ qt. $= 95\frac{1}{2}$ qt. The time required to fill it $= (54\frac{1}{6} \div 95\frac{1}{2})$ min.

8. The number of hours that must elapse before all will again be together at the starting point, is the least common multiple of $\frac{5}{36}, \frac{2}{9}, \frac{35}{99}$. The least common multiple of the numerators is 70. The smallest fraction that will contain the above fractions an exact number of times must have 70 for its numerator, and for the denominator the largest number that will divide 36, 9, and 99 without a remainder; *i.e.* the greatest common divisor of these numbers. The G. C. D. is 9, and the fraction is $\frac{70}{9}$. In $\frac{70}{9}$ hr., therefore, A, B, and C will be at the starting point. A will have walked around the circle $(\frac{70}{9} \div \frac{5}{36})$ 56 times; B, $(\frac{70}{9} \div \frac{2}{9})$ 35 times; C, $(\frac{70}{9} \div \frac{35}{99})$ 22 times.

NOTE. — The scholars will readily understand that the fraction which is the least common multiple of $\frac{5}{36}$, $\frac{2}{9}$, and $\frac{35}{99}$, should have 70 for its numerator. The following may make clear to them why 9 should be the denominator:

$$\frac{70}{x} \div \frac{5}{36}, \text{ or } \frac{2}{9}, \text{ or } \frac{35}{99} \text{ should be a whole number;}$$

i.e.

$$\frac{70}{x} \times \frac{36}{5}, \text{ or } \frac{9}{2}, \text{ or } \frac{99}{35} \text{ should be a whole number.}$$

An examination of the second line, in which the divisors are inverted, will show that 70 contains the three denominators, 5, 2, and 35, an exact number of times; the numerators, 36, 9, and 99, should contain x an exact number of times; x, therefore, must be a divisor of these numbers, etc.

1172. This work may be slightly abbreviated by combining the interest on the annual interest into one item of 6 years' interest instead of the three separate ones of 3 years' interest, 2 years' interest, and 1 year's interest. In beginning a new topic, however, pupils should not be confused by short methods.

2. $\$1200 + \$300 + (4 + 3 + 2 + 1)$ years' interest on $\$60$.

For partial payments on notes bearing annual interest, see Art. 1308. The special rules for New Hampshire and Vermont will be found in Arts. 1309 and 1310.

In states in which the collection of annual interest is not allowed, the teacher should omit this topic.

1173. In the older states, time should not be spent upon this topic.

1176. 1. $\frac{1}{4}$ of $\frac{1}{2}$ a section $= \frac{1}{8}$ of 640 A.

2. $\frac{1}{2}$ of $\frac{1}{4}$ section measures 80 rd. by 160 rd.

3. A line from the southwest corner of Sec. 1, to the northeast corner of Sec. 30 (see township diagram on the opposite page), is the hypotenuse of a right-angled triangle whose perpendicular, the eastern boundaries of Secs. 11, 14, and 23, is 3 mi. long; and whose base, the southern boundaries of Secs. 20, 21, 22, and 23, is 4 mi. long.

5. The number of rods of fence $= 80 + 160 + 80 + 160 = 480$. The number of feet $= 16\frac{1}{2} \times 480 = 7920$. A fence 4 boards high requires 7920 ft. $\times$ 4, or 31680, running feet of boards. If the latter are $\frac{1}{2}$ ft. wide, the number of board feet $= 31680 \times \frac{1}{2} = 15840$.

1184. 2. $26.50 \times .85$. 4. Multiply 135 by 69, and point off two places in the product. 5. Find the base. 6. 8.50 francs $\times (10 \times 1 \times 3.25)$. 7. Each dimension can be expressed in decimeters, $105 \times 80 \times 65$, whose product is the number of liters; or the product of the dimensions in meters — $10.5 \times 8 \times 6.5$ — may be multiplied by 1000. 8. 0*l*.75 means .75*l*., the denomination in France being generally written before the decimal. 9. 1.25 marks $\times [(68 \div 10) \times 36]$; *i.e.* $1\frac{1}{4}$ marks $\times 6.8 \times 36$. 10. The number of liters $= 50 \times 40 \times 30 = 60000$; 92% of this number gives the weight in kilograms (kilos).

1186. These problems are given for practice in obtaining the approximate values of the metric units in terms of our weights and measures. The use of 39.37 in. makes the work too tedious.

13. 4 in. by 4 in. by 4 in. A quart $= \frac{231}{4}$ cu. in.

14. A hectoliter = 100 liters = 6400 cu. in. = (6400 ÷ 2150.4) bu. 6400 cu. in. = (6400 ÷ 231) gal.

15. A liter of water, 64 cu. in., weighs a kilo. 1 cu. in. of water = $\frac{1000}{1728}$ oz.; 64 cu. in. = [(64000 ÷ 1728) ÷ 16] lb. = 4000 lb. ÷ 1728.

16. 400000000 in. = $\frac{1}{4}$ circumference. See Arithmetic, Art. 1177.

17. A square meter = (40 × 40) sq. in.

18. An are = (400 × 400) sq. in. A hectare = [(400 × 400) × 100] sq. in.

19. Hectometer = 100 meters = 4000 in.

20. A stere = (40 × 40 × 40) cu. in. = (64000 ÷ 1728) cu. ft.

21. 1000 grams weigh (4000 lb. ÷ 1728); 1 gram weighs 4 lb. ÷ 1728 = 28000 grains ÷ 1728.

22. A kilometer = 40000 in.; a mile = 63360 in.; a mile = (6.336 ÷ 4) Km.

1197. The average pupil should be permitted to use a pencil for his first solution of these problems.

1. Let x = the value of the second suit. Since $12 and the overcoat = $2x$, the overcoat = $2x - 12$. The second suit (x) and the overcoat ($2x - 12$) = three times the first suit (36).

$$x + 2x - 12 = 36;\ \text{etc.}$$

The second suit is worth $16; the overcoat, $20. *Ans.*

2. $x - 22 + \frac{x - 22}{4} = \frac{x}{3}$; etc.

The arithmetical analysis might assume some such form as this: The remainder + $\frac{1}{4}$ of the remainder, or $\frac{5}{4}$ of the remainder = $\frac{1}{3}$ of original sum. The remainder = $\frac{1}{3}$ of original sum × $\frac{4}{5}$ = $\frac{4}{15}$ of original sum. The sum lost is $1 - \frac{4}{15}$, or $\frac{11}{15}$ of original sum. As this is $22, the original sum = $22 × $\frac{15}{11}$ = $30. *Ans.*

3. Let $x =$ time past noon; $x + 12 =$ time past midnight. $x = \frac{x+12}{5}$; $5x = x + 12$; $4x = 12$; $x = 3$. The time is 3 hr. past noon, or 3 P.M. *Ans.*

4. At 3 o'clock, the hour hand is 15 minute spaces in advance. To be only 5 spaces behind, the minute hand must gain 10 spaces. While the minute hand goes 1 space, the hour hand goes $\frac{1}{12}$ space; so that each minute, the minute hand gains $\frac{11}{12}$ space. To gain the 10 spaces necessary, the minute hand must travel $(10 \div \frac{11}{12})$ minutes $= 120$ min. $\div\ 11 = 10\frac{10}{11}$ min. The time is $10\frac{10}{11}$ min. past 3.

5. $\frac{1}{5}$ A $= \frac{4}{5}$ B; A $= 4$ B; 5 B $= 30$. B's age $= 6$ yr.; A's age $= 24$ yr. *Ans.*

6. A takes \$15 less than $\frac{3}{4}$ of the profits. If his capital is \$30 less than $\frac{3}{4}$ of the whole, the latter must be double the profits, or \$1440. A's capital $=$ \$525 $\times 2 =$ \$1050; B's $=$ \$390. *Ans.*

Or, A takes $\frac{525}{720}$, or $\frac{35}{48}$, of the profits; he owns, therefore, $\frac{35}{48}$ of the capital. If $\frac{35}{48}$ of the capital $+$ \$30 $= \frac{3}{4}$, or $\frac{36}{48}$, of the capital, $\frac{1}{48}$ of the capital $=$ \$30; etc.

7. Let $x =$ the number of sheep; $\frac{80}{x} =$ cost of each; $x - 5$ $=$ number remaining; $\frac{2x-10}{3} =$ number sold; $\frac{2x-10}{3} \times \frac{80}{x} =$ $\frac{160x - 800}{3x} =$ sum received $= 40$.

$160x - 800 = 120x$; $40x = 800$; $x = 20$. *Ans.* 20 sheep.

Or, if he received \$40 for $\frac{2}{3}$ of the remainder, he would have received \$60 for the remaining sheep. \$60 being $\frac{3}{4}$ of \$80, $\frac{3}{4}$ of the sheep remained, and $\frac{1}{4}$ of them died, or 5 sheep. The whole number was, therefore, 20 sheep.

8. Let $x =$ A's age; then $x + 10 =$ B's age;

$$\frac{x}{2} = \frac{x+10}{3}; \text{ etc.}$$

1198. 3. $7\underline{)\ 19 \text{ mi. } 180 \text{ rd. } 2 \text{ yd. } 0 \text{ ft. } 9 \text{ in.}}$

$$\begin{array}{r} 2 \text{ mi. } 254 \text{ rd. } 1 \text{ yd. } 2 \text{ ft. } 8\tfrac{1}{7} \text{ in.} \\ \times 12 \\ \hline 33 \text{ mi. } 172 \text{ rd. } 0 \text{ yd. } 2 \text{ ft. } 1\tfrac{5}{7} \text{ in.} \end{array}$$

4. 18 hr. 24 min. 12 sec. $\div$ 15 = 1 hr. 13 min. $36\frac{4}{5}$ sec.

1 hr. 45 min. time difference = $1° \, 45' \times 15 = 26° \, 15'$ difference in longitude. As the place has the later time, it is more easterly.

7. Calling it 1 in. thick, the number of board feet $= 16 \times \frac{3}{4} = 12$. $\$40 \times .012 = 48¢$. *Ans.*

$\$40 \times (16 \times \frac{3}{4} \times 2\frac{1}{2}) \div 1000 = \$1.20$. *Ans.*

9. [96 (in.) $\times$ 90 (in.) $\times$ 48 (in.)] $\div$ 2150.4.

10. If the strips run lengthwise, their number will be 8 yd. $\div \frac{3}{4}$ yd. $= 10\frac{2}{3}$. The number purchased must be 11, each 9 yd. long, or 99 running yards of carpet.

XVIII

NOTES ON CHAPTER FIFTEEN

While the work contained in this chapter is intended more particularly for use in such schools as extend their instruction beyond the eighth year of the elementary course, it can profitably replace some of the less useful arithmetical topics taught during the eighth school year.

1199. These exercises should be taken up without any preliminary explanations. Their previous work in simple equations has so familiarized the pupils with the use of letters to express numbers, etc., that they need no assistance in the first ten examples. The necessary technical terms should be employed as occasion requires, and their meanings should be made clear; but exhaustive treatment of the different operations should be left for the study of the science of algebra in the high school.

1200. The explanation of the meaning xy, abc, etc., may be deferred until Art. 1238. For the present, the use of the word *coefficient* may be limited to simple numerical ones, as given in the text-book. The teacher should not yet explain that in the expression $5xy$, $5x$ may be considered the coefficient of y; nor that in $9abc$, $9a$ may be considered the coefficient of bc, and $9ab$ the coefficient of c.

1204. So far, the pupils have been required to add only single columns containing the same letters. When the signs are alike throughout, as in Art. 1199, they have found the sum of the coefficients, annexed the letter or letters, and prefixed the

common sign. When the signs are unlike, the difference between the sums of the coefficients of the positive and of the negative terms is written, preceded by the sign of the greater sum.

It will scarcely be necessary to state to pupils that algebraic expressions containing dissimilar terms are added by placing the plus sign between them; thus the sum of $4ab$ and $3ac$, for instance, is written $4ab + 3ac$.

1205. The expressions employed in the preceding exercises are called *monomials*, or algebraic expressions of *one* term. Those of more than one term are called *polynomials*.

A polynomial of two terms is called a *binomial;* one of three terms, a *trinomial.*

6. The scholar will readily see that in the addition of polynomials, each column should contain similar terms; *i.e.* terms containing the same letters. That the letters should also be affected by the same exponents, need not be told him for the present.

1207. From some of the preceding examples, may be seen the use of the plus and of the minus sign to indicate direction north and south, and east and west; past and future time, etc.

In **2**, is required the difference between $-10°$ and $+90°$. In **6**, there is asked the sum of $+40¢$ and $-50¢$. Calling the distance north of the starting point $+50$ miles, in **8**, and the distance south -70 miles, the required location will be $(+50$ miles$) + (-70$ miles$) = -20$ miles, or 20 miles south.

6 and **8** are problems in algebraic addition; **10**, like **2**, is a problem in subtraction. The results of a man's transactions during a month are ascertained by subtracting the value of his possessions at the beginning of the month from their value at the end. In **10**, a man is worth $-\$250$ on Feb. 1; deducting from this $+\$150$, which represents his condition on Jan. 1, we obtain $-\$400$. The operation may be indicated thus: $(-\$250) - (+\$150) = -\$400$, the minus sign in the result indicating a loss.

The algebraic analyses of these problems, if asked at all, should not be required until the pupils have solved them in their own way. The main object of teaching subtraction at this stage, is to enable the scholars to understand the reasons for the change of signs that accompanies the removal of a parenthesis preceded by a minus sign. See Art. 1210.

1210. Considering (*a*) as an example in subtraction, it may be made one in addition (*b*) by changing the signs of the subtrahend. It may then be written $84 - 49 + 25$.

(*a*) From 84
Take $\underline{49 - 25}$

(*b*) 84
$\underline{-49 + 25}$

1211. The pupil will readily ascertain that a parenthesis preceded by a plus sign may be removed without any alteration b—g required in the signs of the quantities enclosed within it. · $(33 - 16) = 74$, may be written $57 + 33 - 16 = 74$. In **2**, signs of the quantities within a parenmust be changed. The first number the parenthesis, being without a sign, is positive; it therefore becomes negative when the parenthesis is taken away. The equation then becomes $92 - 63 - 25 = 4$. In **5**, the multiplier 4 affects only the quantity within thesis. The brackets heretofore used, have been omitted **6**, to make these arithmetical equations resemble more e algebraic equations of Art. 1213. **5** becomes 75 + 95; **6** becomes $75 - 60 + 40 = 55$.

From + 92
Take $\underline{+63 + 25}$

$\underline{25}$
$63 - 25$

It has not been considered necessary to give any previous practice in multiplying simple algebraic polynomials by a ordinary number. The average pupil will readily understa that 6 times two x = twelve x.

1. $12x - 30 = 5x + 12$.

$$12x - 5x = 12 + 30;\ 7x = 42;\ x = 6.$$

Proof. $6\,(12 - 5) = 30 + 12$; *i.e.*, 6 times $7 = 42$.

2. $7x + 14 = 3x + 50$; etc.

3. $15 + 5x + 16 = 61$; etc.

4. $48 - 3x = 52 - 4x$; etc.

7. $2x - 2 - 4x + 38 = 3x - 9$; etc.

8. $12x - 30 - 5x = 12$; etc.

9. $5x - 12x + 30 = -12$; etc.

10. $11 - 3x + 10x = 38$; etc.

1215. 12. $3x - 3 - 2x + 4 = 12.$

13. $6x - 6 - 4x + 8 - 3x + 9 + 24 = 0.$

15. $14x - 16 - 18x - 36 = 12x + 15 - 6x - 12.$

Transposing, $14x - 18x - 12x + 6x = 15 - 12 + 16 + 36.$

Combining, $-10x = 55.$

Changing the signs of both members,

$$10x = -55.$$

Or, $x = -5\frac{1}{2}.$

17. $\frac{39}{4} - \frac{5x}{4} + \frac{x}{2} = \frac{3x}{8} + \frac{15x}{4}$; etc.

18. $2x = 3 + \frac{9x}{4} - 5 - \frac{2x}{5} + \frac{13}{5}.$

19. $\frac{3x}{4} + 9 = 2x + \frac{3x}{5} - \frac{x}{2}.$

22. $x - 20 = \frac{4x}{7} + 60.$

1216. 1. $\frac{3\frac{2}{3}x - 7}{16} = 3.$

Multiplying by 16, $3\frac{2}{3}x - 7 = 48$; or $\frac{11x}{3} - 7 = 48.$

Clearing of fractions, $11x - 21 = 144$; etc.

2. $\frac{3x}{8} = x - 60$; etc.

3. $x+\frac{x}{3}+x+\frac{x}{4}+x+\frac{x}{5}+x+\frac{x}{6}=99.$

5. $x-\frac{x}{6}-\frac{x}{8}+\frac{x}{3}=x+4.$

6. $x+12=$ son's present age; $2x+12=$ father's present age.

$$x+12+2x+12=138;$$
$$3x=138-24=114;$$
$$x=38, \text{ the son's age 12 yr. ago};$$
$$2x=76, \text{ the father's age 12 yr. ago.}$$

The present age of the son is 50 yr. $(x+12)$; the present age of the father is 88 yr. $(2x+12)$. *Ans.*

7. $(80+x)=2\frac{1}{2}(60-x)$; etc.

8. $2(11+x)=25+x.$

9. Let $x=$ the number of gallons originally in the cask. $\frac{x}{4}=$ amount drawn off, leaving $\frac{3x}{4}$ in the cask. $60-\frac{3x}{4}=$ number of gallons required to fill the cask.

$$24=60-\frac{3x}{4};$$
$$96=240-3x; \text{ etc.}$$

10. $\frac{x}{3}-40=104.$

11. $\frac{x+430}{x}=4+\frac{76}{x}.$

Clearing of fractions, $x+430=4x+76.$
Transposing, $x-4x=76-430.$
Combining, $-3x=-354.$
Changing the signs of both members,

$$3x=354.$$

Or, $x=118$, the smaller number;
$x+430=548$, the larger number.

12. Let $x =$ the number of \$2 bills.
Then $29 - x =$ the number of \$5 bills.

$$2x + 5(29 - x) = 103; \text{ etc.}$$

13. $4(x + 4) = x + 34.$

14. $(x + 3) \times \frac{180}{x} = 225.$

Multiplying, $$\frac{180x + 540}{x} = 225.$$

Clearing of fractions, $180x + 540 = 225x$; etc.

15. $33(x + 1) = 40x + 12.$

16. The numbers are x and $x + 17$.
Then $$x + x + 17 = 47; \text{ etc.}$$

17. Let $x =$ number of years.
The mother's age will then be $41 + x$, and the son's $5 + x$.

$$5 + x = \tfrac{1}{3}(41 + x) = \frac{41 + x}{3};$$
$$15 + 3x = 41 + x; \text{ etc.}$$

1218. **1.** Substituting the value of $8x$, the equation becomes
$$16 + 7y = 44; \text{ etc.}$$

2. $9 + 5z = 34$; $5z = 25$; $z = 5$. *Ans.*

1219. **11.**
$$\begin{aligned} 3x + 14y &= 78 \\ 2x + 14y &= 66 \\ \hline \text{Subtracting,} \quad x \qquad &= 12 \end{aligned}$$

Substituting this value of x in the first equation,
$$36 + 14y = 78; \text{ etc.}$$

14. Multiplying the first equation by 2, we have $2x + 2y = 30$. Subtract the new equation from the second one, $2x + 3y = 38$, thus finding the value of y. Substitute this value in either of the original equations.

17. Multiplying the first equation by 3, and the second by 2, we have $6x + 9y = 120$ and $6x + 4y = 70$.

18. Multiply the first by 2, and the second by 7.

19. Multiply the first by 9, and the second by 5.

20. Multiply the first by 8, and the second by 3.

1221. **21.** If we add, we have $2x = 22$. Or, subtracting, $2y = 14$. By adding, y is eliminated; by subtracting, x is eliminated. From this example, pupils should see that either addition or subtraction may be employed to eliminate one of the unknown quantities; and that either of the two may be eliminated, as may be found convenient.

27. $5x - 6y = 5$, $3x - 5y = -4$.

28. $3x + 5y = -8$, $2x - y = 12$.

29. $-10x + y = -1$, (1) $-5x + y = 9$. (2) Subtract (1) from (2).

30. $3x + 8y = 204$, $10x + 5y = 160$.

31. $3x + 2y = 252$, $7x + 5y = 609$.

34. $3x + 7 = 15y - 20$; $3x - 15y = -27$. (1)
$7x - 6 = 10y + 6$; $7x - 10y = 12$. (2)

Multiply (1) by 7 and (2) by 3, to eliminate x; or (1) by 2 and (2) by 3, to eliminate y.

35. $51x + 44y = 804$, (1) $45x - 32y = 72$. (2) Multiply (1) by 8, and (2) by 11. Add.

37. $2x - 22 - 2y + 18 = 6$, $15x + 135 = 32y - 96$. The pupils should be taught to indicate the common denominator of $y - 3$ and 15, as $15(y - 3)$, which contains $y - 3$, 15 times; and 15, $(y - 3)$ times.

39. $2x + 5y + 3 = 18x - 24y - 12$; $-16x + 29y = -15$.
$4x - 7y + 5 = 5x - 10y + 10$; $-x + 3y = 5$.

1222. 1. $x + y = 37$; $2x + 3y = 96$. This problem can also be solved by the use of one unknown quantity, by calling the numbers x and $37 - x$. The equation becomes $2x + 3(37 - x) = 96$; or, $2x + 111 - 3x = 96$.

2. Using one unknown quantity, the numbers are x and $x + 28$. $5x - 2(x + 28) = 197$.

3. $5x + 3y = 37$; $6x - 5y = 10$.

4. $x + y = 65$; $x - y = 19$.
By one unknown quantity, $x + (x + 19) = 65$.

5. $x + y = 32$; $2x + 5y = 103$. See Art. 1216, **12**.

6. $x + y = 25$; $7x + 5y = 145$.

7. $10x + 4y = 38$; $6x + 7y = 32$.

8. $5x + 3y = 375$ (cents); $8x + y = 505$ (cents).

9. $125(x) + 45(4x) + 10(8x) + 5(\frac{1}{2}x) = 1550$.

11. $x + y = 19$; $y + 10x - (x + 10y) = 45$.

12. $13x = 5y$; $x + y = 126$.

13. $15x = 8y$; $x - y = -147$. $\frac{8}{15}$ being a *proper* fraction, any equivalent fraction must have a denominator exceeding the numerator.

1223. 4. Eliminate z by comparing the first and the second equation, multiplying the latter by 5. Multiply the second by 3 and compare with the third equation, eliminating z.

5. First eliminate y.

6. $x + x + y = 42$; $3x + 3y - x + y = 96$.

7. $15x - 25y + 30 = 4x + 2y$; $11x - 27y = -30$.
$96 - 3x + 6y = 6x + 4y$; $-9x + 2y = -96$.

9. $15x - 9 - 9x + 57 = 24 - 6y + 2x$,
$16x + 8y - 18x + 14 = 12y + 36 - 4x - 5y$.

Transposing and combining, $4x + 6y = -24$; $2x + y = 22$. Eliminate x by dividing the first equation by 2; etc.

1224. 1. $x + \frac{x}{30} + \frac{x}{50} = 31600.$

2. Let x = number of B's chestnuts; $x + 18$ = number of A's chestnuts.

$$x + 18 + 4 = 4(x + 4).$$

3. $x + y = 8$; $23x + 17y = 166$.

4. $x + y = 55$; $x + z = 62$; $y + z = 83$.

Comparing the first two, we get $y - z = -7$; adding this to the third eliminates z; etc.

6. $x = \frac{x}{5} + \frac{16x}{45} + \frac{8x}{45} + 24.$

7. $x - \frac{3x}{20} = 510.$ 8. $x - \frac{x}{50} = 147.$

10. $\frac{3x}{7} = \frac{x}{5} + 16.$

11. Let x = value of the clothes. $x + 280$ = yearly wages. $\frac{1}{2}(x + 280)$ = wages for 6 months $= x + 130$.

Clearing of fractions, $x + 280 = 2x + 260$; etc.

1228. 14.

$$\begin{array}{rl} & 3x + 6 \\ & 2x - 3 \\ \hline 2x(3x+6) & 6x^2 + 12x \\ -3(3x+6) & \quad\; - 9x - 18 \\ \hline & 6x^2 + 3x - 18 \;\; \textit{Ans.} \end{array}$$

1236. 5. $5x^2 + 85 - 3x^2 + 63 = 198$; etc.

7. $x^2 + 2x + 1 - x^2 = 49$; etc.

8. $4y^2 + 20 - 6y^2 + 54 = 24$; etc.

9. Clearing of fractions, Art. 1221, 37, we have $(z + 7)(z - 9) = (z - 5)(z - 3)$.

Performing the multiplication indicated,

$$z^2 - 2z - 63 = z^2 - 8z + 15;$$
$$6z = 78;\ z = 13.\ \textit{Ans.}$$

10. $20x(x+1) = 30x(x-1)$; etc. Divide by x.

13. $(x+4)(x+4) = 8x+80$; etc.

15. $6x^2+36 = 5x^2+72$.

16. $x^2-6x+9-(x^2-10x+25) = 12$; removing the parenthesis, $x^2-6x+9-x^2+10x-25 = 12$; etc.

18. The common denominator is $36x$. Clearing of fractions,

$$9x^2+144 = 4x^2+324; \text{ etc.}$$

19. $(x+7)(x-9) = (x-3)(x-5)$.

20. $(y-9)(y+7) = (y-3)(y-5)$.

1237. **1.** Let x = the breadth; $2x$ = the length. The area $= x \times 2x = 2x^2 = 1800$; etc.

2. Let x = the length of one edge. The area of one face $= x^2$; that of six faces is $6x^2$, and is equal to 96 sq. in.

$$6x^2 = 96; \text{ etc.}$$

3. Let x = one number; $\frac{4x}{5}$ = the other.

$$x \times \frac{4x}{5} = \frac{4x^2}{5} = \text{their product} = 80; \text{ etc.}$$

4. $\frac{x}{3} \times \frac{2x}{5} = \frac{2x^2}{15} = 270$; etc.

5. $\frac{30x}{100} \times \frac{40x}{100} = \frac{3x^2}{25} = 300$; etc.

6. 40% of a number $(x) = \frac{2x}{5}$; 30% of $\frac{2x}{5} = \frac{3}{10}$ of $\frac{2x}{5} = \frac{3x}{25}$. $\frac{3x}{25} = 300$; etc.

7. Let x = the length of the perpendicular; $\frac{3x}{4}$ = the length of the base. The area $= \frac{1}{2}\left(x \times \frac{3x}{4}\right) = \frac{3x^2}{8}$.

$$\frac{3x^2}{8} = 96;\ 3x^2 = 768;\ x^2 = 256;\ x = \pm 16.$$

Neglecting the negative result, the perpendicular measures

16 rd., and the base 12 rd. The hypotenuse $=\sqrt{16^2+12^2}$ rd. $=20$ rd.

8. $x^2+\left(\frac{3x}{4}\right)^2=15^2$; $x^2+\frac{9x^2}{16}=225$; $16x^2+9x^2=3600$; etc.

9. $(x+9)^2=x^2+15^2$; $x^2+18x+81=x^2+225$; etc.

10. $(x+1)^2-x^2=49$; etc.

1238. The pupils should be informed that the product of the numbers represented by two letters is represented by writing the letters together; thus a times b is written ab, m times n is written mn, just as 3 times x is written $3x$.

1242. 1. x^2+6x+9.
2. $x^2-12x+36$.
3. $x^2-8x+16$.
6. x^2+2x+1.
7. x^2-4x+4.
8. $x^2-10x+25$.

1244. 1. $x^2+6x+9=49$; $x+3=\pm 7$. *Ans.*

2. $x^2-12x+36=64$; $x-6=\pm 8$. *Ans.*

5. $x^2+18x+81=19+81=100$; $x+9=\pm 10$. *Ans.*

6. $x^2+2x+1=24+1=25$; $x+1=\pm 5$. *Ans.*

7. $x^2-14x+49=15+49$; $x-7=\pm 8$. *Ans.*

1246. 1. $x^2-6x+9=7+9$; $x-3=\pm 4$; etc.

2. $x^2-12x+36=108+36$; etc.

3. $x^2+2x+1=48+1$; etc.

1247. The first member is made a complete square by adding the square of $\frac{1}{2}$ of the coefficient of x.

1248. 1. $x^2+x+\frac{1}{4}=12+\frac{1}{4}=\frac{49}{4}$.

$x+\frac{1}{2}=\pm\frac{7}{2}$; $x=\frac{6}{2}$, or $-\frac{8}{2}=3$ or -4. *Ans.*

2. $x^2-3x+\frac{9}{4}=10+\frac{9}{4}=\frac{49}{4}$;

$x-\frac{3}{2}=\pm\frac{7}{2}$; etc.

3. $x^2+5x+(\frac{5}{2})^2=-4+(\frac{5}{2})^2$; etc.

1249. 1. $x^2 - x = 6$; $x^2 - x + \frac{1}{4} = 6 + \frac{1}{4} = \frac{25}{4}$; etc.

1250. 1. $12x - x^2 = 32.$

Changing the signs, and rearranging,

$$x^2 - 12x = -32.$$

Completing the square, $x^2 - 12x + 36 = -32 + 36 = 4.$

Extracting the square root, $x - 6 = \pm 2.$

Transposing, $x = +2 + 6 = 8$; or $x = -2 + 6 = 4.$

$$12 - x = 12 - 8 = 4;\ \text{or}\ 12 - 4 = 8.$$

8 and 4, or 4 and 8. *Ans.*

2. $x^2 + 50x + 625 = 2400 + 625 = 3025.$

$$x + 25 = \pm 55;$$

$$x = 30\ \text{or}\ -80.$$

Neglecting the negative result, the altitude is 30 ft. *Ans.*

3. $x^2 + 225 + 30x + x^2 = 5625.$

Transposing, $2x^2 + 30x = 5625 - 225 = 5400.$

Dividing both members by 2, $x^2 + 15x = 2700.$

Completing the square, $x^2 + 15x + (\frac{15}{2})^2 = 2700 + (\frac{15}{2})^2$; etc.

4. Perpendicular $= \sqrt{\frac{25}{16}x^2 - x^2} = \sqrt{\frac{9}{16}x^2} = \frac{3}{4}x.$

Area $= \frac{1}{2}\left(x \times \frac{3x}{4}\right) = \frac{3x^2}{8} = 150.$

Clearing of fractions, $3x^2 = 1200,$

$$x^2 = 400,$$

$$x = 20.$$

Base $= 20$ yd.; hypotenuse $= 20$ yd. $\times 1\frac{1}{4} = 25$ yd. *Ans.*

5. The convex surface $= 6(x + x + x + x) = 24x$; the surface of the two bases $= 2x^2$; the entire surface $= 2x^2 + 24x = 170$; $x^2 + 12x = 85$; etc.

6. The area of the walk = area of outside rectangle − area of inner rectangle.

$$(40+2x)(50+2x)-40\times 50=784;$$
$$2000+180x+4x^2-2000=784;$$
$$4x^2+180x=784;$$
$$x^2+45x=196; \text{ etc.}$$

7. 12 acres = 1920 sq. rd.

$$\frac{15x^2}{8}=1920;\ 15x^2=15360;\ x^2=1024;\ x=\pm 32.$$

Base = 32 rd.; perpendicular = 32 rd. $\times 1\frac{7}{8}$ = 60 rd.; hypotenuse = $\sqrt{32^2+60^2}$ rd. = 68 rd. Diagonal = 68 rd. *Ans.*

8. $x^2+30^2=(50-x)^2$.

$$x^2+900=2500-100x+x^2;$$
$$100x=1600;\ x=16.$$

AC = 16 ft., CB, the part broken off = 50 ft. − 16 ft. = 34 ft. *Ans.*

Or, making $BC=x$, $AC=50-x$.

Then, $(50-x)^2+30^2=x^2$.

$$2500-100x+x^2+900=x^2$$
$$-100x=-3400;$$
$x=34$, the length in feet of the part broken off.

9. $60^2+(58-x)^2=56^2+x^2;$

$$3600+3364-116x+x^2=3136+x^2;$$
$$x^2-x^2-116x=3136-3600-3364$$
$$-116x=-3828, \text{ or } 116x=3828;$$
$$x=33=AE.$$

The length of the ladder in feet = $\sqrt{56^2+33^2}=\sqrt{3136+1089}$ $=\sqrt{4225}$. 65 ft. *Ans.*

10. From ABD, the square of $BD=13^2-(15-x)^2$. From BCD, the square of $BD=4^2-x^2$. Therefore

$$13^2-(15-x)^2=4^2-x^2;$$

or,
$$169-(225-30x+x^2)=16-x^2.$$

Removing the parenthesis, $169 - 225 + 30x - x^2 = 16 - x^2$.
Transposing and combining, $30x = 16 - 169 + 225 = 72$;

$$x = 2\tfrac{2}{5}.$$

$$BD = \sqrt{BC - CD} = \sqrt{4^2 - 2\tfrac{2}{5}^2} = \sqrt{16 - \tfrac{144}{25}} = \tfrac{16}{5} = 3\tfrac{1}{5}.$$

Altitude $= 3\tfrac{1}{5}$ ft. *Ans.*

11. $AF = \sqrt{AB^2 - BF^2} = \sqrt{1156 - 256} = \sqrt{900} = 30$;

$FC = \sqrt{BC^2 - BF^2} = \sqrt{400 - 256} = \sqrt{144} = 12$;

$AC = AF + FC = 30 + 12 = 42$. 42 ft. *Ans.*

Let $AE = x$; $EC = 42 - x$;

$$ED^2 = AD^2 - AE^2 = 26^2 - x^2 = 676 - x^2;$$

$$ED^2 = DC^2 - EC^2 = 40^2 - (42 - x)^2 = 1600 - (1764 - 84x + x^2).$$

Therefore $676 - x^2 = 1600 - 1764 + 84x - x^2$

$$-84x = 1600 - 1764 - 676 = -840;$$

$$x = 10.$$

$$ED = \sqrt{26^2 - 10^2} = \sqrt{576} = 24;$$

or, $= \sqrt{40^2 - 32^2} = \sqrt{576} = 24$. 24 ft. *Ans.*

XIX

NOTES ON CHAPTER SIXTEEN

The geometry work contained in this chapter should be commenced not later than the seventh year of school, and should be continued throughout the remainder of the grammar-school course.

1251. No formal definitions of *lines*, *angles*, etc., should be given at the beginning. After drawing angles of various sizes and with lines of different lengths, the pupils will be able to understand that "an angle is the difference in direction of two straight lines that meet in one point, or that would meet if produced."

1255. The semi-circular protractor is better than the common rectangular one for beginners, as they see more clearly by using the former that an angle is measured by the arc of a circle. Two protractors are printed on a fly-leaf in the back of the text-book, for the use of such pupils as cannot procure others. Protractors made of stout manilla paper can be obtained from the Milton Bradley Co., New York, at one cent each in quantities. A large protractor is needed for blackboard use. This can be made of pasteboard; or wooden ones can be bought of the Keuffel & Esser Co., New York.

Many scholars that are able to measure an angle one of whose sides is horizontal, Fig. 1, find it difficult at first to ascertain the number of degrees in an angle formed by two oblique lines, Figs. 2 and 3. They should be permitted to discover the method

for themselves. All that is necessary, is to place the center (A) of the base of the protractor on the vertex of the angle, Figs. 1–3, and the edge of the protractor on one of the sides, the other side cutting the circumference. In Figs. 2 and 3, the number of degrees in the angle XAZ is determined by the number of degrees

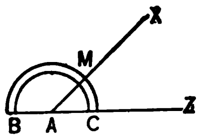


FIG. 1.

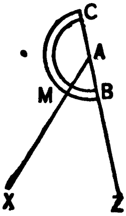


FIG. 2.

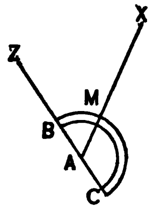


FIG. 3.

in the arc BM, and the upper row of figures is used, having the zero mark at B. In Fig. 1, the number of degrees in the angle is measured by the arc CM, which requires the use of the lower row of figures.

1256. The average class will find the 100 exercises to Art. 1269, inclusive, sufficient for the first year's work. This will give three per week, and leave some time for review. Pupils should work the exercises at home without any preliminary discussion in class. After the exercises are brought in, they should be done on the blackboard, at which time the mistakes made can be pointed out. While first-class drawing cannot be expected from the instruments used by school-children, the teacher should exact the best work possible under the circumstances. A hard pencil, kept sharp, is necessary to secure the requisite fineness of line.

1. In drawing an angle, commence at the vertex.

This exercise is given to remove the impression sometimes formed, that the size of an angle depends upon the length of the lines, instead of their greater or less difference in direction.

In this and all other exercises, the pupils should be encouraged

to commence occasionally with an oblique line. No two results should be exactly alike. If two pupils compare notes, it should be for the purpose of producing a different drawing. One pupil's angle may have its vertex at the right, another at the left; one vertex may be above, another below; etc. The better the teaching, the greater will be the variety of results in exercises that permit of variety.

3. It is expected that the pupils will see for themselves that each arc will contain $\frac{1}{4}$ of 360°.

4. Using the ruler, draw the first line of any convenient length and in any direction. Placing *A* of the protractor at either end, mark off 45°, being careful to use the proper row of figures. Remove the protractor; place the ruler so that its edge just touches the end of the line and the 45° point, and draw the second line. This latter should not be of the same length as the first, unless for some good reason; so that pupils will not consider that the lines forming an angle should be equally long.

Write the number of degrees in each angle.

5. The teacher should not inform the pupils in advance how many degrees they will find in the second angle. They should measure it for themselves, using the protractor.

In drawing these angles, the figure in the book should not be followed. The second line should be drawn to the left in some cases; the lower angle may be made 60°; etc.

When two lines meet to form two angles, it is not at all necessary that the point of meeting should be at the *center* of one line.

1257. Pupils should be taught that horizontal lines are lines parallel to the surface of still water. Floating straws are horizontal, and may point in any direction. A spirit level is used by the carpenter to determine whether or not a beam, for instance, is horizontal. A vertical line is one that has the direction of a plumb line, which is used by a mason to ascertain if a wall is perpendicular.

In drawings, however, lines that will be horizontal when the paper is placed upon the wall, are called horizontal lines; and lines that will be vertical when the paper is placed upon the wall, are called vertical lines.

6. The perpendicular need not be drawn to the center of either of the others, nor need it always be drawn above. The teacher should encourage variety.

8. The pupils should draw these lines, and mark in each angle the number of degrees it contains. Encourage the greatest possible variety in the size of the angles and the direction of the lines.

9. While pupils may be able by this time to give the result without drawing the angles and measuring the second one, the teacher should not fail to give them the necessary practice in constructing angles of a given number of degrees, and in measuring the contents of others.

Many scholars make as ridiculous mistakes in the measurement of angles as they do in their work in numbers, frequently reading the wrong figures, and figures from the wrong row — marking an angle of 45°, for instance, 135°; etc. They should learn to "approximate" the size of an angle, as well as to "estimate" the probable answer to an arithmetical problem. An acute angle should not be marked as containing over 90°; etc.

10. Having learned by observation that the sum of two adjacent angles is 180°, the pupils should now discover that the sum of any number of angles formed on one side of a straight line is 180°. When they have learned this from drawing the first exercise, they may be permitted to calculate the result in the other two, especially as the protractors are not marked for fractions of a degree.

The first exercise should show the same variety in the work of the different pupils as has been recommended for previous work.

12. In constructing a square, the protractor is used to erect a perpendicular at each corner. These perpendiculars are made

equal to each other and to the original line. A fourth line is drawn. The accuracy of the work may be tested by measuring, with the protractor, the two upper angles.

The base lines used by different pupils should be of different lengths. The pupils should be permitted, also, to construct the square in their own way. Some may erect a perpendicular at one end of the given line, and at the extremity of the second line erect another perpendicular. Some pupils may not measure the first line, drawing the second and third lines lightly of indefinite length, and using compasses to make them equal to the first. In this case, the light lines should not be erased; but the square should be marked off by heavier lines.

It is a good practice to have the pupils give a written description of their method of working one of these exercises, which should be accepted as a regular composition. The language should be correct; the proper technical terms should be employed; and there should be sufficient detail to enable any one not familiar with the work to understand just how it was done.

13. Pupils should be permitted to learn for themselves from this exercise and from **14**, that vertical, or opposite, angles are equal.

17. After drawing the required lines, the scholar should mark in each angle its contents in degrees.

19. This exercise should enable the pupil to see that the sum of all of the angles formed about a point will be 360°.

20. The teacher should not give unnecessary assistance. If the scholars have a few days in which to work out an exercise, they should find no difficulty in managing this.

The word "adjacent" in geometry is applied to each of the two angles formed by one straight line meeting another. In **19**, the two lower angles are adjacent; but none of the upper three angles is adjacent to any other, because three straight lines are used to construct two angles in each case. No two angles in **20** are adjacent, and no two are vertical.

21. There are no adjacent angles. They are all vertical, because the lines forming each are produced to form an opposite angle.

22. The pupil is not yet ready to do this in the geometrical way. The teacher should be satisfied if he adds 65° and 25°, and uses the protractor to make an angle of 90°; etc.

24. If the pupil examines a clock, he will see that the number of degrees between 12 and 1 is $\frac{1}{12}$ of 360°, or 30°. He has learned already that the length of the sides has nothing to do with the magnitude of the angle.

25. The minute hand goes 90° in a quarter of an hour. The hour hand goes 30° in an hour; 15° in $\frac{1}{2}$ hr.; $7\frac{1}{2}°$ in $\frac{1}{4}$ hr.

To ascertain the angle at 12:15, the pupils should draw a clock face, locating the hands properly. Some will place the hour hand at 12, forgetting that it has gone $7\frac{1}{2}°$ in $\frac{1}{4}$ hour; and will give the answer as 90° instead of the correct one of $90° - 7\frac{1}{2}°$, or 82° 30′. At 6:30, the minute hand is at 6, and the hour hand is half way between 6 and 7, or $\frac{1}{2}$ of $30° = 15°$. At 8:20, the minute hand is at 4, and the hour hand $\frac{1}{3}$ of the way between 8 and 9 — the number of hour spaces being $4\frac{1}{3}$, corresponding in degrees to $30° \times 4\frac{1}{3} = 130°$.

Pupils should understand that while the angle at 4 o'clock is $30° \times 4$, or 120°, and while the angle at 5 o'clock is $30° \times 5$, or 150°, the angle at 7 o'clock is not $30° \times 7$, or 210°. By making a drawing, they will see that in the last case the angle should be measured on the left, which will make it $30° \times 5$, or 150°.

NOTE. — Angles of 180°, 210°, etc., may be left for more advanced work.

1260. From **26** should be learned that two lines perpendicular to a third line are parallel to each other; and from **27**, that two lines running in the same direction and making the same angle with a third line, are parallel to each other.

29. *DE* will be drawn parallel to *BC* by means of the protractor, an angle of 58° being made at the intersection of *AB* and

DE. Six of the twelve angles will contain 58° each; the remaining six will each measure 180° − 58° = 122°. The pupils should be permitted to ascertain this for themselves.

The card suggested in the note is to be used in schools in which small wooden triangles are not obtainable.

32. To draw from *P*, a line parallel to the oblique line *AB*, place the perpendicular of the triangle on the line *AB*, Fig. 4, and place the ruler *XY* against the base of the triangle. Holding the ruler in position, slide the triangle along it until the perpendicular passes through the point *P*. A line *PE* drawn along this side of the triangle will be parallel to *AB*.

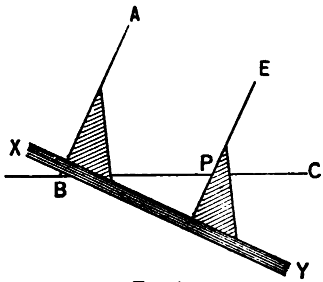


Fig. 4.

Time should be given the pupils to discover this or a similar method of drawing by means of a ruler and a triangle a line parallel to another line. The method may be made the subject of a composition.

33. *QR* and *UV* are drawn parallel by means of the ruler and the triangle. They may lie in any position, care only being taken to cut them by a line making angles of 50° and 130° with one of them. Three of the remaining six angles will measure 50° each; and the others, 130° each.

1261. **35.** If the work is done as it should be, the angle at *C* will measure 80°.

The line *AB* does not need to be horizontal; nor should all the pupils draw *AB* of the same length.

36. The third angle will measure 60°.

37. There are 68° in *a*, and 57° in *c*. In *b*, there are 180° − (68° + 57°), or 55°. In *d*, which is vertical to *b*, there are 55°.

38. The angle *e* should measure 28°, and *f* 120°. There are 180° in *e* (28°) + *g* + *f* (120°).

39. $PRQ = 180° - (70° + 60°) = 50°$. $PRS = 180° - 50° = 130°$. PRS is therefore equal to the sum of the angles P and Q.

40. 180°. *Ans.*

41. $180° - (36° + 65°)$.

45. In measuring the side of a triangle, use the smallest fraction marked on the ruler. When the ruler in use has the denominations of the metric system on one face, that face should be used, and the length of the line given in millimeters. This will not require any teaching of the metric system beyond showing pupils how to read their rulers, and it will do away with the need of using fractions.

46. The length of each side should be marked. If two of them are not found exactly equal, the construction is faulty.

47. The three sides should be equal.

50. Each of the oblique angles will contain 45°.

52. Angles 2 and 3, 90° each; 4, 50°; 5 and 6, 40° each.

53. Angles 1 and 4, $67\frac{1}{2}°$ each; 2 and 3, 90° each; 5 and 6, $22\frac{1}{2}°$ each.

54. The angle p contains 120°; m, 30°; n, 30°.

1266. **56.** Construct this parallelogram by drawing two lines of the given lengths, meeting at an angle of 60°. By means of the ruler and the triangle, construct the other two sides.

If the work is properly done, these two sides will measure 2 in. and 3 in., respectively; and the remaining angles will measure 120°, 60°, and 120°, respectively. From this exercise, the pupils should learn that the opposite sides and the opposite angles of a parallelogram are equal, and that the sum of the four angles is 360°. The angles being oblique, the figure is a rhomboid.

The work of the scholars should show the variety suggested in previous exercises. It is not essential that the longer of the two

given sides should be taken as the base, nor that the base should be parallel to the lower edge of the paper.

57. Different pupils will construct this trapezoid in different ways. Some, seeing that one angle is a right angle, will use the triangle to draw the second side, 3 in.; and at the extremity of this side, will draw, by the same means, an indefinite perpendicular line to form the third side, which is parallel to the base. The fourth side is drawn to make an angle of 60° with the base.

The remaining angles will measure 90° and 120°, respectively; and the sides will measure 5 in., 3 in., nearly 3½ in., and nearly 3½ in.

58. The triangle cut off will form a rhombus when opened out, unless the base and the perpendicular are equal. In this case, the paper, when opened, will form a square.

Making one angle of the triangle 30° (or 40°) will give a rhombus containing 60° (or 80°).

60. When the three sides of a triangle are equal, its three angles are equal; but the rhombus has four equal sides without having equal angles.

61. A triangle that contains three equal angles, has its sides equal; but an oblong has four angles of 90° each, with unequal sides.

62. To construct the rhomboid, draw a base of 2½ in. Two inches above, draw a parallel line 2½ in. long, with the extremities of the latter on the right or the left of the extremities of the base. If the two remaining sides are exactly equal to each other, the work is correctly done.

It is not necessary to draw the altitude, though a broken line may be used. While different methods may be employed, the use of an incorrect one should not be permitted. The work done on the board by a pupil, should be criticised by the class if it be faulty; or a better way may be suggested, if the one employed by the pupil at the board require too much time or unnecessary work.

The lengths of the two remaining sides should be measured, and marked on the papers. Those of different pupils should be different, there being no limit except the size of the paper: they must, however, be longer than 2 in. each; and they should not be just 2½ in., which would make the figure a rhombus.

63. If the line that shows the altitude of the rhomboid can be drawn within the figure, a right-angled triangle can be cut from one side and transferred to the other, making the figure a rectangle. Arithmetic, Art. 929, 5, last parallelogram.

In the case of a rhomboid whose altitude does not fall

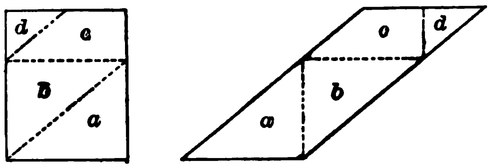


FIG. 5.

within the figure, several cuts will be necessary to change it into a rectangle. See Fig. 5.

65. The areas will be equal, because each rhomboid is equal in area to a rectangle 3 in. by 2 in.

66. The three rhomboids in Fig. 6 have their respective sides equal each to each, but their angles are unequal; hence their altitudes and their areas are unequal.

To construct these rhomboids, draw base lines of three inches,

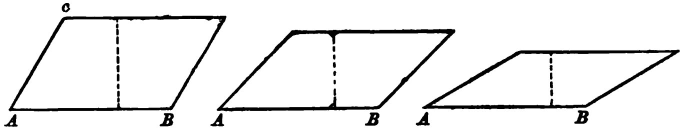


FIG. 6.

and inclined to each, at any angle except one of 90°, a two-inch line. Use the ruler and the triangle to complete the figures.

68. As the altitudes are not given, the areas of the figures drawn by different pupils should vary. The protractor or the triangle should be used in drawing the altitudes, which should then be carefully measured.

69. See **62**, making the upper side $2\frac{1}{2}$ in. The fourth trapezoid in Arithmetic, Art. 929, **6**, shows the manner of determining the dimensions of the required rectangle.

1267. 78. The diameters of these circles should be such as not to admit of the use of the protractor in drawing them. To ascertain the extremities of an arc of 120°, two lines are drawn meeting in the center of the circle at an angle of 120°. The portion of the circumference intercepted by these lines will constitute an arc of 120°. The remainder of this circumference will form an arc of 240°.

79. The pupil should be permitted to make the first attempts in his own way. He will doubtless soon discover that the distance between the points of his compasses in drawing the circle, is the length of the chord required, and that by placing one point of the compasses on any portion of the circumference of the circle just drawn, the other point will indicate on the circumference the other extremity of the chord.

To measure the length of the arc in degrees, draw radii to its extremities, and use the protractor to determine the angle made by these radii, which may be produced, if necessary. If the work is properly done, the angle should measure 60°, which is the length of the arc.

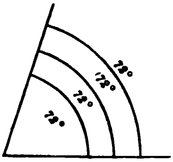


Fig. 7.

80. Draw two light lines meeting at an angle of 72°. Using the vertex of the angle as a center, and any radius, draw an arc between the lines. This arc will measure 72°. Darken the lines from the arc to the vertex to show the radii of the circle of which the arc is a part.

81, 82. Figs. 8 and 9 show sectors of 90° and 270°, respectively; Figs. 10 and 11, segments of 120° and 240°, respectively.

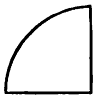


FIG. 8.

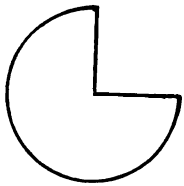


FIG. 9.

FIG. 10.

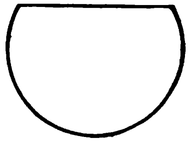


FIG. 11.

1268. **86.** A diameter will divide the circle into two equal parts. A second diameter perpendicular to the first, drawn by means of the protractor or the triangle, will divide the circle into four equal parts.

To divide the circumference into four equal parts, it will not be necessary to draw the diameters. When he has the ruler in the proper place to draw the first diameter, the pupil needs to mark only the two points where the ruler cuts the circumference. The third point can be indicated when the triangle is placed at the center of the circle and against the ruler; etc.

While the scholars may be permitted in the beginning of this work to draw a number of unnecessary lines, and while it may be an advantage to even require it, they should gradually learn to make as few lines as possible. The construction lines that are employed, should be drawn very lightly and should not be erased. Other lines should be made more conspicuous. Careful pupils may be allowed to use ink for this purpose.

1269. It is not intended that the methods here suggested should be communicated in advance to pupils. Each should be allowed to try the problem in his own way. The discussion of the method employed afterwards on the blackboard, will suggest other and possibly better modes of procedure.

88. To inscribe a regular pentagon in a circle, it will be necessary to divide the circumference into five arcs of 72° each. The

protractor should be used to obtain the first arc; the remaining ones can be set off by the compasses, the first being used as a measure. Careful work should be exacted by the teacher.

89. Many pupils will have learned in their drawing lessons the regular method of inscribing a hexagon in a circle. Those unfamiliar with this way, should not be shown it until after they have constructed the hexagon by means of the protractor.

It is a pedagogical mistake to suggest "short-cuts" to pupils before they thoroughly understand a general method. For this reason, the teacher should permit the members of her class to use the protractor in the construction of the inscribed triangle, leaving it to themselves to discover a simpler way. She should encourage, also, the employment of a variety of methods even if some of them are not very direct. The experiments made by pupils to discover a new mode of constructing a polygon, will help them in their geometrical study.

The chord of 60° being equal in length to the radius (**79**), the shortest method of inscribing a hexagon is to apply the radius as a chord six times. Two of these divisions of the circumference will make arcs of 120°, the chords of three of these forming the sides of an inscribed equilateral triangle.

90. Each of the six angles at the center contains 60°. Since the two sides AC and AB, enclosing any central angle, are radii of the circle, and therefore equal to each other, the angles opposite

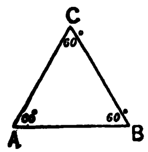


Fig. 12.

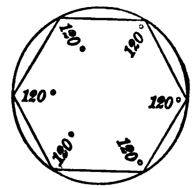


Fig. 13.

site those sides are equal; that is, angle $A =$ angle B. The angle at C being 60°, $A + B = 180° - 60° = 120°$, and $A = B = 60°$. Angles 1 and 2 (see Arithmetic), therefore, measure 60°

each, and the whole angle contained between two adjoining sides of the hexagon, measures 120°. After determining by this method the number of degrees in each angle of a regular hexagon, the pupils should be required to construct one, and to mark in each angle its contents in degrees, as in Fig. 13, verifying the result by using the protractor.

91. A careless scholar, measuring the number of degrees in each angle of a regular pentagon (Arithmetic, Art. 1268), will sometimes read from the wrong row of numbers on the protractor, getting the result 72°, instead of the correct one of 108°. As there are 72° in each division of the circumscribing circle, he will have no doubt of the correctness of his answer, unless he has been trained to estimate the size of an angle. In this case, he will see that each angle of a regular pentagon is obtuse, and, therefore, greater than 90°.

One method of calculating the number of degrees, is to divide the pentagon into five equal triangles, one of which is shown in Fig. 14. The angle at C is 72°. The sides CA and CB, being radii, are equal; which makes equal angles at A and B, each of which is $\frac{1}{2}$ of (180° − 72°), or 54°. Each of these angles is the half of one of the angles of the pentagon, so that these latter angles measure 108° each.

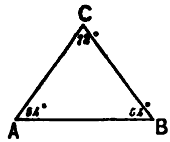


Fig. 14.

92. A circumscribed square touches the circle at four points, each side constituting a *tangent*. A tangent being perpendicular to the radius drawn to the point of contact, the square may be constructed by drawing perpendiculars to two diameters intersecting at right angles, using the triangle or the protractor for the purpose. The ingenious pupil will discover other ways; drawing, for instance, at each extremity of the two diameters, a line parallel to the intersecting diameter, by means of the ruler and the triangle; etc. No method should be permitted that merely approximates accuracy, such as determining that a line is parallel or perpendicular by the eye alone. The

average class will contain many members intelligent enough to pass upon the correctness of a given method, and they should be called upon to give reasons for any criticisms they may have to offer.

In circumscribing some polygons, a hexagon for instance, many pupils prefer locating points X and Y, instead of using the triangle and the ruler to draw a tangent XF. After dividing the circumference (Fig. 15) into six equal parts at 1, 2, 3, 4, 5, and 6, they draw a diameter from 1 to 4. Through 2 and 6 they draw a secant XA, making MX and MA each equal to the radius. Through 3 and 5 a second secant is drawn, and NY and NB are also made equal to the radius. A line drawn through X and Y will form one side of the circumscribed hexagon. One extremity of this side can be determined by producing the diameter $3C6$ to F, and the other by a line through $2C5$.

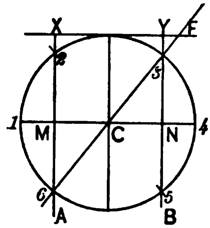


Fig. 15.

Note. — A *secant* is a line that cuts a circle at two points; a *tangent* is a line that touches a circle at one point.

93. The smallest number of triangles into which a pentagon can be divided, is three (Fig. 16). The three angles of each triangle contain 180°, making the sum of angles 1–9, 540°. Since $1 = A$, $2 + 4 = B$, $6 + 7 = C$, $8 = D$; $3 + 5 = E$, the sum of the five equal angles (A, B, C, D, and E) of a regular pentagon $= 540°$, and each equals 108°. This is the result that was found in **91** by another method.

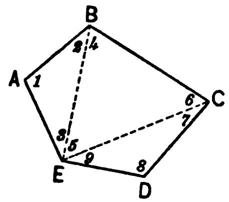


Fig. 16.

94. The hexagon is divisible, as above, into four triangles, containing $180° \times 4$, or 720°; making each angle $720° \div 6$, or 120°.

95. A quadrilateral is divisible into 2 triangles; a pentagon,

into 3; a hexagon, into 4; a heptagon, into 5; an octagon, into 6, being 2 triangles less in each case than the number of sides in the polygon.

96. The number of degrees in each angle of a regular octagon $= [180° \times (8-2)] \div 8$.

97. At each end of the 2-inch line, draw a 2-inch line at an angle of 108°. At the farther extremity of each of those lines draw a line at an angle of 108°. These last lines meet at an angle of 108° if the work is correctly done, and are each two inches long to the point of their intersection.

98. A line drawn to each extremity of the base at an angle of 60° will form an equilateral triangle. Use angles of 90° for the square, 108° for the pentagon, 120° for the hexagon, nearly 129° for the heptagon, 135° for the octagon, 140° for the nonagon, etc.

The first line should be placed near the center of the bottom of the paper to give room for the successive polygons. See Fig. 17. Bright pupils will calculate the necessary angles and continue to construct polygons as far as the space will permit.

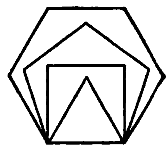


FIG. 17.

99. By drawing the diameters AB and XY (Fig. 18), the inscribed square will be divided into four triangles, while the circumscribed square contains eight, being double the area of the inscribed square.

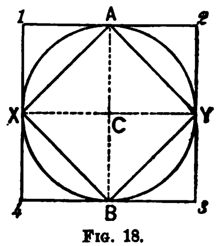


FIG. 18.

1270. These problems are given to enable the pupils to learn how to bisect lines, erect perpendiculars, construct angles, etc., by means of the ruler and the compasses, and incidentally to learn a number of geometrical facts. The use of other instruments is unnecessary, and should, therefore, not be tolerated.

1. The distance between the centers = $1\frac{1}{2}$ in. + 1 in.

2. $1\frac{1}{2}$ in. − 1 in.

4. The line XY joining the two points of intersection of the equal circles (Fig. 19), bisects the line AB connecting the centers. The radii AX, BX, AY, BY are each 2 inches long.

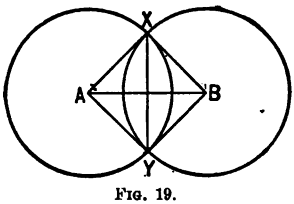


FIG. 19.

5. The previous exercise should lead the pupils to see the steps necessary to the construction of the required triangle. The 3-inch base AB is first drawn. The next requirement is to find a point X (Fig. 19) 2 inches from A and from B. A circle of 2-inches radius with B as a center, will contain every point that is 2 inches from B. A similar circle with A as a center, contains every point 2 inches from A. The intersections, X and Y, are each 2 inches from both points A and B. Using either one as a vertex, draw lines to A and B, forming the required triangle.

Authorities differ somewhat as to the advisability of requiring pupils to employ circles rather than arcs in geometrical problems. While it may be better at first to use circles, the point to be finally reached is the employment of the shortest lines possible. This should not be inconsistent with the acquirement of geometrical knowledge. A scholar should know after a very few exercises that each point of an arc is 2 inches from the center, as well as he does when he draws the entire circle.

In his later construction of an isosceles triangle, the pupil should know that the vertex is above (or below) the center of the base. For this reason the first arc need not be a very long one. The second should be still shorter.

6. Fig. 19 will suggest the necessary steps. Using each end of the $3\frac{1}{2}$-inch base as a center, and with a radius of 4 inches, draw two circles. Draw lines corresponding to XA and XB for the required triangle. Placing the ruler on X and Y will give the perpendicular, which should not extend below the base.

If arcs are used, the upper intersection determines the position of the vertex. A lower intersection is used to determine the

direction of the perpendicular. The pupil will gradually learn that while a definite radius, 4 inches, is required to locate the vertex of the triangle, intersecting arcs, each of 3 inches or 5 inches or any other radius, will serve to locate the second point, used with the vertex to determine the direction of the perpendicular. The point of their intersection may be above the base or below it, according to convenience. A point below secures greater accuracy, by being probably at a greater distance from the vertex than one above is likely to be.

7. This is a variation of **6**, but without directions as to length of sides. If the perpendicular is correctly drawn, it will bisect the base. See Exercise 48, Art. 1263. The compasses should be employed to determine the equality or inequality of the segments of the base.

8. The same procedure is required as in **7**, except that the sides of the triangle are not drawn. The bisecting line should be extremely short.

9. With a 2-inch radius, draw intersecting arcs, using as centers the two extremities of a 2-inch line.

11. Either side may be used as the base; and different pupils should use a different base, although the greatest number will probably take the longest side.

Using the 2-inch side as a base, draw from one end, as a center, an arc with a radius of 1 inch; and from the other, an arc with a radius of $1\frac{1}{2}$ inches. The intersection of the arcs will be the vertex of the required triangle.

With the 3-inch line as a base, the radius of the arcs will be 2 inches and $2\frac{1}{2}$ inches, respectively.

Besides employing different bases, the pupils should use oblique lines and vertical lines as bases, and the vertex in some instances should be below the base; etc.

12. A scholar should be permitted to discover for himself that the intersection of the 2-inch arcs will be at the center of the 4-inch base. After endeavoring, also, to make a triangle whose

sides shall measure 1, 2, and 3 inches, respectively, he will understand that the third side of a triangle must be shorter than the combined lengths of the other two.

13. If the pupil, in drawing arcs to locate the bisecting line, employs the radius used in drawing the circle, he will discover that one intersection will take place at the center of the circle. This will lead him to see that only one set of intersecting arcs is necessary — the one beyond the circle, the center of the circle serving as a second point.

The teacher should be in no hurry to inform the pupil using two sets of arcs, that a single set will be sufficient; knowledge that comes in some other way than merely by direct telling, is apt to last longer.

14. Before dividing a sector (Figs. 8 and 9) into two equal parts, some scholars may consider it necessary to draw the chord. Permit them to do so at first.

15. The pupils have already learned that a line drawn from the vertex of an isosceles triangle to the center of the base, is perpendicular to the base. The method employed in bisecting a line is practically to consider it the common base of two isosceles triangles having their vertices on opposite sides of the base, although the equal sides of the triangles are not drawn. This bisecting line is perpendicular to the assumed base. The pupils have probably discovered that the perpendicular line bisecting the chord in **13**, would, if produced, pass through the center of the circle. From this previous knowledge, they will doubtless be able to answer the last question of **15**.

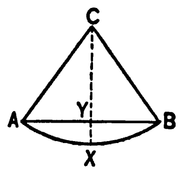


Fig. 20.

The scholars that have drawn the chord in dividing the sector in **14** into two equal parts, will be likely to see that the radius CX (Fig. 20) is perpendicular to the bisected chord AB.

16 shows that a perpendicular that bisects a chord, AB (Fig. 20), also bisects the arc AXB.

It will readily be seen by the pupils that an arc AXB can be bisected without drawing the chord AB.

17. This problem is the same as **8**. The drawing, however, should show a longer line, and one that does not cut the given line, the requirement being that the perpendicular be drawn *to* the latter. A perpendicular drawn to a horizontal or to an oblique line from below it, would be correct; although it might not be accepted as satisfactory if the more common wording of the problem were employed: *Erect* a perpendicular at the middle point of a line.

18 requires no explanation.

19. Bisecting one of the four divisions of a circumference gives the dividing point between two one-eighths. A ruler placed upon this point and on the center of the circle will indicate the location of another. The distance between two points can be ascertained by the compasses, which can then be used to locate the remaining two dividing points.

The lines used to divide a circumference should not be too long.

The thoughtless pupil sometimes fails to see, when he employs his compasses to measure the distance between two points on a circumference, that he is measuring the chord of an arc, and not the arc itself. It may be necessary for the teacher to explain this in connection with **25**.

22. Unless the pupils have learned in their drawing work the method of erecting a perpendicular at the end of a line, some of them may experience a little difficulty in solving this problem. One way of drawing the circumscribed square is to construct on AY (Fig. 18) an isosceles triangle $A2Y$ equal to ACY. Produce $2A$ to 1, making $A1$ equal to $2A$. Produce $2Y$ to 3 in the same manner. Lines from 1 through X, and 3 through B, will intersect at 4, which completes the square.

24. The object of **23** and **24** is to lead the pupils to see again that the side of an inscribed hexagon is equal in length to the radius of the circle.

26. The four triangles will together constitute a circumscribed equilateral triangle.

27. Draw lightly an arc less than 90° in length. After cutting off 60°, darken this portion. No radii should be drawn.

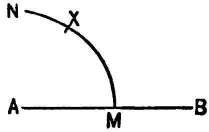


FIG. 21.

To construct an angle of 60°, draw an indefinite line AB (Fig. 21). With any convenient radius, as AM, draw an arc MN. With M as a center, and using the same radius, cut the arc at X. This makes MX an arc of 60°. From A draw a line through X.

28. An arc of 30° is constructed by the bisection of an arc of 60°.

To construct an angle of 30°, draw AB (Fig. 21) and the arc MN; and cut off MX, as in **27**. With the same, or any other convenient radius, and using M and X as centers, draw two arcs intersecting on the right of MX. A line drawn from A through this intersecting point will make with AB an angle of 30°.

29. To construct an angle equal to $60° + 30°$ (or $30° + 60°$), bisect the arc XM (Fig. 21), and from the bisecting point, which is 30° from M, lay off an additional 60°. A line drawn from A through this point will make with AB an angle of 90°.

30. One method of erecting a perpendicular at the end of a line is given in **29**. A somewhat similar method consists in prolonging the arc, and marking off at Q and R two divisions of 60° each. Using these two points as centers, draw arcs intersecting at F. From D draw a line through F.

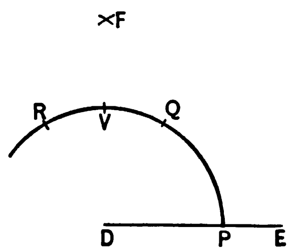


FIG. 22.

The line DF bisects the arc QR, which makes the angle $FDE = 60° \, (PQ) + 30° \, (QV) = 90°$.

31. The bisection of VP gives an angle of 45°. Do not draw DF.

The recommendation so often made as to getting a variety of drawings from the various members of the class, applies to these problems. The erection of a perpendicular at the end of a vertical line should call forth at least four different results. The perpendiculars may be erected at either end, and may run to the right or to the left. In constructing an angle of 45°, a greater variety is possible. The first line may be horizontal, vertical, or oblique; the second line may start from either end; and it may extend above or below, to the right or to the left; the lines forming the angles need not be of the same length on all papers, nor need both of them be of the same length on any one paper.

$22\frac{1}{2}° = \frac{1}{2}$ of $45°$; $135° = 90° + 45°$; $15° = \frac{1}{2}$ of $30°$; $75° = 60° + 15°$.

33. The preceding problem should give a hint as to the method of solving the present one. In **32**, a circle was drawn with A as a center and AX as a radius. This circle cut XY (Fig. 23) in M. To erect a perpendicular at A, the center of the circle, the extremities X and M of the diameter XM were used as centers to draw arcs intersecting at J. A perpendicular was then drawn from A through J.

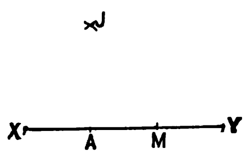


FIG. 23.

This problem does not furnish a circle, nor is one necessary. The line XY is given, and the point A at which the perpendicular is to be erected. With A as a center, and a radius equal to AX, cut off AM. This gives us the diameter of the circle employed in the preceding problem.

It will be noticed that a second set of intersecting arcs on the opposite side of XM is not needed, the point A answering instead. This problem amounts to the bisection of an arc of 180°, of which only the chord XM is drawn, A being the center of the circle.

34. The first two divisions of this problem are reviews of parts of **17** and **30**. The erection of a perpendicular at a point between the end and the center, is a variation of the preceding problem. Let *RS* (Fig. 24) be the required line and *A* the point. With *A* as a center, and any convenient radius, lay off the points *X* and *M*, which will be equidistant from *A*. Using *X* and *M* as centers, draw arcs intersecting at *J*. Draw the required perpendicular from *A* through *J*.

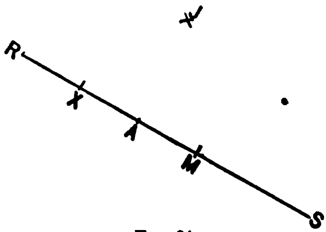


FIG. 24.

In problem **33** the point *M* was located at a distance from *A*, equal to *AR*; and while greater accuracy is obtained by having the points *M* and *X* as far apart as possible, the present method is suggested to show that the only essential requirement as to their location is to have them equidistant from *A*.

35. The base need not be a horizontal line.

36. Proceed as in the construction of a right-angled triangle, base 2 inches, perpendicular 2 inches; but do not draw the hypotenuse. With *B* and *H* as centers (Fig. 25), and a radius of 2 inches (*BP* or *PH*), draw arcs intersecting at *O*. *BO* and *HO* will form the remaining sides.

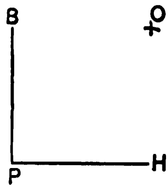


FIG. 25.

To construct the rectangle, *PH* (Fig. 25) should be made 3 inches long, *BP* measuring 2 inches. With *B* as a center, and a radius of 3 inches, draw an arc. Intersect this by a second arc drawn with *H* as a center, and a radius of 2 inches. From the point of intersection, draw lines to *B* and *H*.

Some scholars may prefer to erect a perpendicular at each end of the base, etc.

38. Draw a base line, *WV* (Fig. 26), 3 inches long. At any convenient point, *M*, erect a perpendicular, *MP*, $2\frac{1}{2}$ inches long

At its extremity P, draw the perpendicular PO. With W as a center and a radius of 3 inches, locate the point X on the line PO; WX will be the second side of the rhombus. On PO draw XN 3 inches; and connect NV. This last line should measure 3 inches if the work is properly done.

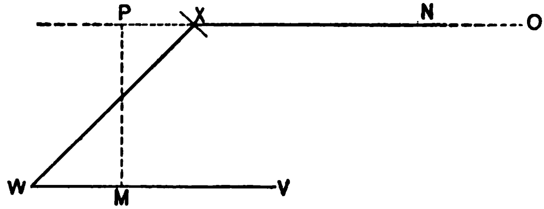


FIG. 26.

To construct a 3-inch rhombus containing an angle of 60°, draw WV; at W construct an angle of 60°, and make WX 3 inches. At X or at V, draw a 3-inch line making an angle of 120° with XW or VW, etc.; or, using X and V as centers, and with a radius of 3 inches, draw arcs intersecting at N; draw NX and NV.

39. Proceed as in the first problems of **38**; but make WV and XN each 4 inches long.

40. See BPH (Fig. 25); draw BH.

Some pupils, preferring to commence with a horizontal base, calculate the number of degrees in each angle at the base, $\frac{1}{2}(180° - 90°)$, or 45°. To each end of a base line of any length (Fig. 27), they draw a line making with the base an angle of 45°.

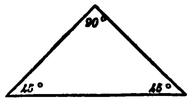


FIG. 27.

If one angle is 120°, the angles at the base will be 30° each; if one is 135°, the angles at the base will measure $22\frac{1}{2}°$ each.

The method shown in Fig. 27 requires the construction of two angles; and is, therefore, not so direct as drawing two equal lines meeting at the given angle.

41. Erect a 3-inch perpendicular at the middle of a 3-inch line.

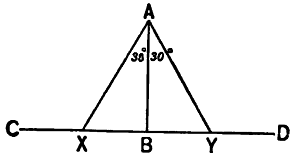


FIG. 28.

A perpendicular, AB (Fig. 28), bisects the angle at A of the equilateral triangle. Erect a 3-inch perpendicular at B, on an indefinite line CD; at A, construct two angles of 30° each, as shown in the figure; draw AX and AY, forming the required equilateral triangle AXY. Test the work by measuring XY.

42. To construct a scalene triangle of the required dimensions, erect the 3-inch perpendicular at a point not in the center of the 3-inch base, nor at either extremity.

To construct the obtuse-angled triangle, produce the 3-inch base by a dotted line, and erect the 3-inch altitude at some point outside of the base.

43. This problem may puzzle the pupils at first. If the triangle were isosceles, XYM, for instance, there would be no difficulty. The solution of the problem consists practically in making an isosceles triangle, although the line XM is not drawn. With X as a center (Fig. 29), and XY as a radius, the point M is located (without drawing the arc shown in the figure). Using M and Y as centers, and with any convenient radius, arcs are drawn intersecting at R. XR gives the direction of the altitude.

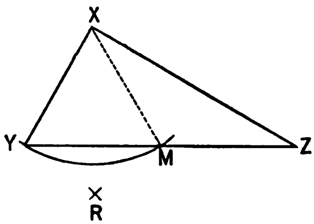


FIG. 29.

44. The given triangle in this case will be XMZ (Fig. 29). Produce ZM indefinitely towards Y, and with XM as a radius, cut the base at Y; etc. XY is, of course, not drawn; MY should be a light line or a broken line; etc.

45. This is the same problem in another form. Let YZ (Fig. 29) be the given line, and X the given point, from which a perpendicular is to be let fall upon YZ. In **43–44**, the arc described with X as a center, passes through one extremity of the line. The pupils should see that this is not essential, and that it would be very inconvenient in the case of a very long line. All that is necessary is to intersect the line YZ at two points sufficiently far apart to secure accuracy.

47. Each angle will measure 60°; the arc on which it stands measures 120°; an angle at the circumference, therefore, is measured by one-half the arc intercepted by its sides.

Apply this to the regular pentagon (Arithmetic, Art. 1268). The angle at the top of the figure stands on an arc containing 72° + 72° + 72°, or 216° (made up of three arcs); it contains, therefore, $\frac{1}{2}$ of 216°, or 108°. See Fig. 30.

An angle of a square stands on an arc of 180°; its contents are, therefore, $\frac{1}{2}$ of 180°, or 90°; etc.

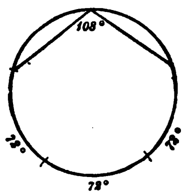


FIG. 30.

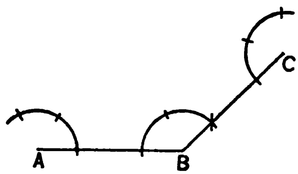


FIG. 31.

48. The number of degrees in each angle of a regular hexagon is 120° (**94**, Art. 1269). From each end of the given line AB (Fig. 31) draw a line equal in length to AB, and making with it an angle of 120°; etc.

While this is not the best way to construct a regular hexagon, it gives the pupils an opportunity to try the general method on a polygon of six sides.

49. For the construction of an octagon, the angles at A, B, C, etc., should measure 135° (Problem **31**).

By a later problem, the pupils will learn how to find the center of the circumscribing circle after two sides of the octagon are drawn; for the present, however, they should be required to repeat the construction of the angle of 135° at the required number of corners. The work should be as accurate as possible, considering the necessary imperfections of the tools employed.

50. A right angle whose vertex is at the circumference, subtends an arc of 180°, or a semi-circumference.

51. By means of a ruler, placed on the center of the circle, mark points A and B (Fig. 32) the extremities of a diameter (not drawn). The arc AMB is one-half of the circumference, or 180°; and the angles X, Y, and Z, whose sides XA and XB, YA and YB, ZA and ZB, subtend this arc, are each measured by one-half of it, and contain, therefore, $\frac{1}{2}$ of 180°, or 90°.

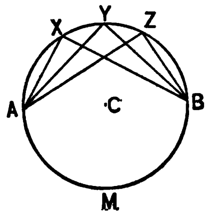


FIG. 32.

52. The hypotenuse of an inscribed right-angled triangle is always the diameter of the circle.

In Fig. 32 draw the diameter ACB (or any other). Bisect the arc $AXYZB$, which gives the vertex of the required triangle. Connect this point with the points A and B.

NOTE. — It may be necessary to have the scholars understand that all the vertices of an inscribed polygon must lie in the circumference; an inscribed triangle, therefore, has three vertices in the circumference.

53. The diameter of a circle having a radius of 2 inches, will be the diagonal of the inscribed square. The location of the remaining two corners is easily determined.

The square may also be constructed by drawing 4-inch diagonals bisecting each other at right angles; etc.

The rhombus will consist of two 3-inch equilateral triangles having a common base.

54. Every angle being on the circumference, each will be measured by one-half the intercepted arc, and the sum of the

three angles will contain one-half the number of the degrees contained in the three arcs; *i.e.*, $\frac{1}{2}$ of 360°.

55. See the rhombus in **53.**

56. The two triangles will be equal in all respects, as will those in **57.**

58. From **56** and **57** the pupils should learn that two triangles are equal in every respect (*a*) when two sides and the included angle of one are equal to two sides and the included angle of the other, each to each; and (*b*) when two angles and the included side of one are equal to two angles and the included side of the other, each to each. From **59**, they should learn that when two triangles have the three sides of the one equal to three sides of the other, each to each, they are equal throughout. From the present problem, they should learn that any number of triangles can be constructed having their corresponding angles, equal each to each, but whose corresponding sides are unequal, each to each.

60. Impossible, because the sum of three angles of 75° each is greater than 180°.

61. The radius of each circle must be one-half of the hypotenuse, or $1\frac{1}{2}$ in. See Problem **52.**

Any inscribed triangle having one side the diameter of the circle will answer the conditions of the first case.

The 2-inch base of the second triangle is obtained by taking A as a center (Fig. 32), and with a radius of 2 inches, drawing an arc at, say, Y; AY will be the required base, YB the perpendicular, and AB the hypotenuse.

Using a radius of $1\frac{1}{2}$ inches, as above, will give a perpendicular, AX, of $1\frac{1}{2}$ inches, etc.

NOTE.—The term *base* does not necessarily imply a horizontal line; nor *perpendicular*, a vertical one.

63. To construct an angle at Y (Fig. 33) equal to the angle at X, take X as a center and any convenient radius, and draw the arc Aa; then take Y as a center and the same radius, and draw the arc Cc. The angle X is measured by the arc AB, the

length of whose chord is determined by means of the compasses. Lay off the same on *Cc*, making the arc *CD* equal to the arc *AB*. An angle formed by drawing a line from *Y* through *D*

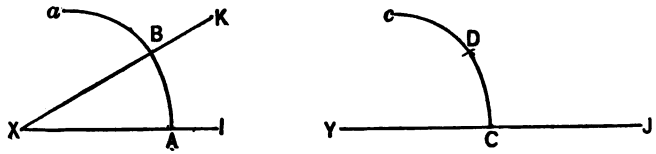


FIG. 33.

will be equal to the angle at *X*, as each angle is measured by an arc of the same number of degrees. See note to Problem **19**, Art. 1270.

64. Draw the first line at any angle; and by the preceding problem, draw the two intermediate lines, making, with the horizontal line, angles equal to that made by the first line. The pupil will have no difficulty with the oblique line at the other extremity of the horizontal line.

65. Proceed as in the second part of **64**, using a horizontal 3-inch line for the diagonal, drawing from one end a $2\frac{1}{2}$-inch oblique line running upward at any angle, and from the other end a similar line running downward at the same angle.

The lines divide the 3-inch diagonal into 5 equal parts, each part measuring $\frac{3}{5}$ inch.

66. The scholars should gradually learn that it is unnecessary to measure the oblique lines, *Km* and *Ln* (Fig. 34). In setting off the divisions *K*1, 1 2, etc., *La*, *ab*, etc., it will be found unnecessary to use a definite measure; all that is required is that they be equal. This equality is obtained by opening the compasses slightly and marking off the divisions with them. There is no need of completing the

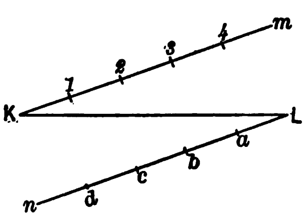


FIG. 34.

rhomboid, nor of drawing the lines $4a$, $3b$, $2c$, etc. Place the ruler on 4 and on a, and use a short line on KL to mark off one division; etc.

It is expected that the teacher will not give this method at the outset.

To draw a line exactly $\frac{3}{8}$ of an inch in length, a construction line, 3 inches long, KL (Fig. 34), is drawn lightly, also Km and Ln of indefinite length. Only the first division is required on Km. Placing the ruler on 1 and d, mark off on KL $\frac{3}{8}$ inch, and darken this portion

One-seventh of a 5-inch line $= \frac{5}{7}$ inch.

67. When the base of a right-angled triangle measures 3 inches, and the perpendicular measures 2 inches, the hypotenuse will measure $\sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$ inches.

68. A line $\sqrt{13}$ inches long is drawn by constructing a right-angled triangle having a 3-inch base and a 2-inch perpendicular. The hypotenuse is the required line.

A base of 2 inches and a perpendicular of 1 inch give a hypotenuse of $\sqrt{5}$ inches.

A hypotenuse of 4 inches and another side of 3 inches give a remaining side of $\sqrt{7}$ inches. For the construction of this triangle, see Problem **61**.

69. The side opposite the angle of 30° is one-half as long as the side opposite the angle of 90°.

71. The chord of an arc of 60° is 2 inches long; the chord of an arc of 180° is the diameter, and is 4 inches long. The chord of an arc of 300° is 2 inches long.

72. The perpendicular "erected" at one end of the chord should be turned downwards if the chord is drawn above the center of the circle.

The perpendicular and the diameter meet at the circumference. See Problem **61**.

73. The triangle will retain its shape, because only one triangle can be drawn with sides of a given length. Problem **59**. The rectangle will not retain its shape, because an indefinite vari-

ety of parallelograms can be constructed having their corresponding sides equal, each to each. See Exercise 66, Art. 1266.

77. From Problem **15**, the pupils have learned that a perpendicular bisecting a chord, passes through the center. If there are two lines passing through the center, the latter must be located at their intersection.

78. To inscribe a circle in any triangle, draw lines bisecting the three angles. See problem **28**. The intersection of these three lines will be the center of the circle.

In practice, only two lines are drawn; but the third serves to test the accuracy of a pupil's work.

79. The sides of an inscribed triangle are chords of the circumscribing circle. In Problem **77**, we have learned that the intersection of two perpendiculars that bisect chords, locates the center of the circle. With this center, and with a radius equal to the distance from the center to the vertex of any angle of the triangle, draw the circle.

80. The larger the circle the better for beginners. The average scholar may consider it necessary to draw two adjacent chords, but every purpose will be served by cutting the circumference by three short lines to mark the boundaries of two adjoining arcs. The bisection of these arcs by perpendiculars will locate the center of the circle.

82. Use a cup to draw the arc. Divide it at random into two parts. Bisect each, as above.

83. The altitude is 2 in. Problem **70**.

84. Experienced draughtsmen save time by changing the radius as infrequently as possible. To construct a square on a 3-inch line AB (Fig. 35), take a radius of 3 inches, and with A as a center draw the arc Bm. With B as a center, mark off $Bn = 60°$. Bisect Bn,

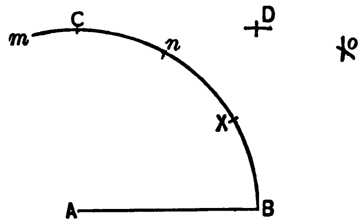


Fig. 35.

using the same radius to obtain the arcs intersecting at *o*, with *B* and *n* as centers. $BX = 30°$; place the compasses on *X*, and cut the first arc at *C*, which makes *XC* 60°, and *BC* 90°. *C* is the third corner of the square. Using *B* and *C* as centers, and with a radius of 3 inches, draw arcs intersecting at D, the fourth corner of the square.

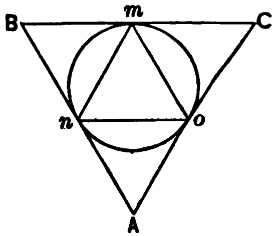


FIG. 36.

86. We have found in Problem **26**, that constructing an equilateral triangle on each side of an inscribed equilateral triangle gives a circumscribed equilateral triangle. Locate *m*, *n*, and *o*, 120° apart. With each as a center, and with a radius equal to *mo*, draw arcs intersecting at *A*, *B*, and *C*. Do not draw the triangle *mno*, shown in the figure.

87. See Problem **41**, second part.

89. See Arithmetic, Art. 1089 (Fig. 2).

90. Construct a right-angled triangle having a base and a perpendicular measuring 2 inches and 3 inches, respectively. The hypotenuse will be the side of the required square.

Do not construct squares on the other two sides.

91. The hypotenuse is 3 inches, etc. See Problem **68.**

92. The square constructed on the hypotenuse of a right-angled triangle whose base and perpendicular measure 3 inches each, will be double the area of a 3-inch square.

When the 3-inch square is constructed, measure the length of the diagonal (without drawing it); and on a line of this length as a base, construct the required square.

93. Four. See Fig. 36.

94. Nine; sixteen; twenty-five.

95. 1, $1\frac{1}{2}$, 2 inches.

96. 3, $4\frac{1}{2}$, 6 inches.

97. The radius of the second circle is 2 inches.

98. A square described on the base of an isosceles right-angled triangle will be one-half the size of the square described on the hypotenuse. $A = a + a = 2a$; $a = \frac{1}{2}A$. The area of an equilateral triangle whose base is YZ, is one-half that of one whose base is XY. The area of a circle whose radius is YZ (or XZ), is one-half that of a circle whose radius is XY.

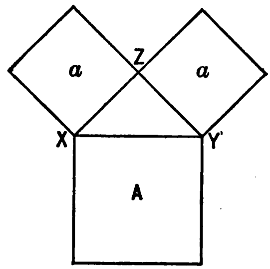


FIG. 37.

NOTE.—If the square A in Fig. 37 is drawn above XY instead of below it, the ratio of the large square to that of each small one will be more apparent.

Construct an isosceles right-angled triangle having a hypotenuse of 2 inches (Problem **52**). The base of this triangle will be the radius of the required circle.

The area of a circle of 2 inches radius $= 2^2 \times \pi = 4\pi$.

The radius of the other circle $= \sqrt{2}$; its area is $(\sqrt{2})^2 \times \pi = 2\pi$, which is one-half of 4π.

99. The side of the required triangle measures $\sqrt{2}$ inches. See **98.**

100. *Cab* (Fig. 38) is one-sixth of an inscribed regular hexagon, and *CAB* is one-sixth of a circumscribed regular hexagon. Calling the radius of the circle 2 inches, the altitude CX of the large triangle is 2 inches. Since ab is 2 inches, $ax = 1$ inch; $Ca = 2$ inches. In the right-angled triangle Cax, $\overline{Cx}^2 + \overline{ax}^2 = \overline{Ca}^2$; $\overline{Cx}^2 + 1 = 4$; $\overline{Cx}^2 = 4 - 1 = 3$; $Cx = \sqrt{3}$. The areas of the two triangles will be proportional to the squares of their respective altitudes. $\overline{Cx}^2 = 3$ and $\overline{CX}^2 = 4$. That is, the area of the larger triangle is $1\frac{1}{3}$ times the area of the smaller; hence the area of the circumscribed hexagon is $1\frac{1}{3}$ times the area of the inscribed hexagon.

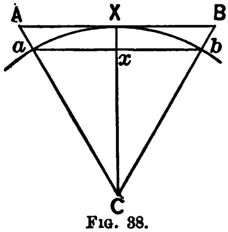


FIG. 38.

The area of the circumscribed square is double the area of the

inscribed square; and the area of the circumscribed equilateral triangle is four times the area of the inscribed equilateral triangle.

1271. 3. At the middle point X of the perpendicular of the right-angled triangle (Fig. 39), cut Xm parallel to the base. Re-arranging the two parts gives a rectangle whose dimensions are 4 inches and $1\frac{1}{2}$ inches. Figs. 40 and 41 show how the

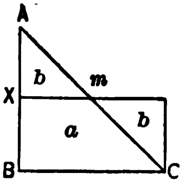


FIG. 39.

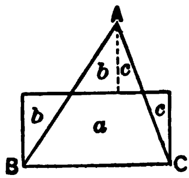


FIG. 40.

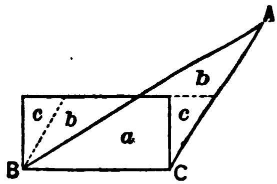


FIG. 41.

other two triangles are divided to make rectangles of the same dimensions.

4–6. See **56–59**, Art. 1270.

7. The two triangles have two sides of one, AC and BC, equal to two sides of the other, CE and CD; and the angle ACB equal to its opposite angle DCE; hence the third side, DE, of one triangle is equal to the third side, AB, of the other.

9. The area of the larger triangle is four times that of the smaller.

11. Another method of dividing a line into equal parts. See Problem **66**, Art. 1270.

1273. Much interest is added to this work by employing the method here given, in calculating heights and distances in the neighborhood of the school. A comparison between the results obtained by calculations and those obtained by actual measurements, will be useful in teaching the pupils the necessity of great accuracy in their preliminary work.

2. The hypotenuse of each triangle represents a ray of light from the sun, etc.

3. $VT = 110$ ft.

4. $CD : DE :: CB : BA$; $50 : 90 :: 1200 : BA$; $BA = 2160$ ft. The width of the river $= 2160$ ft. $- 100$ ft. $= 2060$ ft. *Ans.*

5. $CD = CG - DG = 6$ ft. $- 4$ ft. $= 2$ ft.; $EH = 177$ ft. $+ 3$ ft. $= 180$ ft; $AH = 120$ ft.; $AB = 120$ ft. $+ 4$ ft. $= 124$ ft.

6. The two triangles are similar, because the angles of one are equal to the angles of the other. Angle $D =$ angle $A = 90°$. The two angles at C are vertical angles, and, therefore, equal to each other. The remaining angle B must be equal to the remaining angle E, because the sum of the angles in each triangle is the same.

$$CD : DE :: AC : AB;$$
$$3.25 : \ 5 \ :: (12 - 3.25) : AB.$$

7. $10\frac{1}{2} : (10\frac{1}{2} + 195 + 15) :: (12 - 4\frac{1}{2}) : hf.$ $hf = 157\frac{1}{2}$ ft.; $fi = 157\frac{1}{2}$ ft. $+ 4\frac{1}{2}$ ft. $= 162$ ft. *Ans.*

8. $RP : RN :: PQ : MN$; $6 : (120 + 6) :: 4 : MN$. $MN =$ 84 ft. *Ans.*

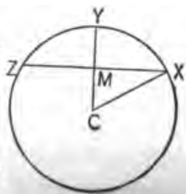
9. The tree is 3 ft. $\times$ 36, or 108 ft. high. *Ans.* $AC = AB$, because angle $C =$ angle $B = 45°$.

10. $AC = AB$. A line CB makes an angle of 45° with AC, angle $A = 90°$; angle $ABC = 45°$.

NOTE.—In the succeeding problems, the triangle and the protractor may be employed when necessary.

1274. **1.** The circumference of the circle, or $360° = 4$ in. $\times 3.1416$. The arc of 60° is $\frac{1}{6}$ of the circumference; the arc of $120° = \frac{1}{3}$ of it; etc.

2. Draw the circle and the various chords. The chord of 60° $=$ chord of $300° =$ radius $= 2$ inches; the chord of $180° =$ diameter $= 4$ inches.



If the pupils draw the chord of 120°, ZX (Fig. 42), they will find that it bisects the radius YC, making MC 1 inch. $CX = 2$ inches; therefore $MX = \sqrt{4 - 1} = \sqrt{3} = 1.732+$; $ZX = 1.732+$ in. $\times 2 = 3.464+$ in. $=$ chord of $120° =$ chord of 240°.

Diagrams should be employed, unless the pupils can do satisfactory work without them. Since the arc of 180° is three times as long as the arc of 60°, the more careless members of the class may jump to the conclusion that the chord of 180° is three times as long as the chord of 60°. This mistake cannot be made if the circle is constructed, and the chords are drawn and measured.

5. See Exercise **98**, Art. 1269.

6. Each side of the hexagon measures 1 inch; the perimeter $= 1$ inch $\times 6 = 6$ inches. *Ans.*

Circumference $= 2$ in. $\times 3.1416 = 6.2832$ inches. *Ans.*

7. The pupil should use the ruler to ascertain the length of the apothem, which is about $\frac{7}{8}$ in. It can be calculated as follows: —

Cb (Fig. 38) is the radius $= 1$ inch; $bx =$ one-half the side of the inscribed hexagon $= \frac{1}{2}$ inch; $Cx =$ apothem. $\overline{Cx}^2 = \overline{Cb}^2 - \overline{bx}^2 = 1 - \frac{1}{4} = .75$; $Cx = \sqrt{.75} = .866+$, or nearly $\frac{7}{8}$.

8. The base of each triangle measures one inch, so that AB (Fig. 43) $= 3$ in. AX (apothem) $= \frac{7}{8}$ in. nearly.

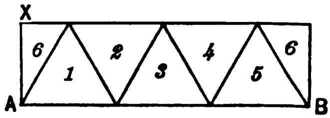


FIG. 48.

9. The base of each triangle measures about $\frac{3}{4}$ in. so that the base of the rectangle (the half perimeter) will be nearly 3 inches and the apothem about $\frac{15}{16}$ in. Area about $3 \times \frac{15}{16}$ sq. in. $=$ about $2\frac{13}{16}$ sq. in.

10. $AB \times AX = 3 \times \frac{7}{8} = 2\frac{5}{8}$. *Ans.* $2\frac{5}{8}$ sq. in.

11. See **9**.

12. The perimeter of a 16-sided polygon will be greater than that of an octagon. The 16-sided polygon will have the greater apothem.

13. As the number of sides increases, the perimeter approaches more and more closely the circumference, 6.2832 inches; and the apothem approaches the radius, 1 inch.

14. The base of the rectangle will be 3.1416 inches, one-half the perimeter (circumference); the apothem will be 1 inch (radius). Area $= 3.1416$ sq. in. *Ans.*

15. One-half the circumference $= 2 \times 3.1416 = 6.2832$, multiplied by the radius, 2, gives answer in square inches, 12.5664 sq. in.

16. 78.54 sq. in. *Ans.*

17. 3.1416 sq. in. *Ans.*

18. 314.16 sq. in. $\div 6 =$ *Ans.*

19. Subtract from the area of the outer circle, 113.0976 sq. in., the area of the inner circle, 28.2744 sq. in. *Ans.* 84.8232 sq. in.

1282. *Right* prisms, cylinders, etc., are meant when the word *oblique* is not used.

1. See Arithmetic, Art. 818, Problem **20.** The upper squares need not be drawn, as only the convex surface is required.

2. Three rectangles and two triangles. See **1.**

3. Three rectangles, each 3 inches high, bases 1, $1\frac{1}{2}$, and 2 inches, respectively.

5. A hollow paper cylinder, without bases, can be opened out into a rectangle whose base is the circumference of the base of the cylinder, and whose height is the altitude of the cylinder.

6. The slant height of a square pyramid is the distance from the apex to the center of one side of the base. This problem requires the pupil to draw, side by side, four isosceles triangles, the base of each being 2 inches and the altitude 3 inches. After constructing the first (Fig. 44), by erecting a 3-inch perpendicular at the center of a 2-inch base, he should use B'' and A as centers, and radii equal to $B''B'$ and AB'' to locate B'''. On AB' construct a third triangle, using A and B' as centers and the radii previously given. Upon one side of this triangle construct a fourth.

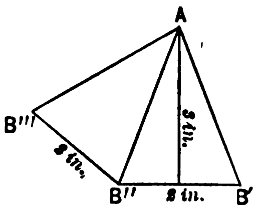


FIG. 44.

The pupils can discover for themselves the method of drawing geometrically the required convex surface.

The altitude should be carefully measured. It will be equal in length to the perpendicular of a right-angled triangle whose hypotenuse is the slant height of a pyramid, 3 inches; and whose base is the distance from the center of one edge of the base of the pyramid (the foot of the slant height) to the foot of the altitude, 1 inch. *Ans.* $\sqrt{8}$ inches $= 2.83$ inches nearly $=$ about $2\frac{13}{16}$ inches.

7. The area of each convex face of a regular pyramid is found by multiplying one side of the base by one-half the slant height; therefore, the area of all the faces forming the convex surface, is obtained by multiplying the sum of all the sides of the base, that is, the perimeter of the base, by one-half the slant height.

8. The pupil requires the slant height in order to proceed as in **6**; and he should obtain it by drawing it rather than by calculating it. He should be able to see that the altitude is a line drawn from the vertex to the center of the base, and that its foot is 1 inch distant from the foot of the altitude. Constructing a right-angled triangle whose base is 1 inch, and whose perpendicular is 3 inches, will give a hypotenuse equal to the required slant height.

Some scholars will bring in a prism whose slant height is 3 inches. The mistake should be pointed out, but not the mode of correcting it; and a pyramid of the required dimensions should be insisted upon.

1283. When the diameter of the base of a cone is 2 inches, the arc *BDC*, which forms the circumference when folded, measures 2π inches, or 6.2832 inches.

NOTE. — The Greek letter π (pi) represents 3.1416.

9. The semi-circumference of paper $= 3\pi$ inches, which is the circumference of base of cone. The diameter of base of cone $= 3\pi$ inches $\div \pi = 3$ inches. The radius of base $= 1\frac{1}{2}$ inches.

The slant height = radius of paper = 3 inches = diameter of base of cone = twice radius of base of cone.

10. The area of any sector of a circle = radius $\times \frac{1}{2}$ length of arc. The arc when folded becomes the circumference of base, and the radius becomes the slant height; so that convex surface $= \frac{1}{2}$ circumference $\times$ slant height = circumference $\times \frac{1}{2}$ slant height.

11. Length of arc of $90° = \frac{1}{4}$ circumference $= \frac{1}{4}$ of 6π inches $= 1\frac{1}{2}\pi$ inches. This is the circumference of the base of the cone, which makes the diameter $= 1\frac{1}{2}\pi$ inches $\div \pi = 1\frac{1}{2}$ inches. The slant height is 3 inches.

Length of arc of $60° = \frac{1}{6}$ circumference $= \frac{1}{6}$ of 6π inches $= \pi$ inches. The diameter of the base of the cone $= \pi$ inches $\div \pi$ $= 1$ inch; slant height, 3 inches.

12. The circumference of the base of the cone $= 3\pi$. This equals the length of the arc of the required sector. If the slant height is 5 inches, the circumference of which the sector is a part $= 10\pi$. The arc of the sector is, therefore, $\frac{3}{10}$ of the circumference; and its length is $\frac{3}{10}$ of $360° = 108°$.

13. XY represents the base of the pyramid; and AC, its altitude. The slant height of two faces, AM (Fig. 45), is the hypotenuse of a right-angled triangle, base $1\frac{1}{2}$ in., perpendicular 4 in. Using this as the perpendicular of a new triangle, with a base MY, gives as the hypotenuse AY (Figs. 45 and 46), one of the edges.

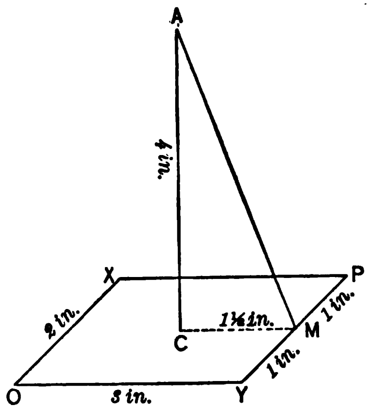


FIG. 45.

To draw the development, take AY as a radius, and draw an arc. On this lay off succes-

sive chords of 3 in., 2 in., 3 in., and 2 in., connecting the extremity of each with the center A. These four triangles

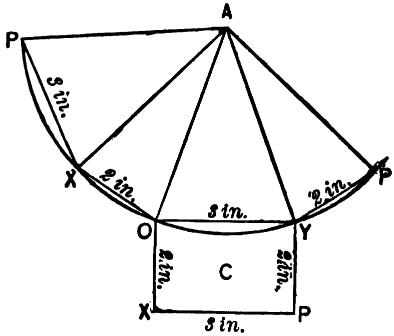


FIG. 46.

constitute the convex faces of the pyramid. On one of them, construct a rectangle 3 inches by 2 inches for the base.

14. In this pyramid, AC (Fig. 45) measures 12 inches, CM measures $\frac{1}{2}$ of 18 inches, or 9 inches, making AM, one slant height, $= \sqrt{144 + 81}$ inches $= 15$ inches. The other slant height, drawn to the center of OY, $= \sqrt{144 + 25}$ inches $= 13$ inches.

The pupils should be encouraged to construct stout paper pyramids and cones of required dimensions, making the necessary calculations themselves. The previous fourteen problems will present no difficulty whatever to pupils that are interested. As examples in dry calculation, they may prove somewhat tiresome. Many scholars will, of themselves, construct models of solids much more complicated than the foregoing.

1284. If there are no solids at hand, the pupils should construct a supply of paper ones, or make them of modeling clay, turnips, etc. Drawings are not sufficient for effective instruction.

15. The short method of ascertaining the convex surface should not be given until **18**. Each of the convex faces is a trapezoid, whose parallel sides measure 4 inches and 8 inches, respectively, the altitude being 10 inches.

16. Two inches apart, draw two parallel lines, AB and CD (Fig. 47), measuring 1 inch and 2 inches, respectively, a 2-inch perpendicular, XY, connecting the middle point of each. Draw AC and BD, and produce them until they meet in M. With this as a center, and MC as a radius, draw an arc; on which three other chords equal to CD are laid off, as in Fig. 46. With M as a center and a radius MA, lay off another arc, on which chords are laid off equal to AB; etc. See Arithmetic, Art. 1296.

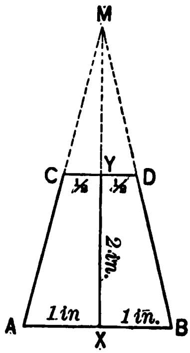


FIG. 47.

17. The entire surface = convex surface + 4 sq. ft. + 9 sq. ft.

18. The pupils can readily understand this rule, using the frustums of problems **15** and **17** as illustrations.

19. The pupil should first locate the apex of the cone. This he can do by following the method shown in **16**; making AB (Fig. 47) $2\frac{1}{2}$ inches; CD, $1\frac{1}{2}$ inches; and AC, 2 inches. This makes MA 5 inches, the slant height of the whole cone. The circumference of the base of the cone $= 2\frac{1}{2}\pi$. The slant height, 5 inches, is the radius of the required sector, its circumference being 10π. $2\frac{1}{2}\pi$, the length of the arc of the sector, being one-fourth of 10π, shows that the required sector is a quadrant.

20. The number of square inches in the convex surface = [circumference (perimeter) of upper base (9×3.1416) + circumference of lower base (6×3.1416)] $\times \frac{1}{2}$ slant height (2). Adding to this, the area of the bottom (3^2) 9 sq. in. $\times$ 3.1416 (Art. 1124, 2) gives the number of square inches of material required.

22.	Circumference of upper base	$= \ \ 6 \times 3.1416$
	Circumference of lower base	$= 10 \times 3.1416$
	One-half sum	$= \ \ 8 \times 3.1416$
	Multiplying by slant height gives	48×3.1416
	Add to this the area of the upper base,	9×3.1416
	And the area of the lower base,	25×3.1416
	Total in square yards,	82×3.1416

or $([(3+5)\times 6]+9+25)\times 3.1416$.

23.

$$EA : EC :: 6 : 8$$
$$x : x+9 :: 6 : 8$$
$$8x = 6x + 54$$
$$2x = 54$$
$$x = 27.$$

Ans. 27 in., $2\frac{1}{4}$ ft.

The slant height of the whole cone $= 27$ in. $+ 9$ in. $= 3$ ft.

24. The convex surface of the whole cone $= \frac{1}{2}(8 \times 3.1416 \times 3)$ sq. ft.; of the part cut off $= \frac{1}{2}(6 \times 3.1416 \times 2\frac{1}{4})$ sq. ft.

1287. A sphere (a croquet ball, for instance) and a hemisphere should be used to illustrate these problems. On the plane face of the latter can be drawn the lines AD, FG, HI, CI, etc.; while on the curved face can be drawn HYI, FXG, etc.

25. $\frac{1}{6}$ of 25,000 miles.

26. $IH =$ chord of 60° of the great circle $=$ radius of the great circle $= 4000$ miles. $IB = \frac{1}{2}$ of $IH = 2000$ miles.

27. The diameter, HI, of the small circle is $\frac{1}{2}$ diameter FG of the great circle; the circumference $HYI = \frac{1}{2}$ of 25,000 miles, or 12,500 miles.

28. The length of a degree of longitude on the 60th parallel is about one-half of the length of a degree on the equator. (See Art. 995, Problem **10.**)

29. On the plane face of the hemisphere suggested above (Art. 1287), draw diameters AD and FG at right angles; and 45° from G, a chord NM parallel to FG. (This chord will not *bisect* AC.) As $MCG = 45°$, $WCM = 45°$; and the triangle WCM is a right-angled isosceles triangle, and $\overline{WM}^2 = \frac{1}{2}\overline{CM}^2$; $WM = .7071 CM$. (See Art. 995, Problem 12.) If CG measures 4000 miles, $CM = 4000$ miles, and $WM = \sqrt{8,000,000} = 2828.4$ miles.

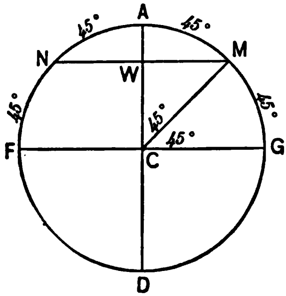


FIG. 48.

1289. The pupils have already learned that the volume of a rectangular prism is equal to the area of the base multiplied by its altitude; these problems are intended to show that the same is true of all prisms and of the cylinder (6).

1292. 8. The volume of the frustum is obtained by deducting the volume of the part cut off from the volume of the whole pyramid (Problem 7). The rule is given later.

10. Fig. 49 gives the method of calculating the slant height. The illustration shows a section of the frustum formed by passing a plane through the center of the frustum perpendicular to the base. The center of the upper base of the frustum of a right pyramid is directly above the center of the lower base, so that a perpendicular let fall from A will fall on CD at a point X, 5 in. from C. In the right-angled triangle AXC, $AX =$ altitude $= 12$ in. $\overline{AC}^2 = \overline{AX}^2 + \overline{XC}^2 = 144 + 25 = 169$; $AC = \sqrt{169} = 13$.

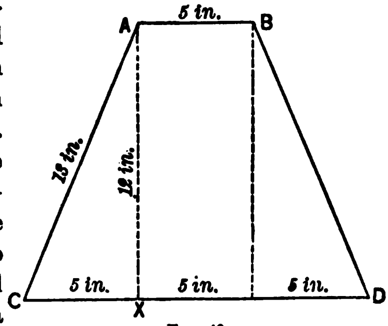


FIG. 49.

XX

NOTES ON THE APPENDIX

1306. A person that contracts to receive a rate of interest greater than is permitted by law, is liable to a penalty, except in Connecticut. In Delaware, Minnesota, etc., the penalty is the forfeiture of the contract; in New York, the forfeiture of the contract, $1000 fine, and 6 months' imprisonment; in Indiana, Kansas, Kentucky, Maryland, Michigan, Mississippi, Ohio, Pennsylvania, Tennessee, Vermont, Virginia, and West Virginia, the forfeiture of the excess of interest; in North Carolina, the forfeiture of double the amount of interest; in Georgia and New Hampshire, the forfeiture of three times the excess of interest; in Alabama, Delaware, Florida, Illinois, Iowa, Louisiana, Missouri, Nebraska, New Jersey, South Carolina, Texas, and Wisconsin, the forfeiture of all interest; in Arkansas and Oregon, the forfeiture of principal and interest.

1307. 3.

Amount of $1000, June 1, 1896, to June 1, 1897	$1,060.00
Amount of $150, Sept. 16, 1896, to June 1, 1897, 8½ mo.	156.37
Due June 1, 1897	$903.63
Interest to settlement, Apr. 16, 1898, 10½ mo.	47.44
Amount Apr. 16, 1898	$951.07
Amount of $50, Sept. 16, 1897, to Apr. 16, 1898, 7 mo.	51.75
Balance due	$899.32

4.

Amount of $500, July 25, 1896, to Apr. 1, 1897, 8 mo. 6 da.		$520.50
Amount of $100, Sept. 18, 1896, to Apr. 1, 1897, 6 mo. 13 da.	$103.22	
Amount of $200, Feb. 5, 1897, to Apr. 1, 1897, 1 mo. 26 da.	201.86	305.08
Balance due		$215.42

5. Amount of $870.50, Jan. 2, 1894, to March 18, 1896, 2 yr. 2 mo. 16 da.	$985.99
Less payments of $35 and $200	235.
New principal	$750.99
Interest March 18, 1896, to Jan. 2, 1897, 9 mo. 14 da.	35.54
Amount	$786.53
Amount of $250, Aug. 24, 1896, to Jan. 2, 1897, 4 mo. 8 da.	255.33
Due at maturity	$531.20

Below will be found answers to the partial payments examples of Chapters XIII and XIV, according to the Connecticut rule.

NOTE.—Although the legal rate in Connecticut is 6%, the rates given in the examples should be used.

Art. 1008. $70.51. Art. 1009. **1.** $224.22. **2.** $261.21. **3.** $1278.15. Art. 1011. **4.** $771.24. **5.** $899.32 (see Art. 1307, **3**). Art. 1013. **5.** $1088.31. Art. 1015. **7.** $700.50. Art. 1023. **7.** $1232.26. Art. 1051. **7.** $77.07. Art. 1090. **3.** $649.13. **4.** $224.64. Art. 1107. **2.** $1089.64.

The time in each of the preceding examples was found by compound subtraction, taking 30 days to each month. In the examples in Art. 1110, the Connecticut rule has been followed. In these, the time is taken in days. (See Art. 1111.)

1308. 7. Principal		$700.00
Interest to June 15, 1897, 2 yr.		84.00
1 year's interest on $42, unpaid interest		2.52
Amount June 15, 1897		$786.52
Amount of $20, Nov. 15, 1895, to June 15, 1897	$21.90	
Amount of $80, Feb. 15, 1897, to June 15, 1897	81.60	103.50
New principal June 15, 1897		$683.02
Interest to Oct. 15, 1899, 2 yr. 4 mo.		95.62
Interest for 1 yr. 4 mo. on $40.98, unpaid interest		3.28
Interest for 4 mo. on $40.98, unpaid interest		.82
Amount Oct. 15, 1899		$782.74
Amount of $15, Sept. 15, 1898, to Oct. 15, 1899		15.98
Due Oct. 15, 1899		$766.76

NOTE.—If four places of decimals are used, the answer to **6** is $367.6442, or $367.64 +; to **7**, $766.7658, or $766.77 −.

The teacher that wishes other examples of this kind can use **9** and **10** of Art. 1309, and **12**, **14**, and **15** of Art. 1310, the answers to which are as follows: Art. 1309. **9.** $1734. **10.** $738.29. Art. 1310. **12.** $1901.18. **14.** $3432.20. **15.** $1010.95.

1309. Pupils in New Hampshire should not be taught the preceding method, but should be confined to the rule laid down by the courts of their own state.

An examination of 8 will show the manner of ascertaining the balance. The interest for 1 year is $36. As no interest is due except that which accrued during the year, and as the sum paid is less than the interest due, no interest is allowed on the payment of $30, made during the year. This payment being $6 less than the interest due at the end of the year, the interest on $6 is added to the interest on the principal at the end of the next year, making a total of $642.36 then due less the *amount* of $100 for 11 months. Interest is allowed on the $100 payment because it is in excess of the interest then due.

9. Principal		$2500.00
Annual interest due May 1, 1897	$150.00	
Payment (no interest) Oct. 1, 1896	100.00	
Balance of interest	$ 50.00	
Annual interest due May 1, 1898	150.00	
Interest on balance of interest, 1 yr.	3.00	203.00
Amount May 1, 1898		$2703.00
Amount of $1000, June 1, 1897, to May 1, 1898		1055.00
New principal May 1, 1898		$1648.00
Interest on $1648 to May 1, 1899	$98.88	
Payment (no interest) Nov. 1, 1898	50.00	
Balance of interest	$48.88	
Interest on $1648 May 1 to Oct. 1	41.20	
Interest on balance of interest, 5 mo.	1.22	91.30
Due Oct. 1, 1899		$1739.30

10. Principal		$1000.00
Annual interest due Jan. 3, 1896	$60.00	
Payment (no interest) June 1, 1895	10.00	
Balance of interest	$50.00	

Amounts brought forward	$ 50.00	$ 1000.00
Annual interest due Jan. 3, 1897	60.00	
Interest on balance of interest, 1 yr.	3.00	
Interest due Jan. 3, 1897	$ 113.00	
Amount of payment March 14, 1896, 9 mo. 19 da.	10.48	
Balance of interest	$ 102.52	
Annual interest due Jan. 3, 1898	60.00	
Interest on balance of interest, 1 yr.	6.15	
Annual interest due Jan. 3, 1899	60.00	
Interest for 1 year on $ 102.52 and $ 60	9.75	238.42
Amount Jan. 3, 1899		$ 1238.42
Amount of $ 500, Sept. 30, 1898, to Jan. 3, 1899		507.75
New principal Jan. 3, 1899		$ 730.67
Interest on $ 730.67 to March 11, 2 mo. 8 da.		8.28
Due March 11, 1899		$ 738.95

NOTE. — The *amount* of the payment of March 14 canceled the $ 3 interest on interest and $ 7.48 additional, leaving $ 102.52 interest still unpaid Jan. 3, 1897 on which two years' interest is taken to Jan. 3, 1899. Two years' interest on the principal is also taken, and one year's interest on the annual interest due Jan. 3, 1898 and unpaid. It will be noticed that no interest is taken on the interest upon interest, $ 3 and $ 6.15. See **14** of Art. 1310, as calculated by the N.H. rule: —

Principal			$ 3000.00
Interest to March 17, 1900, 4 yr.		$ 720.00	
Interest on $ 180 (3 + 2 + 1) yr.	$ 64.80		
Amount of $ 20, 10 mo.	21.00		
Unpaid interest on interest	$ 43.80		
Interest to March 17, 1903, 3 yr.		540.00	
Interest on $ 180 (3 + 3 + 3 + 3 + 2 + 1) yr.	162.00	205.80	
Total interest due March 17, 1903		$ 1465.80	
Amount of $ 1000, 6 mo.		1030.00	435.80
Due March 17, 1903			$ 3435.80

NOTE. — Interest is not taken on $ 43.80.

As additional examples, the teacher may use **7** of Art. 1308, and **12**, **14**, and **15** of Art. 1310. The answers by the N.H. method are **7**, $ 767.60; **12**, $ 1901.27; **14**, $ 3435.80; **15**, $ 1011.40.

1310. Vermont pupils should omit Arts. 1307, 1308, and 1309.

12. Principal		$2500.00
Annual interest due June 1, 1897		309.00
Amount June 1, 1897		$2809.00
Amount of $100 to June 1, 1897	$104.00	
Payment June 1, 1897	1000.00	1104.00
New principal		$1705.00
Annual interest to June 1, 1899	$210.74	
Amount of $50, 7 mo.	51.75	
Unpaid interest	$158.99	
Interest on $1705 to settlement	34.10	
Interest on $158.99 to settlement	3.18	196.27
Due Oct. 1, 1899		$1901.27

13. Principal Jan. 3, 1895		$1000.00
Interest due Jan. 3, 1896	$60.00	
Amount of $10, 7 mo. 2 da.	10.35	
Balance of interest	$49.65	
Interest due Jan. 3, 1897	60.00	
1 year's interest on $49.65	2.98	
Total interest	$112.63	
Amount of $10, 9 mo. 19 da.	10.48	
Balance of interest	$102.15	
Interest on $1000 for 2 years	120.00	
2 years' interest on $102.15	12.26	
1 year's interest on $60	3.60	238.01
Amount Jan. 3, 1899		$1238.01
Amount of $500, Sept. 30, 1898, to Jan 3, 1899		507.75
New principal Jan. 3, 1899		$730.26
Interest to March 11, 1899		8.28
Due March 11, 1899		$738.54

14. Principal March 17, 1896			$3000.00
Interest to March 17, 1900, 4 yr.		$720.00	
Interest on yearly interest (3 + 2 + 1) yr.	$64.80		
Amount of $20, 10 mo.	21.00		
Unpaid interest on yearly interest	$43.80		

Amounts brought forward . .	$43.80	$720.00	$3000.00
3 years' interest to March 17, 1903 . .		540.00	
Interest on yearly interest, 3 yr. on $720 (2 + 1) yr. on $180	162.00	205.80	
Total interest due March 17, 1903 .		$1465.80	
Amount of $1000 to March 17, 1903 . .		1030.00	435.80
Due March 17, 1903 . . .			$3435.80

15. Principal Feb. 25, 1893		$1200.00
Annual interest to Feb. 25, 1897, 4 yr. . . .		313.92
Amount Feb. 25, 1897		$1513.92
Amount of $400, 11 mo. 29 da.		423.93
New principal Feb. 25, 1897		$1089.99
Annual interest on $1089.99, 2 yr.	$134.72	
Amount of $10, 8 mo. 14 da.	10.42	
Unpaid interest Feb. 25, 1899 . . .	$124.30	
Annual interest on $1089.99, 2 yr.	134.72	
Interest for 2 yr. on $124.30	14.91	273.93
Amount Feb. 25, 1901		$1363.92
Amount of $400, 4 mo. 29 da.		409.93
New principal Feb. 25, 1901	•	$953.99
Annual interest on $953.99, 1 yr. 1 da. . . .		57.41
		$1011.40

As additional examples, **7** and **9** may be used. The answer to **7** by the Vermont rule is $766.75; to **9**, $1733.73.

1311. 1. $[(\$8500 + \$600) \times .0155] + \$2 = \$141.05$. *Ans.*

2. The amount to be raised on property $= \$2500 - (\$2 \times 150) = \$2200$. Rate on $\$1 = \$2200 \div 275000 = \$.008$, or 8 mills. 8 mills on $1, or $\frac{4}{5}\%$. *Ans.*

4. Mr. Simmons' grand list $= \$95 + \$2 = \$97$. Tax $= \$2.45 \times 97 = \$237.65$. *Ans.*

5. $1500 + 2x =$ grand list. The grand list multiplied by the rate gives the amount to be raised; $(1500 + 2x) \times 2 \quad 3600$; $3000 + 4x = 3600$; $4x = 600$; $x = 150$. 150 taxable polls. *Ans.*

6. Mr. Hallock's grand list $= \$120 + \$6 = \$126$. Since his taxes are $\$252$, the rate $= \$252 \div 126 = \$2$.

Let $x =$ appraised value of property.

$$\frac{x}{100} + 400 = \text{grand list.}$$

$$2\left(\frac{x}{100} + 400\right) = \text{total levy} = 6800.$$

$$\frac{2x}{100} + 800 = 6800.$$

$$2x + 80000 = 680000.$$

$$x = 300000.$$

Appraised value of property is $\$300000$. *Ans.*

SUPPLEMENT

DEFINITIONS, PRINCIPLES, AND RULES

A **Unit** is a single thing.

A **Number** is a unit or a collection of units.

The **Unit of a Number** is one of that number.

Like Numbers are those that express units of the same kind.

Unlike Numbers are those that express units of different kinds.

A **Concrete Number** is one in which the unit is named.

An **Abstract Number** is one in which the unit is not named.

Notation is expressing numbers by characters.

Arabic Notation is expressing numbers by figures.

Roman Notation is expressing numbers by letters.

Numeration is reading numbers expressed by characters.

The **Place of a Figure** is its position in a number.

A figure standing alone, or in the first place at the right of other figures, expresses *ones*, or *units of the first order*.

A figure in the second place expresses *tens*, or *units of the second order*.

A figure in the third place expresses *hundreds*, or *units of the third order*; and so on.

A **Period** is a group of three orders of units, counting from right to left.

RULE FOR NOTATION. — *Begin at the left, and write the hundreds, tens, and units of each period in succession, filling vacant places and periods with ciphers.*

RULE FOR NUMERATION. — *Beginning at the right, separate the number into periods.*

Beginning at the left, read the numbers in each period, giving the name of each period except the last.

ADDITION

Addition is finding a number equal to two or more given numbers.

Addends are the numbers added.

The **Sum**, or **Amount**, is the number obtained by addition.

PRINCIPLE. — *Only like numbers, and units of the same order can be added.*

RULE. — *Write the numbers so that units of the same order shall be in the same column.*

Beginning at the right, add each column separately, and write the sum, if less than ten, under the column added.

When the sum of any column exceeds nine, write the units only, and add the ten or tens to the next column.

Write the entire sum of the last column.

SUBTRACTION

Subtraction is finding the difference between two numbers.

The **Subtrahend** is the number subtracted.

The **Minuend** is the number from which the subtrahend is taken.

The **Remainder**, or **Difference**, is the number left after subtracting one number from another.

PRINCIPLES. — *Only like numbers and units of the same order can be subtracted.*

The sum of the difference and the subtrahend must equal the minuend.

RULES. — I. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

Beginning at the right, find the number that must be added to the first figure of the subtrahend to produce the figure in the corresponding order of the minuend, and write it below. Proceed in this way until the difference is found.

If any figure in the subtrahend is greater than the corresponding figure in the minuend, find the number that must be added to the former to produce the latter increased by ten; then add one to the next order of the subtrahend and proceed as before.

II. *Beginning at the units' column, subtract each figure of the subtrahend from the corresponding figure of the minuend and write the remainder below.*

If any figure of the subtrahend is greater than the corresponding figure in the minuend, add ten to the latter and subtract; then, (a) *add one to the next order of the subtrahend and proceed as before; or,* (b) *subtract one from the next order of the minuend and proceed as before.*

MULTIPLICATION

Multiplication is taking one number as many times as there are units in another number.

The **Multiplicand** is the number taken or multiplied.

The **Multiplier** is the number that shows how many times the multiplicand is taken.

The **Product** is the result obtained by multiplication.

PRINCIPLES. — *The multiplier must be an abstract number.*

The multiplicand and the product are like numbers.

The product is the same in whatever order the numbers are multiplied.

RULE. — *Write the multiplier under the multiplicand, placing units of the same order in the same column.*

Beginning at the right, multiply the multiplicand by the number of units in each order of the multiplier in succession. Write the

figure of the lowest order in each partial product under the figure of the multiplier that produces it. Add the partial products.

To multiply by 10, 100, 1000, etc.

RULE. — *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.*

DIVISION

Division is finding how many times one number is contained in another, or finding one of the equal parts of a number.

The **Dividend** is the number divided.

The **Divisor** is the number contained in the dividend.

The **Quotient** is the result obtained by division.

PRINCIPLES. — *When the divisor and the dividend are like numbers, the quotient is an abstract number.*

When the divisor is an abstract number, the dividend and the quotient are like numbers.

The product of the divisor and the quotient, plus the remainder, if any, is equal to the dividend.

RULE. — *Write the divisor at the left of the dividend with a line between them.*

Find how many times the divisor is contained in the fewest figures on the left of the dividend, and write the result over the last figure of the partial dividend. Multiply the divisor by this quotient figure, and write the product under the figures divided. Subtract the product from the partial dividend used, and to the remainder annex the next figure of the dividend for a new dividend.

Divide as before until all the figures of the dividend have been used.

If any partial dividend will not contain the divisor, write a cipher in the quotient, and annex the next figure of the dividend.

If there is a remainder after the last division, write it after the quotient with the divisor underneath.

FACTORING

An **Exact Divisor** of a number is a number that will divide it without a remainder.

An **Odd Number** is one that cannot be exactly divided by two.

An **Even Number** is one that can be exactly divided by two.

The **Factors** of a number are the numbers that multiplied together produce that number.

A **Prime Number** is a number that has no factors.

A **Composite Number** is a number that has factors.

A **Prime Factor** is a prime number used as a factor.

A **Composite Factor** is a composite number used as a factor.

Factoring is separating a number into its factors.

To find the Prime Factors of a Number.

RULE. — *Divide the number by any prime factor. Divide the quotient, if composite, in like manner; and so continue until a prime quotient is found. The several divisors and the last quotient will be the prime factors.*

CANCELLATION

Cancellation is rejecting equal factors from dividend and divisor.

PRINCIPLE. — *Dividing dividend and divisor by the same number does not affect the quotient.*

GREATEST COMMON DIVISOR

A **Common Factor** (divisor or measure) is a number that is a factor of each of two or more numbers.

A **Common Prime Factor** is a prime number that is a factor of each of two or more numbers.

The **Greatest Common Factor** (divisor or measure) is the largest number that is a factor of each of two or more numbers.

Numbers are prime to each other when they have no common factor.

The greatest common divisor of two or more numbers is the product of their common prime factors.

PRINCIPLES. — *A common divisor of two numbers is a divisor of their sum, and also of their difference.*

A divisor of a number is a divisor of every multiple of that number; and a common divisor of two or more numbers is a divisor of any of their multiples.

To find the Common Prime Factors of Two or More Numbers.

RULE. — *Divide the numbers by any common prime factors, and the quotients in like manner, until they have no common factor; the several divisors are the common prime factors.*

To find the Greatest Common Divisor of Numbers that are Easily Factored.

RULE. — *Separate the numbers into their prime factors; the product of those that are common is the greatest common divisor.*

To find the Greatest Common Divisor of Numbers that are not Easily Factored.

RULE. — *Divide the greater number by the less; then divide the last divisor by the last remainder, continuing until there is no remainder. The last divisor is the greatest common divisor.*

If there are more than two numbers, find the greatest common divisor of two of them; then of that divisor and another of the numbers until all of the numbers have been used. The last divisor is the greatest common divisor.

LEAST COMMON MULTIPLE

A **Multiple** of a number is a number that exactly contains that number.

A **Common Multiple** of two or more numbers is a number that is a multiple of each of them.

The **Least Common Multiple** of two or more numbers is the smallest number that is a common multiple of them.

PRINCIPLES. — *A multiple of a number contains all the prime factors of that number.*

A common multiple of two or more numbers contains each of the prime factors of those numbers.

The Least Common Multiple of two or more numbers contains only the prime factors of each of the numbers.

To find the Least Common Multiple of Two or More Numbers.

RULE. — *Divide by any prime number that is an exact divisor of two or more of the numbers, and write the quotients and undivided numbers below. Divide these numbers in like manner, continuing until no two of the remaining numbers have a common factor. The product of the divisors and remaining numbers is the least common multiple.*

FRACTIONS

A **Fraction** is one or more of the equal parts of anything.

The **Unit of a Fraction** is the number or thing that is divided into equal parts.

A **Fractional Unit** is one of the equal parts into which the number or thing is divided.

The **Terms of a Fraction** are its numerator and its denominator.

The **Denominator** of a fraction shows into how many parts the unit is divided.

The **Numerator** of a fraction shows how many of the parts are taken.

A fraction indicates division; the numerator being the dividend and the denominator the divisor.

The **Value of a Fraction** is the quotient of the numerator divided by the denominator.

Fractions are divided into two classes — **Common** and **Decimal.**

A **Common Fraction** is one in which the unit is divided into any number of equal parts.

A common fraction is expressed by writing the numerator above the denominator with a dividing line between.

Common fractions consist of three principal classes — **Simple, Compound,** and **Complex.**

A **Simple Fraction** is one whose terms are whole numbers.

A **Proper Fraction** is a simple fraction whose numerator is less than its denominator.

An **Improper Fraction** is a simple fraction whose numerator equals or exceeds its denominator.

A **Compound Fraction** is a fraction of a fraction.

A **Complex Fraction** is one having a fraction in its numerator, or in its denominator, or in both.

A **Mixed Number** is a whole number and a fraction written together.

The **Reciprocal of a Number** is one divided by that number.

The **Reciprocal of a Fraction** is one divided by the fraction, or the fraction inverted.

PRINCIPLES. — *Multiplying the numerator or dividing the denominator multiplies the fraction.*

Dividing the numerator or multiplying the denominator divides the fraction.

Multiplying or dividing both terms of a fraction by the same number does not alter the value of the fraction.

Reduction of fractions is changing their terms without altering their value.

To reduce a Fraction to Higher Terms.

RULE. — *Multiply both numerator and denominator by the same number.*

To reduce a Fraction to its Lowest Terms.

RULE. — *Divide both terms of the fraction by their greatest common divisor.*

A fraction is in its lowest terms when the numerator and the denominator are prime to each other.

To reduce a Mixed Number to an Improper Fraction.

RULE. — *Multiply the whole number by the denominator; to the product add the numerator; and place the sum over the denominator.*

To reduce an Improper Fraction to a Whole or to a Mixed Number.

RULE. — *Divide the numerator by the denominator.*

A **Common Denominator** is a denominator common to two or more fractions.

The **Least Common Denominator** is the smallest denominator common to two or more fractions.

To reduce Fractions to a Common Denominator.

RULE. — *Find the least common multiple of all the denominators for the least common denominator. Divide this multiple by the denominator of each fraction, and multiply the numerator by the quotient.*

ADDITION OF FRACTIONS

PRINCIPLE. — *Only like fractions can be added.*

RULE. — *Reduce the fractions, if necessary, to a common denominator, and over it write the sum of the numerators.*

If there are mixed numbers, add the fractions and the whole numbers separately, and unite the results.

SUBTRACTION OF FRACTIONS

PRINCIPLE. — *Only like fractions can be subtracted.*

RULE. — *Reduce the fractions, if necessary, to a common denominator, and over it write the difference between the numerators.*

If there are mixed numbers subtract the fractions and the whole numbers separately, and unite the results.

MULTIPLICATION OF FRACTIONS

RULE. — *Reduce whole and mixed numbers to improper fractions; cancel the factors common to numerators and denominators, and write the product of the remaining factors in the numerators over the product of the remaining factors in the denominators.*

DIVISION OF FRACTIONS

RULES. — I. *Reduce whole and mixed numbers to improper fractions. Reduce the fractions to a common denominator. Divide the numerator of the dividend by the numerator of the divisor.*

II. *Invert the divisor and proceed as in multiplication of fractions.*

To reduce a Complex Fraction to a Simple One.

RULES. — I. *Multiply the numerator of the complex fraction by its denominator inverted.*

II. *Multiply both terms by the least common multiple of the denominators.*

DECIMALS

A **Decimal Fraction** is one in which the unit is divided into tenths, hundredths, thousandths, etc.

A **Decimal** is a decimal fraction whose denomination is indicated by the number of places at the right of the decimal point.

The **Decimal Point** is the mark used to locate units.

A **Mixed Decimal** is a whole number and a decimal written together.

A **Complex Decimal** is a decimal with a common fraction written at its right.

To write Decimals.

RULE. — *Write the numerator; and from the right, point off as many decimal places as there are ciphers in the denominator, prefixing ciphers, if necessary, to make the required number.*

To read Decimals.

RULE. — *Read the numerator, and give the name of the right-hand order.*

PRINCIPLES. — *Prefixing ciphers to a decimal diminishes its value.*

Removing ciphers from the left of a decimal increases its value.

Annexing ciphers to a decimal or removing ciphers from its right does not alter its value.

To reduce a Decimal to a Common Fraction.

RULE. — *Write the figures of the decimal for the numerator, and* 1, *with as many ciphers as there are places in the decimal, for the denominator, and reduce the fraction to its lowest terms.*

To reduce a Common Fraction to a Decimal.

RULE. — *Annex decimal ciphers to the numerator, and divide it by the denominator.*

To reduce Decimals to a Common Denominator.

RULE. — *Make their decimal places equal by annexing ciphers.*

ADDITION AND SUBTRACTION OF DECIMALS

Decimals are added and subtracted the same as whole numbers.

MULTIPLICATION OF DECIMALS

RULE. — *Multiply as in whole numbers, and from the right of the product, point off as many decimal places as there are decimal places in both factors.*

DIVISION OF DECIMALS

RULE. — *Make the divisor a whole number by removing the decimal point, and make a corresponding change in the dividend. Divide as in whole numbers, and place the decimal point in the quotient under (or over) the new decimal point in the dividend.*

ACCOUNTS AND BILLS

A **Debtor** is a person who owes another.

A **Creditor** is a person to whom a debt is due.

An **Account** is a record of debits and credits between persons doing business.

The **Balance** of an account is the difference between the debit and credit sides.

A **Bill** is a written statement of an account.

An **Invoice** is a written statement of items, sent with merchandise.

A **Receipt** is a written acknowledgment of the payment of part or all of a debt.

A bill is receipted when the words, "Received Payment," are written at the bottom, signed by the creditor, or by some person duly authorized.

DENOMINATE NUMBERS

A **Measure** is a standard established by law or custom, by which distance, capacity, surface, time, or weight is determined.

A **Denominate Unit** is a unit of measure.

A **Denominate Number** is a denominate unit or a collection of denominate units.

A **Simple Denominate Number** consists of denominate units of one kind.

A **Compound Denominate Number** consists of denominate units of two or more kinds.

A **Denominate Fraction** is a fraction of a denominate number.

A denominate fraction may be either **common** or **decimal.**

Reduction of denominate numbers is changing them from one denomination to another without altering their value.

Reduction Descending is changing a denominate number to one of a lower denomination.

RULE. — *Multiply the highest denomination by the number required to reduce it to the next lower denomination, and to the product add the units of that lower denomination, if any. Proceed in this manner until the required denomination is reached.*

Reduction Ascending is changing a denominate number to one of a higher denomination.

RULE. — *Divide the given denomination successively by the numbers that will reduce it to the required denomination. To this quotient annex the several remainders.*

To find the Time between Dates.

RULE. — *When the time is less than one year, find the exact number of days; if greater than one year, find the time by compound subtraction, taking* 30 *days to the month.*

PERCENTAGE

Per Cent means hundredths.

Percentage is computing by hundredths.

The elements involved in percentage are the **Base, Rate, Percentage, Amount,** and **Difference.**

The **Base** is the number of which a number of hundredths is taken.

The **Rate** indicates the number of hundredths to be taken.

The **Percentage** is one or more hundredths of the base.

The **Amount** is the base increased by the percentage.

The **Difference** is the base diminished by the percentage.

To find the Percentage when the Base and Rate are Given.

RULE. — *Multiply the base by the rate expressed as hundredths.*

To find the Rate when the Percentage and Base are Given.

RULE. — *Divide the percentage by the base.*

To find the Base when the Percentage and Rate are Given.

RULE. — *Divide the percentage by the rate expressed as hundredths.*

To find the Base when the Amount and Rate are Given.

RULE. — *Divide the amount by* 1 + *the rate expressed as hundredths.*

To find the Base when the Difference and Rate are Given.

RULE. — *Divide the difference by* 1 — *the rate expressed as hundredths.*

PROFIT AND LOSS

Profit or **Loss** is the difference between the buying and selling prices.

In Profit and Loss,

The buying price, or cost, is the *base.*

The rate per cent profit or loss is the *rate.*

The profit or loss is the *percentage.*

The selling price is the *amount* or *difference,* according as it is more or less than the buying price.

COMMERCIAL DISCOUNT

Commercial Discount is a percentage deducted from the list price of goods, the face of a bill, etc.

The **Net Price** of goods is the sum received for them.

In Commercial Discount,

$\left.\begin{array}{l}\text{The list price, or}\\ \text{The face of the bill}\end{array}\right\}$ is the *base.*

The rate per cent discount is the *rate.*

The discount is the *percentage.*

The list price diminished by the discount is the *difference.*

In successive discounts, the first discount is made from the list price or the face of the bill; the second discount, from the list price or face of the bill diminished by the first discount; and so on.

COMMISSION

Commission is a percentage allowed an agent for his services.

A **Commission Agent** is one who transacts business on commission.

A **Consignment** is the merchandise forwarded to a commission agent.

The **Consignor** is the person who sends the merchandise.

The **Consignee** is the person to whom the merchandise is sent.

The **Net Proceeds** is the sum remaining after all charges have been deducted.

In buying, the commission is a percentage of the *buying price;* in selling, a percentage of the *selling price;* in collecting, a percentage of the *sum collected;* hence:

$\left.\begin{array}{l}\text{The sum invested, or}\\ \text{The sum collected}\end{array}\right\}$ is the *base.*

The rate per cent commission is the *rate.*

The commission is the *percentage.*

The sum invested increased by the commission is the *amount.*

The sum collected diminished by the commission is the *difference.*

INSURANCE

Insurance is a contract of indemnity.

Insurance is of three kinds — **Fire, Marine,** and **Life.**

Fire Insurance is indemnity against loss of property by fire.

Marine Insurance is indemnity against loss of property by the casualities of navigation.

Life Insurance is indemnity against loss of life.

The **Insurance Policy** is the contract setting forth the liability of the insurer.

The **Policy Face** is the amount of insurance.

The **Premium** is the price paid for insurance.

The **Insurer,** or **Underwriter,** is the company issuing the policy.

The **Insured** is the person for whose benefit the policy is issued.

In Insurance,

The policy face is the *base.*

The rate per cent premium is the *rate.*

The premium is the *percentage.*

TAXES

A **Tax** is a sum of money levied on persons or property for public purposes.

A **Personal**, or **Poll Tax**, is a tax on the person.

A **Property Tax** is a tax of a certain per cent on the assessed value of property.

Property may be either personal or real.

Personal Property consists of such things as are movable.

Real Property is that which is fixed, or immovable.

In Taxes,

The assessed value is the *base*.

The rate of taxation is the *rate*.

The tax is the *percentage*.

DUTIES

Duties are taxes on imported goods.

Duties are either **Specific** or **Ad Valorem**.

A **Specific Duty** is a tax on goods without regard to cost.

An **Ad Valorem** duty is a tax of a certain per cent on the cost of goods.

In Ad Valorem Duties,

The cost of the goods is the *base*.

The rate per cent duty is the *rate*.

The ad valorem duty is the *percentage*.

INTEREST

Interest is the sum paid for the use of money.

The **Principal** is the sum loaned.

The **Amount** is the sum of the principal and interest.

The **Rate of Interest** is the rate per cent for one year.

The **Legal Rate** is the rate fixed by law.

Usury is interest at a higher rate than that fixed by law.

Simple Interest is interest on the principal only.

To find the Interest when the Principal, Time, and Rate are Given.

RULE. — *Multiply the principal by the rate expressed as hundredths, and this product by the time expressed in years.*

To find the Time when the Principal, Interest, and Rate are Given.

RULE. — *Divide the given interest by the interest for one year.*

To find the Rate when the Principal, Interest, and Time are Given.

RULE. — *Divide the given interest by the interest at one per cent.*

To find the Principal when the Interest, Rate, and Time are Given.

RULE. — *Divide the given interest by the interest on* $ 1.

To find the Principal when the Amount and Time and Rate are Given.

RULE. — *Divide the given amount by the amount of* **$ 1.**

INTEREST BY ALIQUOT PARTS.

To find the Interest for Years, Months, and Days.

RULE. — *Find the interest for one year and take this as many times as there are years.*

Take the greatest number of the given months that equals an aliquot part of a year and find the interest for this time. Take aliquot parts of this for the remaining months.

In the same manner find the interest for the days.

The sum of these interests will be the interest required.

To find the Interest when the Time is Less than a Year.

RULE. — *Find the interest for the time in months or days that will gain one per cent of the principal.*

Find by aliquot parts, as in the first rule, the interest for the remaining time.

The sum of these interests will be the interest required.

INTEREST BY SIX PER CENT METHOD.

To find the Interest at 6%.

RULE.—**For Years:** *Multiply the principal by the rate expressed as hundredths, and that product by the number of years.*

For Months: *Move the decimal point two places to the left, and multiply by one-half the number of months.*

For Days: *Move the decimal point three places to the left, and multiply by one-sixth the number of days.*

To find the interest at any other rate per cent, divide the interest at 6% by 6, and multiply the quotient by the given rate.

To find Exact Interest.

RULE.—*Multiply the principal by the rate expressed as hundredths, and that product by the time expressed in years of* 365 *days.*

ANNUAL INTEREST

Annual Interest is interest payable annually. If not paid when due, annual interest draws simple interest.

To find the Amount Due on a Note with Annual Interest, when the Interest has not been Paid Annually.

RULE.—*Find the interest on the principal for the entire time, and on each annual interest for the time it remained unpaid. The sum of the principal and all the interest is the amount due.*

COMPOUND INTEREST

Compound Interest is interest on the principal and on the unpaid interest, which is added to the principal at regular intervals. The interest may be compounded annually, semi-annually, quarterly, etc., according to agreement.

To find Compound Interest.

RULE.—*Find the amount of the given principal for the first period. Considering this as a new principal, find the amount of*

it for the next period, continuing in this manner for the given time.

Find the difference between the last amount and the given principal, which will be the compound interest.

PARTIAL PAYMENTS

Partial Payments are part payments of a note or debt. Each payment is recorded on the back of the note or the written obligation.

UNITED STATES RULE.—*Find the amount of the principal to the time when the payment or the sum of two or more payments equals or exceeds the interest.*

From this amount deduct the payment or sum of payments.

Use the balance then due as a new principal, and proceed as before.

MERCHANTS' RULE.—*Find the amount of an interest-bearing note at the time of settlement.*

Find the amount of each credit from its time of payment to the time of settlement; subtract their sum from the amount of the principal.

BANK DISCOUNT

Bank Discount is a percentage retained by a bank for advancing money on a note before it is due.

The **Sum Discounted** is the face of the note, or if interest-bearing, the amount of the note at maturity.

The **Term of Discount** is the number of days from the day of discount to the day of maturity.

The **Bank Discount** is the interest on the sum discounted for the term of discount.

The **Proceeds** of a note is the sum discounted less the bank discount.

Problems in bank discount are calculated as problems in interest.

In Bank Discount,

The sum discounted is the *principal.*

The rate of discount is the *rate of interest.*

The term of discount is the *time.*

The bank discount is the *proceeds.*

EXCHANGE

Exchange is making payments at a distance by means of drafts or bills of exchange.

Domestic Exchange is exchange between places in the same country.

Foreign Exchange is exchange between different countries.

Exchange is *at par* when a draft, or bill, sells for its face value; *at a premium* when it sells for more than its face value; *at a discount* when it sells for less.

The cost of a sight draft is the face of the draft increased by the premium, or diminished by the discount.

The cost of a time draft is the face of the draft increased by the premium, or diminished by the discount, and this result diminished by the bank discount.

To find the Cost of a Draft.

RULE.—*Find the cost of* $1 *of the draft; multiply this by the face of the draft.*

To find the Face of a Draft.

RULE.—*Divide the cost of the draft by the cost of* $1 *of the draft.*

EQUATION OF PAYMENTS

Equation of Payments is a method of ascertaining at what time several debts due at different times may be settled by a single payment.

The **Equated Time** of payment is the time when the several debts may be equitably settled by one payment.

The **Term of Credit** is the time the debt has to run before it becomes due.

The **Average Term of Credit** is the time the debts due at different times have to run, before they may be equitably settled by one payment.

To find the Equated Time of Payment when the Terms of Credit begin at the Same Date.

RULE. — *Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit.*

Add the average term of credit to the date of the debts, and the result will be the equated time of payment.

To find the Equated Time when the Terms of Credit begin at Different Dates.

RULE. — *Find the date at which each debt becomes due. Select the earliest date as a standard.*

Multiply each debt by the number of days between the standard date and the date when the debt becomes due, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit from the standard date.

Add the average term of credit to the standard date, and the result will be the equated time of payment.

RATIO

Ratio is the relation one number bears to another of the same kind.

The **Terms** of the ratio are the numbers compared.

The **Antecedent** is the first term.

The **Consequent** is the second term.

The antecedent and consequent form a *couplet.*

PRINCIPLES. — See Fractions.

PROPORTION

A **Proportion** is formed by two equal ratios.

The **Extremes** of a proportion are the first and last terms.

The **Means** of a proportion are the second and third terms.

PRINCIPLES. — *The product of the means is equal to the product of the extremes.*

Either mean equals the product of the extremes divided by the other mean.

Either extreme equals the product of the means divided by the other extreme.

RULE FOR PROPORTION. — *Represent the required term by x.*

Arrange the terms so that the required term and the similar known term may form one couplet, the remaining terms the other.

If the required term is in the extremes, divide the product of the means by the given extreme.

If the required term is in the means, divide the product of the extremes by the given mean.

PARTNERSHIP

Partnership is an association of two or more persons for business purposes.

The **Partners** are the persons associated.

The **Capital** is that which is invested in the business.

The **Assets** are the partnership property.

The **Liabilities** are the partnership debts.

To find the Profit, or Loss, of Each Partner when the Capital of Each is Employed for the Same Period of Time.

RULE. — *Find the part of the entire profit, or loss, that each partner's capital is of the entire capital.*

To find the Profit, or Loss, of Each Partner when the Capital of Each is Employed for Different Periods of Time.

RULE. — *Find each partner's capital for one month, by multiplying the amount he invests by the number of months it is employed; then find the part of the entire profit, or loss, that each partner's capital for one month is of the entire capital for one month.*

INVOLUTION

A **Power** of a number is the product obtained by using that number a certain number of times as a factor.

The **First Power** of a number is the number itself.

The **Second Power** of a number, or the **Square**, is the product of a number taken twice as a factor.

The **Third Power** of a number, or the **Cube**, is the product of a number taken three times as a factor.

An **Exponent** is a small figure written a little to the right of the upper part of a number to indicate the power.

Involution is finding any power of a number.

To find the Power of a Number.

RULE.— *Take the number as a factor as many times as there are units in the exponent.*

EVOLUTION

A **Root** is one of the equal factors of a number.

The **Square Root** of a number is one of its two equal factors.

The **Cube Root** of a number is one of its three equal factors.

Evolution is finding any root of a number.

Evolution may be indicated in two ways: by the *Radical Sign*, $\sqrt{\ }$, or by a *fractional exponent.*

The **Index** of a root is a small figure placed a little to the left of the upper part of the radical sign, to indicate what root is to be found. In expressing square root, the index is omitted.

In the fractional exponent, the numerator indicates the power to which the number is to be raised; the denominator indicates the root to be taken of the number thus raised.

To find the Square Root of a Number.

RULE.—*Point off in periods of two figures, commencing at units. Find the greatest square in the first period and place the root in the quotient. Subtract this square from the first period, and bring down the next period.*

Multiply the quotient figure by two, and use it as a trial divisor. Place the second figure in the quotient, and annex it also to the trial divisor. Then multiply the figures in the trial divisor by the second quotient figure, and subtract.

Bring down the next period, and proceed as before until the square root is found.

To find the Square Root of a Fraction.

RULE. — *Reduce the fraction to its simplest form, and find the square root of each term separately.*

To find the Cube Root of a Number.

RULE. — *Point off in periods of three figures each, beginning at units.*

Find the greatest cube in the first period and place the root in the quotient. Subtract this cube from the first period, and bring down the next period.

Multiply the square of the first quotient figure by three and annex two ciphers for a trial divisor. Place the second figure in the quotient. Then, to the trial divisor add three times the product of the first and second figures, also the square of the second. Multiply this sum by the second figure and subtract.

Bring down the next period, and proceed as before until the cube root is found.

To find the Cube Root of a Fraction.

RULE. — *Reduce the fraction to its simplest form, and find the cube root of each term separately.*

STOCKS AND BONDS.

Capital Stock is the money or property employed by a corporation in its business.

A **Share** is one of the equal divisions of capital stock.

The **Stockholders** are the owners of the capital stock.

The **Par Value** of stock is the face value.

The **Market Value** of stock is the sum for which it may be sold.

Stock is at a *premium* when the market value is above the par value; at a *discount*, when below par.

Bonds are interest-bearing notes issued by a government or a corporation.

A **Dividend** is a percentage apportioned among the stockholders.

A **Stock Broker** is a person who deals in stocks.

Brokerage is a percentage allowed a stock broker for his services.

In Stocks and Bonds,

The par value is the *base*.

The rate per cent premium, or discount, is the *rate*.

The premium, discount, or dividend } is the *percentage*.

The market value is the { *amount*, or *difference*.

NOTES, DRAFTS, AND CHECKS.

A **Promissory Note** is a written promise to pay a specified sum on demand, or at a specified time.

The **Face** of a note is the sum named in the note.

The **Maker** is the person who signs it.

The **Payee** is the person to whom the sum specified is to be paid.

The **Indorser** is the person who signs his name on the back of the note, thus becoming liable for its payment in case of default of the maker.

An **Interest-bearing Note** is one payable with interest.

If the words "with interest" are omitted, interest cannot be collected until after maturity.

A **Demand Note** is one payable when demand of payment is made.

A **Time Note** is one payable at a specified time.

A **Joint Note** is one signed by two or more persons who jointly promise to pay.

A **Joint and Several Note** is one signed by two or more persons who jointly and severally promise to pay.

In a joint note, each person is liable for the whole amount, but they must all be sued together. In the joint and several note, each is liable for the whole amount, and may be sued separately.

A **Negotiable Note** is one that may be transferred or sold. It contains the words "or bearer," or "or order."

A **Non-negotiable Note** is one not payable to the bearer, nor to the payee's order.

The **Maturity** of a note is the day on which it legally falls due.

A **Draft**, or **Bill of Exchange**, is a written order directing the payment of a specified sum of money.

The **Face** of a draft is the sum named in it.

The **Drawer** is the person who signs the draft.

The **Drawee** is the person ordered to pay the sum specified.

The **Payee** is the person to whom the sum specified is to be paid.

A **Sight Draft** is one payable when presented.

A **Time Draft** is one payable at a specified time.

An **Acceptance** of a time draft is an agreement by the drawee to pay the draft at maturity, which he signifies by writing across the face of the draft the word "accepted" with the date and his name.

A **Check** is an order on a bank or banker to pay a specified sum of money.

ANSWERS. — PART I.

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3. 120.
4. 160.
5. 168.
6. 176.
7. 184.
8. 192.
9. 200.
10. 248.
11. 264.
12. 280.
13. 328.
14. 344.
15. 416.
16. 488.
17. 584.
18. 640.
19. 728.
20. 232.
21. 304.
22. 376.
23. 448.
24. 520.
25. 592.
26. 664.
27. 736.
28. 808.
29. 1624.
30. 2440.
31. 3256.
32. 4072.
33. 4904.
34. 5736.
35. 6568.
36. 7480.
37. 8400.
38. 17,080.
39. 27,648.
40. 32,664.
41. 41,336.
42. 49,872.
43. 60,544.
44. 66,320.
45. 72,536.
46. 75,264.
47. 81,872.
48. 96,368.
49. 92,056.
50. 82,760.

Page 91.

1. 13.
2. 14.
3. 15.
4. 20.
5. 21.
6. 22.
7. 23.
8. 24.
9. 25.
10. 31.
11. 29.
12. 38.
13. 47.
14. 56.
15. 65.
16. 74.
17. 83.
18. 92.
19. 101.
20. 203.
21. 305.
22. 407.
23. 509.
24. 613.
25. 717.
26. 821.
27. 935.
28. 1050.
29. 2135.
30. 3456.

Page 94.

1. 117.
2. 126.
3. 135.
4. 180.
5. 189.
6. 198.
7. 207.
8. 216.
9. 225.
10. 288.
11. 306.
12. 324.
13. 369.
14. 387.
15. 486.
16. 558.
17. 675.
18. 738.
19. 837.
20. 252.
21. 351.
22. 414.
23. 495.
24. 576.
25. 657.
26. 729.
27. 828.
28. 918.
29. 1836.
30. 6552.
31. 7488.
32. 8514.
33. 9459.
34. 19,287.
35. 31,185.
36. 36,837.
37. 37,944.
38. 39,924.
39. 45,225.
40. 46,557.
41. 54,162.
42. 57,078.
43. 69,165.
44. 75,330.
45. 81,603.
46. 93,105.
47. 99,603.
48. 94,401.
49. 94,707.
50. 95,922.
51. 13.
52. 14.
53. 15.
54. 20.
55. 21.
56. 22.
57. 23.
58. 24.
59. 25.
60. 31.
61. 33.
62. 35.
63. 41.
64. 43.
65. 46.
66. 61.

67. 73.
68. 80.
69. 91.
70. 29.
71. 43.
72. 47.
73. 56.
74. 65.
75. 74.
76. 83.
77. 92.
78. 101.
79. 203.
80. 305.
81. 407.
82. 509.
83. 613.
84. 717.
85. 821.
86. 935.
87. 1050.
88. 2125.
89. 5091.
90. 7246.

Page 95.

91. $17\frac{4}{8}$.
$15\frac{5}{9}$.
92. $27\frac{1}{8}$.
$24\frac{1}{9}$.
93. $45\frac{1}{8}$.
$40\frac{1}{9}$.
94. $60\frac{7}{8}$.
$53\frac{5}{9}$.
95. $84\frac{3}{8}$.
75.
96. $97\frac{1}{8}$.
$86\frac{4}{9}$.
97. $100\frac{4}{8}$.
$89\frac{3}{9}$.
98. $112\frac{6}{8}$.
$100\frac{2}{9}$.
99. $50\frac{5}{8}$.
45.
100. $78\frac{3}{8}$.
$69\frac{6}{9}$.
101. $114\frac{3}{8}$.
$101\frac{5}{9}$.
102. $46\frac{7}{8}$.
$41\frac{6}{9}$.

Page 97.

1. 73,188.
2. 92,345.
3. 67,172.
4. 98,789.
5. 24,246.
6. 22,941.
7. 14,286.
8. 282.
9. 547.

Page 98.

1. 28.
2. 45.
3. 64.
4. 85.
5. 108.
6. 32.
7. 60.
8. 88.
9. 116.
10. 144.
11. 172.
12. 228.
13. 300.
14. 273.
15. 420.
16. 4300.
17. 690.
18. 2886.
19. 4329.
20. 1221.

Page 101.

1. $897.42.
2. $740.87.
3. $1226.38.
4. $7700.88.
5. $86,322.53.
6. $168.64.
7. $116.93.
8. $599.93.
9. $81.89.
10. $497.27.
11. $386.08.
12. $95.07.
13. $98.01.

Page 102.

14. $20.95.
15. $256.91.
16. $13.20.
17. $136.50.
18. $406.
19. $418.75.
20. $207.41.
21. $9.90.
22. $21.50.
23. $38.35.
24. $40.13.
25. $18.59.
26. $112.75.

Page 103.

1. 378.
2. 210.
3. 496.
4. 609.
5. 744.
6. 1088.
7. 1944.
8. 2175.
9. 3139.
10. 1200.
11. 4104.
12. 944.
13. 819.
14. 2583.
15. 775.
16. 2180.
17. 3402.
18. 5376.
19. 5238.
20. 1342.

Page 111.

1. 50.
2. $68\frac{3}{4}$.
3. $96\frac{3}{4}$.
4. 72.
5. 100.
6. $96\frac{3}{4}$.
7. $85\frac{1}{4}$.
8. $70\frac{1}{4}$.
9. $86\frac{1}{4}$.
10. $43\frac{1}{4}$.
11. 78.

Page 113.

1. 143.
156.
2. 154.
168.
3. 165.
180.
4. 176.
192.
5. 187.
204.
6. 231.
252.

7. 242.
264.
8. 253.
276.
9. 264.
288.
10. 275.
300.
11. 341.
372.
12. 352.
384.
13. 363.
396.
14. 374.
408.
15. 451.
492.
16. 462.
504.
17. 473.
516.
18. 550.
600.
19. 561.
612.
20. 572.
624.
21. 671.
732.
22. 682.
744.
23. 803.
876.
24. 924.
1008.
25. 1023.
1116.
26. 1221.
1332.
27. 2343.
2556.
28. 3564.
3888.
29. 5016.
5472.
30. 5577.
6084.
31. 6765.
7380.
32. 7920.
8640.
33. 8976.
9792.
34. 10,164.
11,088.
35. 11,682.
12,744.
36. 25,795.
28,140.
37. 34,364.
37,488.
38. 49,676.
54,192.
39. 63,019.
68,748.
40. 70,224.
76,608.
41. 86,988.
94,896.
42. 90,970.
99,240.
43. 88,935.
97,020.
44. 86,768.
94,656.
45. 82,137.
89,604.
46. 69,520.
75,840.
47. 64,988.
70,896.
48. 50,237.
54,804.
49. 34,045.
37,140.
50. 32,989.
35,988.

Page 114.

51. 12.
11.
52. 24.
22.
53. 36.
33.
54. 48.
44.
55. 60.
55.
56. 72.
66.
57. 84.
77.
58. 96.
88.
59. 108.
99.
60. 120.
110.
61. 144.
132.
62. 168.
154.
63. 252.
231.
64. 360.
330.
65. 492.
451.
66. 672.
616.
67. 828.
759.
68. $170\frac{6}{11}$.
$156\frac{4}{12}$.
69. $213\frac{2}{11}$.
$195\frac{5}{12}$.
70. $333\frac{7}{11}$.
$305\frac{10}{12}$.
71. $389\frac{2}{11}$.
$356\frac{9}{12}$.
72. 482.
$441\frac{10}{12}$.
73. $613\frac{6}{11}$.
$562\frac{5}{12}$.
74. $712\frac{3}{11}$.
$652\frac{11}{12}$.
75. $811\frac{5}{11}$.
$743\frac{10}{12}$.
76. $857\frac{4}{11}$.
$785\frac{11}{12}$.
77. 936.
858.
78. $1110\frac{10}{11}$.
$1018\frac{4}{12}$.
79. 1284.
1177.
80. 1560.
1430.
81. 3300.
3025.
82. 4272.
3916.
83. 7116.
6523.
84. 8484.
7777.
85. $981\frac{5}{11}$.
$899\frac{5}{12}$.

86. 1109$\frac{10}{11}$.
1017$\frac{5}{12}$.
87. 1362$\frac{6}{11}$.
1249.
88. 1518$\frac{2}{11}$.
1391$\frac{8}{12}$.
89. 1637$\frac{1}{11}$.
1500$\frac{8}{12}$.
90. 1910$\frac{2}{11}$.
1751.
91. 2243$\frac{7}{11}$.
2056$\frac{8}{12}$.
92. 2520$\frac{7}{11}$.
2310$\frac{7}{12}$.
93. 2782$\frac{8}{11}$.
2550$\frac{5}{12}$.
94. 3301$\frac{7}{11}$.
3026$\frac{6}{12}$.
95. 4545$\frac{4}{11}$.
4166$\frac{7}{12}$.
96. 4893$\frac{4}{11}$.
4485$\frac{7}{12}$.
97. 5726.
5248$\frac{10}{12}$.
98. 6391$\frac{8}{11}$.
5858$\frac{8}{12}$.
99. 54.
100. 41.
101. 67.
102. 81.
103. 75.
104. 72.
105. 4$\frac{1}{4}$.
106. 8.
107. 7.
108. 11.
109. 124.
110. 124.
111. 243.
112. 129.
113. 420.
114. 810.
115. 183.
116. 412.
117. 504.
118. 336.

Page 115.

119. 405.
120. 255.
121. 1089.
122. 1056.
123. 30.
124. 50.
125. 70.
126. 90.
127. 25.
128. 35.
129. 40.
130. 50.
131. 80.
132. 55.
133. 28.
134. 52.
135. 76.
136. 55.
137. 115.
138. 610.
139. 25$\frac{1}{4}$.
140. $\frac{2}{3}$.
141. 1$\frac{3}{4}$.
142. 32$\frac{1}{4}$.
143. 49$\frac{1}{4}$.
144. 19$\frac{1}{3}$.
145. 58$\frac{1}{2}$.
146. 19$\frac{1}{2}$.
147. 24$\frac{2}{3}$.
148. 52$\frac{1}{4}$.

Page 118.

1. 450.
2. 640.
3. 850.
4. 1080.
5. 1330.
6. 1600.
7. 1890.
9. 2530.
10. 2880.
11. 2460.
12. 7020.
13. 13,800.
14. 22,800.
15. 34,020.
16. 47,460.
17. 63,120.
18. 80,100.
20. 96,360.
21. 91,800.
22. 99,240.
23. 99,150.
24. 99,040.
35. 15.
36. 15.
37. 14.
38. 16.
39. 26.
40. 58.
41. 143.
42. 236.
43. 469.
44. 616.
45. 572.
46. 906.
47. 1307.
48. 1923.

Page 119.

58. 36$\frac{9}{20}$.
59. 28$\frac{3}{30}$.
60. 132$\frac{1}{40}$.
61. 127$\frac{3}{50}$.
62. 126$\frac{5}{60}$.
63. 129$\frac{4}{70}$.
64. 142$\frac{7}{80}$.
65. 149$\frac{5}{90}$.
67. 184$\frac{1}{110}$.
68. 199$\frac{7}{120}$.
69. 356$\frac{9}{110}$.
70. 534$\frac{2}{90}$.
71. 786$\frac{3}{80}$.
72. 1142$\frac{1}{70}$.
89. 54$\frac{21}{80}$.
90. 125$\frac{14}{70}$.
91. 168$\frac{48}{60}$.
92. 469$\frac{28}{50}$.
93. 866$\frac{32}{40}$.
94. 1541$\frac{28}{30}$.
95. 2640$\frac{14}{20}$.
96. 2422$\frac{21}{30}$.
97. 2370$\frac{35}{40}$.
98. 336$\frac{7}{50}$.
99. 555$\frac{36}{60}$.
100. 21$\frac{8}{70}$.
101. 287$\frac{45}{80}$.
102. 109$\frac{66}{90}$.
103. 770$\frac{65}{90}$.
104. 905$\frac{68}{80}$.
105. 617$\frac{20}{70}$.
106. 571$\frac{20}{60}$.
107. 1084$\frac{10}{50}$.
108. 82.
109. 144.
110. 339$\frac{10}{20}$.
111. 59$\frac{94}{110}$.
112. 60$\frac{80}{120}$.
113. 156$\frac{75}{110}$.
114. 155$\frac{95}{120}$.
115. 1036$\frac{6}{90}$.
116. 1014$\frac{68}{80}$.
117. 176$\frac{25}{70}$.

Page 120.

1. 34$\frac{1}{4}$.

2. $48\frac{1}{4}$.
3. $75\frac{1}{4}$.
4. $88\frac{1}{4}$.
5. 83.
6. $42\frac{1}{2}$.
7. $62\frac{3}{4}$.
8. $26\frac{1}{2}$.
9. $83\frac{1}{4}$.
10. 78.
11. $53\frac{1}{4}$.
12. $84\frac{1}{2}$.
13. $83\frac{1}{4}$.
14. $93\frac{1}{4}$.
15. $90\frac{3}{4}$.
16. 96.
17. $94\frac{1}{2}$.
18. $99\frac{3}{4}$.
19. 81.
20. 96.
21. $3\frac{1}{4}$.
22. $17\frac{1}{2}$.
23. 5.
24. $37\frac{1}{4}$.
25. $42\frac{1}{2}$.
26. $52\frac{1}{4}$.
27. $7\frac{1}{3}$.
28. $58\frac{2}{3}$.
29. $7\frac{1}{3}$.
30. $43\frac{3}{4}$.
31. $46\frac{1}{4}$.
32. 13.
34. 15.
35. $36\frac{1}{2}$.
36. $19\frac{1}{4}$.
37. $37\frac{1}{4}$.
38. $13\frac{1}{3}$.
39. $7\frac{1}{2}$.
40. $34\frac{1}{4}$.

Page 123.

1. 182.
2. 299.
3. 793.
4. 715.
5. 196.
6. 322.
7. 854.
8. 770.
9. 315.
10. 504.
11. 1302.
12. 1155.
13. 864.
14. 1476.
15. 2193.
16. 810.
17. 3965.
18. 1224.
19. 5751.
20. 936.
21. 756.
22. 840.
23. 912.
24. 918.
25. 968.
26. 736.
27. 576.
28. 858.
29. 1024.
30. 1485.
31. 1496.
32. 1575.
33. 1806.
34. 2408.
35. 2860.
36. 3510.
37. 4212.
38. 4558.
39. 4428.
40. 3630.
41. 4464.
42. 5544.
43. 6656.
44. 6630.
45. 8208.
46. 9782.
47. 9620.
48. 9375.
49. 9512.
50. 8964.
51. 10,332.
52. 11,220.
53. 10,028.
54. 10,323.
55. 9776.
56. 9975.
57. 3870.
58. 4370.
59. 8262.
60. 11,396.
61. 15,167.
62. 22,104.
63. 11,433.
64. 15,580.
65. 20,709.
66. 28,980.
67. 10,998.
68. 16,560.
69. 22,050.
70. 31,360.
71. 5814.
72. 11,948.
73. 18,408.
74. 27,456.
75. 35,041.
76. 43,112.
77. 55,752.
78. 69,160.
79. 78,925.
80. 93,912.
81. 95,590.
82. 90,300.
83. 95,961.
84. 91,696.
85. 93,005.
86. 99,648.
87. 99,425.
88. 99,078.
89. 99,891.
90. 96,480.
91. 91,350.
92. 89,642.
93. 97,363.
94. 96,768.
95. 72,415.
96. 86,773.
97. 99,975.
98. 99,970.
99. 99,171.
100. 95,168.

Page 124.

1. 13.
2. 21.
3. 22.
4. 23.
5. 112.
6. 211.
7. 123.
8. 222.
9. 11.
10. 12.
11. 21.
12. 22.
13. 111.
14. 212.
15. 12.
16. 21.
17. 121.
18. 122.
19. 201.
20. 222.
21. 11.
22. 12.

23. 13.
24. 23.
25. 31.
26. 112.
27. 213.
28. 313.
29. 211.
30. 122.
31. 311.
32. 213.
33. 311.
34. 231.
35. 33.
36. 113.
37. 23.
38. 34.
39. 31.
40. 34.
41. 31.
42. 32.

Page 126.

1. 200 feet.
2. $20.70.
3. 11 sheep.
4. 900 inches.
5. 480 ounces.
6. 96 cents.
7. 16 pages.
8. 96 packages.
9. 25 gallons.
10. 7 ounces.
11. $50.
12. $100.
13. $175.
14. 15 miles.

Page 130.

1. $41\frac{3}{4}$.
2. $62\frac{1}{8}$.
3. $57\frac{3}{8}$.
4. $66\frac{5}{8}$.
5. $81\frac{7}{8}$.
6. $98\frac{7}{8}$.
7. $91\frac{3}{4}$.
8. $99\frac{5}{8}$.
9. $99\frac{3}{8}$.
10. $52\frac{5}{8}$.

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11. $42\frac{7}{8}$.
12. $46\frac{7}{8}$.
13. 78.
14. 46.
15. 19.
16. 40.
17. 73.
18. 95.
19. 89.
20. 80.
21. $3\frac{1}{2}$.
22. $2\frac{3}{4}$.
27. $3\frac{1}{4}$.
28. $18\frac{1}{4}$.
29. $35\frac{1}{8}$.
30. $53\frac{1}{2}$.
31. $88\frac{1}{8}$.
32. $7\frac{1}{4}$.
33. $37\frac{3}{8}$.
34. $15\frac{1}{4}$.
35. $21\frac{3}{8}$.
36. $27\frac{1}{2}$.
37. $25\frac{5}{8}$.
38. $36\frac{1}{4}$.
39. $8\frac{1}{8}$.
40. $39\frac{3}{4}$.
41. $17\frac{3}{8}$.
42. $37\frac{5}{8}$.
43. $15\frac{7}{8}$.
44. $45\frac{1}{8}$.
45. 47.
50. $4\frac{3}{8}$.

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51. $916.61.
52. $778.91.
53. $1780.53½.
54. $3431.48½.
55. $76.11.
57. $657.66.
58. $4.17.
59. $76.50.
61. $58.20.
62. $457.12.
63. $977.67.
64. $963.69.
65. $725.04.
66. $31.25.
67. $.62.
68. $108.06.
69. $37.12½.
70. $.34.

Page 134.

1. $4.14.
2. $22.50.
3. $14.
4. 21 cents.
5. $252.
6. $750.
7. 5 yards.
8. 96 jars.
9. 9 gallons.
10. 75 cents.
11. $1620.
12. $95.
13. $80.

Page 135.

14. 68 days.
15. 36 bushels.
16. $2.56.
17. $240.
18. 260 feet.
19. 60 miles.
20. 150 days.

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1. 31.
2. 31.
3. 24.
4. 31.
5. 42.
6. 23.
7. 41.
8. 11.
9. 21.
10. 21.
11. 11.
12. 21.
13. 11.
14. 21.
15. 31.
16. 41.
17. 23.
18. 31.
19. 111.
20. 111.
21. 321.
22. 322.
23. 300.
24. $302\frac{1}{15}$.

Page 136.

25. $20\frac{7}{24}$.
26. $40\frac{14}{21}$.
27. $50\frac{9}{81}$.
28. 203.
29. 202.
30. $202\frac{3}{14}$.
31. $202\frac{5}{14}$.
32. 101.
33. $101\frac{5}{15}$.
34. 203.
35. 304.
36. $200\frac{4}{21}$.

37. 304.
38. $430\frac{4}{21}$.
39. 203.
40. $431\frac{8}{21}$.
41. 202.
42. $120\frac{14}{81}$.
43. $221\frac{4}{21}$.
44. 123.
45. 325.
46. $231\frac{1}{32}$.
47. $101\frac{4}{52}$.
48. $34\frac{4}{71}$.
49. $122\frac{14}{81}$.
50. 103.
51. $20\frac{6}{33}$.

Page 141.

1. 47.
2. 84.
3. 32.
4. 73.
5. $81\frac{1}{4}$.
6. $68\frac{3}{4}$.
7. $38\frac{1}{8}$.
8. $20\frac{5}{8}$.
9. $70\frac{5}{8}$.
10. $83\frac{1}{8}$.
11. $6\frac{1}{2}$.
12. $18\frac{3}{8}$.
13. $19\frac{3}{4}$.
14. 17.
15. 50.
16. 26.
17. $31\frac{1}{8}$.
18. $13\frac{1}{8}$.
19. $10\frac{7}{8}$.
20. $22\frac{7}{8}$.
21. $2\frac{1}{4}$.
22. $13\frac{1}{8}$.
23. 21.
24. $30\frac{1}{4}$.
25. 39.
26. $1\frac{1}{8}$.
27. $10\frac{5}{8}$.
28. $10\frac{1}{8}$.
29. $10\frac{1}{8}$.
30. 10.
31. $9\frac{3}{4}$.
32. $20\frac{1}{2}$.
33. $41\frac{1}{4}$.
34. $18\frac{1}{2}$.
35. $26\frac{1}{4}$.
36. $31\frac{3}{8}$.
37. $67\frac{5}{8}$.
38. $19\frac{1}{8}$.
39. $1\frac{1}{8}$.
40. $19\frac{3}{8}$.

Page 142.

41. 185.
42. 48.
43. 203.
44. 19.
45. 90.
46. 29.
47. 70.
48. 4.
49. 428.
50. 17,376.
51. 1000.
52. 5600.
53. 5600.
54. 78.
55. 126.
56. 168.
57. 144.
58. 144.
59. 144.
60. 84.
61. 10.
62. 4410.
63. 15.
64. 15.
65. 13.
66. 200.
67. 48.
68. 60.
69. 32.
70. 60.

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1. $5.
2. 150 stamps.
3. 62 cows.
4. $16.
5. 28 pounds.

Page 143.

6. 144 pieces.
7. $12.80.
8. 40 boxes.
9. 49 inches.
10. 234 eggs.
11. $60.
12. 5 cents.
13. 15 cents.
14. $2.94.
15. 67 cents.
16. 7 packages.
17. $3.
18. 750 pounds.
19. 15 cents.
20. 46 cents.
21. 20 cents.

Page 144.

22. 30 cents.
23. 5 yards.
24. 196 pounds.
25. 20 pieces.

---

1. 70,800.
2. 71,200.
3. 67,000.
4. 48,300.
5. 78,300.
6. 98,400.
7. 98,800.
8. 91,000.
9. 72,000.

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10. 90,000.
11. 88,800.
12. 84,150.
15. 95,000.
16. 77,400.
17. 83,700.
18. 89,100.
19. 93,000.
20. 67,200.
21. 95,370.
22. 99,540.
23. 81,480.
24. 88,480.

Page 147.

1. $7\frac{2}{3}$.
2. $9\frac{1}{2}$.
3. $16\frac{1}{3}$.
4. $19\frac{1}{3}$.
5. $20\frac{1}{6}$.
6. $19\frac{1}{6}$.
7. 48.
8. 37.
9. $22\frac{7}{8}$.
10. $45\frac{5}{6}$.
11. $84\frac{1}{2}$.
12. $89\frac{1}{2}$.
13. $11\frac{5}{6}$.
14. $39\frac{5}{6}$.
15. $48\frac{1}{6}$.
16. 7.
17. $22\frac{1}{3}$.
18. $19\frac{2}{3}$.

19. $30\frac{1}{2}$.
20. $36\frac{1}{2}$.
21. $35\frac{3}{8}$.
22. $25\frac{5}{6}$.
23. $12\frac{5}{6}$.
24. $4\frac{1}{6}$.
25. $97\frac{1}{2}$.
26. $15\frac{7}{8}$.
27. $91\frac{1}{8}$.
28. $31\frac{5}{8}$.
29. 43.
30. $88\frac{3}{8}$.
31. $9\frac{1}{3}$.
32. $21\frac{1}{6}$.
33. $7\frac{1}{3}$.
34. $22\frac{1}{2}$.
35. $5\frac{3}{8}$.
36. $11\frac{1}{8}$.
37. $15\frac{1}{6}$.
38. $14\frac{1}{8}$.
39. $16\frac{1}{2}$.
40. $17\frac{2}{3}$.
41. $19\frac{5}{6}$.
42. $28\frac{5}{6}$.
43. $38\frac{1}{6}$.
44. $47\frac{1}{6}$.
45. $50\frac{1}{6}$.
46. $8\frac{1}{2}$.
47. $28\frac{2}{3}$.
48. $21\frac{3}{4}$.
49. $7\frac{5}{6}$.

Page 149.

1. 54.
2. 54.
3. 54.
4. 54.
5. 44.
6. 32.
7. 24.
8. 33.
9. 32.
10. 45.
11. $45\frac{6}{34}$.
12. 45.
13. 43.
14. 56.
15. 65.
16. 78.
17. 81.
18. 86.
19. 82.
20. 66.
21. 72.
22. 88.
23. 123.
24. 138.
25. 77.
26. 133.
27. 37.
28. 77.
29. 115.
30. 97.
31. $85\frac{71}{84}$.
32. $97\frac{17}{85}$.
33. 92.
34. $65\frac{19}{93}$.
35. 68.
36. 95.
37. 143.
38. 541.
39. 328.
40. 216.
41. 304.
42. 271.
43. 74.
44. 143.
45. 206.
46. 184.
47. 136.
48. 108.
49. 204.
50. 162.
51. 143.
52. 152.
53. 84.
54. $143\frac{13}{66}$.
55. 123.
56. $135\frac{36}{65}$.
57. 96.
58. $121\frac{33}{76}$.
59. 126.
60. 103.
61. 109.
62. 112.
63. 107.
64. $97\frac{41}{55}$.
65. $89\frac{29}{35}$.
66. 54.
67. 97.
68. 102.
69. 92.
70. 350.
71. $232\frac{4}{35}$.
72. $108\frac{29}{45}$.
73. $174\frac{33}{54}$.
74. $139\frac{6}{65}$.
75. $136\frac{13}{72}$.
76. $119\frac{77}{81}$.
77. $131\frac{45}{64}$.
78. $86\frac{15}{56}$.
79. $120\frac{24}{43}$.
80. $206\frac{17}{32}$.
81. $304\frac{8}{25}$.
82. $206\frac{8}{35}$.
83. $234\frac{15}{42}$.
84. $178\frac{43}{49}$.
85. $116\frac{41}{63}$.
86. $120\frac{45}{64}$.
87. $400\frac{9}{24}$.
88. $545\frac{15}{16}$.
89. $555\frac{9}{13}$.
90. 355.
91. 91.
92. 142.
93. 113.
94. $112\frac{59}{86}$.
95. $87\frac{69}{95}$.
96. $51\frac{55}{96}$.
97. $103\frac{43}{92}$.
98. $102\frac{51}{88}$.
99. $103\frac{69}{77}$.

Page 150.

1. 136.
2. 32.
3. 34.
4. 122.
5. 87.
6. 75.
7. 24.
8. 65.
9. 25.
10. 49.

Page 151.

1. $136\frac{90}{300}$.
2. $93\frac{25}{400}$.
3. $119\frac{32}{500}$.
4. $107\frac{180}{600}$.
5. $89\frac{400}{700}$.
6. $90\frac{184}{800}$.
7. $92\frac{616}{900}$.
8. $80\frac{685}{1100}$.
9. $81\frac{1180}{1200}$.
10. $8\frac{602}{1300}$.

Page 152.

2. 3783.
3. 8587.
4. 7488.
5. $335\frac{5}{8}$.
6. $3000.
7. $3750.
8. 160 feet.

Page 153.

3. 8964.
4. 1637.
5. 216.
6. 39,204.

---

3. 9042.
4. 8856.
5. 37.
6. 14,030.
7. $58.50.

Page 155.

1. $27\frac{5}{9}$.
2. $47\frac{7}{9}$.
3. $51\frac{5}{9}$.
4. $38\frac{4}{9}$.
5. $66\frac{7}{9}$.
6. $99\frac{5}{9}$.
7. $68\frac{7}{9}$.
8. $94\frac{4}{9}$.
9. $95\frac{5}{9}$.
10. 85.
11. 99.
12. 23.
13. $44\frac{1}{9}$.
14. $70\frac{2}{9}$.
15. $27\frac{1}{9}$.
16. $42\frac{2}{9}$.
17. $53\frac{5}{6}$.
18. $98\frac{1}{2}$.
19. $83\frac{2}{9}$.
20. $35\frac{4}{9}$.
21. $6\frac{7}{9}$.
22. $23\frac{2}{9}$.
23. $22\frac{7}{9}$.
24. $31\frac{5}{9}$.
25. $31\frac{1}{3}$.
26. $12\frac{7}{8}$.
27. 10.
28. $27\frac{2}{9}$.
29. $16\frac{4}{9}$.
30. $53\frac{2}{9}$.
31. $3\frac{1}{3}$.
32. $26\frac{1}{9}$.
33. $21\frac{8}{9}$.
34. $4\frac{1}{9}$.
35. $44\frac{4}{9}$.
36. $11\frac{1}{9}$.
37. $8\frac{5}{9}$.
38. $16\frac{7}{9}$.
39. $24\frac{7}{9}$.
40. $32\frac{3}{4}$.
41. $40\frac{2}{9}$.
42. $48\frac{5}{9}$.
43. $56\frac{1}{2}$.
44. $64\frac{4}{9}$.
45. $72\frac{2}{9}$.
46. $79\frac{1}{9}$.
47. $27\frac{1}{3}$.
48. $17\frac{1}{6}$.
49. $29\frac{1}{6}$.
50. $62\frac{5}{6}$.

Page 157.

1. 99,684.
2. 85,731.
3. 95,772.
4. 94,770.
5. 94,095.
6. 89,622.
7. 96,882.
8. 95,914.
9. 99,507.
10. 91,344.
11. 86,592.
12. 97,020.
13. 93,832.
14. 79,328.
15. 91,464.
16. 96,266.
17. 94,520.
18. 94,518.
19. 90,750.
20. 93,396.
21. 96,170.
22. 97,908.
23. 89,159.
24. 87,472.
25. 97,768.
26. 95,918.
27. 43.
28. 78.
29. 32.
30. 24.
31. 14.
32. 13.
33. 14.
34. 12.
35. 11.
36. 9.

Page 158.

37. 8.
38. 13.
39. 15.
40. 13.
41. 24.
42. 23.
43. 45.
44. 75.
45. 33.
46. 22.
47. 8.
48. 4.
49. 6.
50. 4.
51. 68,580.
52. 96,621.
53. 96,859.
54. 96,740.
55. 92,640.
56. 85,181.
57. 82,926.
58. 64,684.
59. 76,020.
60. 97,768.
61. 90,752.
62. 70,455.
63. 98,049.
64. 86,592.
65. 98,245.
66. 71,604.
67. 99,770.
68. 98,802.
69. 81,804.
70. 95,081.
71. 98,245.
72. 98,245.
73. 92,486.
74. 75,072.

Page 159.

75. 88.
76. 105.
77. 99.
78. 98.
79. 69.
80. 69.
81. 168.
82. 95.
83. 90.
84. 93.
85. 186.
86. 154.
87. 232.
88. 368.
89. 297.
90. 100.
91. 102.
92. 205.
93. 255.

94. 320.
95. 456.
96. 675.
97. 880.
98. 615.

Page 160.

1. 360.
2. 1125.
3. 800.
4. 1200.
5. 1770.
6. 2800.
7. 2331.
8. 16,044.
9. 14,883.
10. 39,234.
11. 22,243.
12. 4400.

Page 161.

13. 6578.
14. 23,922.
15. 43,190.
16. 49,260.
17. 17,922.
18. 61,479.
19. 85,200.
20. 82,810.
21. 6888.
22. 13,552.
23. 39,528.
24. 51,968.
25. 14,610.
26. 25,280.
27. 50,904.
28. 65,400.
29. 84,252.
30. 96,560.
31. 79,380.
32. 30,537.
33. 42,372.
34. 107,028.
35. 96,444.
36. 92,376.
37. $337\frac{1}{2}$.
38. $673\frac{2}{3}$.
39. $1237\frac{1}{4}$.
40. $1897\frac{1}{5}$.
41. $1683\frac{1}{3}$.
42. $2880\frac{3}{4}$.
43. $1869\frac{3}{8}$.
44. $4575\frac{5}{6}$.
45. $6286\frac{1}{2}$.
46. $21,441\frac{3}{4}$.
47. 40,138.
48. $44,500\frac{4}{5}$.
49. $42,274\frac{3}{7}$.
50. $99,682\frac{2}{9}$.
51. $65,166\frac{4}{5}$.
52. 4231.
53. 3152.
54. 2405.
55. 1600.
56. 1623.
57. 1405.
58. 1234.
59. 1035.
60. 2305.
61. 2046.
62. 1653.
63. 1408.
64. 1305.
65. 1060.
66. 1003.
67. 3265.
68. 907.
69. 807.
70. 486.
71. 325.
72. 247.
73. 189.
74. 155.
75. 138.
76. 123.
77. 109.
78. 406.
79. 308.
80. 203.
81. 170.
82. 146.
83. 123.
84. 105.
85. 104.
86. 98.
87. 48.
88. $33\frac{80}{3002}$.
89. 24.
90. 19.
91. 16.
92. 14.
93. 12.
94. 11.
95. 98.
96. 87.
97. 75.
98. 33.
99. 23.
100. 9.
101. 4.
102. 8.

Page 164.

1. $7.
2. $2.82.
3. 192 pints.
4. 20 yards.
5. $2.40.
6. 40 cents.

Page 165.

7. $3.
8. $2.25.
9. $2\frac{1}{3}$ yards.
10. 108 quarters.
11. $72.50.
12. 3 cents.
13. 86 feet.
14. $7.80.
15. $2.40.
16. 95 cents.
17. 99 cents.
18. 48 eggs; 144 eggs.
19. 40 bushels.
20. $60.
21. 413 butterflies.
22. 99 cents.

Page 166.

23. 130 yards.
24. $7\frac{3}{4}$ acres.
25. $1.75.

---

1. 91,448.
2. 86,400.
3. 97,886.
4. 90,288.
5. 89,415.
6. 88,971.
7. 89,208.
8. 82,766.
9. 99,696.
10. 73,140.
11. 82,602.
12. 99,960.
13. 97,633.
14. 96,348.
15. 180.
16. 232.
17. 348.
18. 567.
19. 864.

20. 1120.
21. 777.
22. 945.
23. 1100.
24. 1343.
25. 496.
26. $1454\frac{2}{5}$.
27. 96,000.
28. 99,712.
29. 96,888.
30. 99,328.
31. 77,608.
32. 99,450.
33. 99,902.
34. 95,841.
35. 61,845.
36. 99,102.
37. 96,696.
38. 92,976.
39. 99,051.
40. 93,345.
41. 96,744.
42. 88,920.
43. 99,601.
44. 99,485.
45. 2000.
46. 2800.
47. 24,000.
48. 24,500.
49. 99,000.
50. 96,000.
51. 81,081.

Page 167.

1. $129\frac{1}{4}$.
2. $92\frac{7}{8}$.
3. $79\frac{7}{8}$.
4. $97\frac{3}{8}$.
5. $69\frac{1}{6}$.
6. $27\frac{8}{9}$.
7. $47\frac{3}{8}$.
8. $99\frac{3}{8}$.
9. $82\frac{3}{8}$.
10. $108\frac{1}{8}$.
11. $64\frac{3}{8}$.
12. 91.
13. 95.
14. $37\frac{7}{8}$.
15. 26.

Page 168.

16. 86.
17. $81\frac{1}{8}$.
18. 49.
19. $83\frac{1}{3}$.
20. $87\frac{1}{2}$.
21. $64\frac{1}{8}$.
22. $28\frac{1}{8}$.
23. $37\frac{1}{8}$.
24. $35\frac{1}{8}$.
25. $54\frac{1}{4}$.
26. $69\frac{3}{8}$.
27. $30\frac{5}{8}$.
28. $81\frac{1}{8}$.
29. $\frac{1}{8}$.
30. $75\frac{1}{6}$.
31. $8\frac{2}{3}$.
32. $6\frac{1}{2}$.
33. $18\frac{1}{2}$.
34. $43\frac{2}{3}$.
35. $12\frac{3}{4}$.
36. $11\frac{5}{6}$.
37. $24\frac{7}{8}$.
38. $18\frac{8}{9}$.
39. $27\frac{7}{9}$.
40. $40\frac{2}{9}$.

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1. $2857\frac{8}{21}$.
2. $3134\frac{14}{31}$.
3. $1225\frac{14}{41}$.
4. $1622\frac{45}{51}$.
5. $990\frac{6}{61}$.
6. $538\frac{70}{71}$.
7. $713\frac{78}{81}$.
8. $282\frac{29}{91}$.
9. $1636\frac{16}{33}$.
10. $1966\frac{15}{25}$.
11. $811\frac{5}{52}$.
12. $787\frac{27}{43}$.
13. $478\frac{37}{74}$.
14. $222\frac{12}{62}$.
15. $279\frac{24}{92}$.
16. $711\frac{77}{85}$.
17. $182\frac{25}{102}$.
18. $432\frac{205}{207}$.
19. $153\frac{184}{301}$.
20. $86\frac{80}{408}$.
21. $181\frac{81}{503}$.
22. $113\frac{363}{604}$.
23. $104\frac{170}{705}$.
24. $111\frac{487}{809}$.
25. $70\frac{210}{906}$.
26. $709\frac{4}{112}$.
27. $219\frac{89}{219}$.
28. $132\frac{81}{318}$.
29. $42\frac{177}{417}$.
30. $182\frac{425}{516}$.
31. $157\frac{138}{615}$.
32. $49\frac{375}{714}$.
33. $25\frac{678}{818}$.
34. $40\frac{58}{912}$.
35. $30\frac{88}{1009}$.
36. $38\frac{232}{2008}$.
37. $5\frac{595}{3007}$.
38. $12\frac{3166}{4006}$.
39. $19\frac{3427}{5005}$.
40. $14\frac{2777}{6004}$.
41. $2\frac{3578}{7003}$.
42. $7\frac{759}{8002}$.
43. $3\frac{8442}{9001}$.
44. $17\frac{528}{1234}$.
45. $3\frac{1}{13235}$.
46. $4\frac{2}{23174}$.
47. 6.
48. 3.

Page 170.

1. 31 barrels.
2. 6 yards.
3. 380 inches.

Page 171.

4. $1\frac{1}{2}$ yards.
5. 25 cents; $1.
6. 16 cents.
7. 98 cents.
8. $3\frac{1}{2}$ pounds.
9. 39 pints.
10. 145 sheep.
11. $2.
12. 93 cents.
13. 6 weeks.
14. 35 gallons.
15. 10 lb. 5 oz.
16. $2.

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17. 41 pounds.
18. $3.60.
19. 150 days; 3 days.
20. $2.05.
21. $39.
22. $225.
23. $2.25.
24. $8.22.
25. $40.

Page 175.

1. 603,275.
2. 678,456.
3. 759,795.
4. 641,426.
5. $2714.42.

6. $8502.43.
7. $7269.80.
8. $9885.02.
9. 300,424.
10. 913,092.
11. $220,119.
12. $1912.09.
13. $359,809.
14. 414,867.
15. $161,715.
16. 173,929.
17. $2952.51.
18. 399,952.
19. $1624.43.

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20. 868,980.
21. 895,048.
22. 954,048.
23. 996,450.
24. 592,320.
25. 864,128.
26. 970,485.
27. 940,215.
28. 967,890.
29. 954,087.
30. 906,205.
31. 968,464.
32. 886,730.
33. 864,565.
34. 941,408.
35. 948,708.
36. 972,930.
37. 761,472.
38. 955,320.
39. 969,855.
40. 976,372.
41. 926,328.
42. 925,245.
43. 856,674.
44. 977,724.
45. 963,976.
46. 887,112.
47. 629,405.
48. 890,765.
49. 933,725.
50. 2123.
51. 1203.
52. 1303.
53. 1203.
54. 1031.
55. 2402.
56. 3002.
57. 3030.
58. 10,444.
59. 1060.
60. 1011.
61. 1012.
62. 1013.
63. 1011.
64. 1101.
65. 1102.
66. 220.
67. 303.
68. 150.
69. 606.
70. 222.
71. $300\frac{2438}{2572}$.
72. $306\frac{2018}{3006}$.
73. 219.
74. $101\frac{4688}{5041}$.
75. $154\frac{5285}{6056}$.
76. $112\frac{5376}{7102}$.
77. $112\frac{1554}{8103}$.
78. $861\frac{8}{22}$.
79. $833\frac{8}{24}$.
80. $903\frac{8}{26}$.
81. $982\frac{18}{28}$.
82. $1313\frac{10}{32}$.
83. $2196\frac{22}{35}$.
84. $2218\frac{34}{38}$.
85. $2279\frac{10}{44}$.
86. $3134\frac{44}{49}$.
87. $3142\frac{9}{55}$.
88. $3009\frac{8}{63}$.
89. $3034\frac{56}{66}$.
90. $3050\frac{67}{69}$.
91. $3071\frac{4}{75}$.
92. $2016\frac{16}{124}$.
93. $1234\frac{147}{256}$.
94. $1132\frac{288}{361}$.
95. $1355\frac{52}{423}$.
96. 504.
97. 306.
98. 203.
99. 105.
100. $109\frac{41147}{90235}$.

Page 177.

101. $59\frac{1}{4}$.
102. $19\frac{1}{2}$.
103. 9639.
104. 12,141.
105. 96.
106. 96.
107. $12\frac{1}{4}$.
108. $60\frac{3}{4}$.
109. $61\frac{1}{4}$.
110. 300.
111. 300.
112. 2.
113. 8.
114. 78.
115. 162.
116. 231.
117. 36.
118. 648.
119. $46\frac{1}{6}$.
120. $70\frac{2}{3}$.
121. 12,126.
122. 187,440.
123. 68.
124. 975.
125. 34.
126. 125.
127. 5.
128. 138.
129. 150.
130. 78.

Page 178.

1. 760 yards.
2. 240 hf. pt.
3. $3.45.
4. $248.
5. $210.
6. $4\frac{3}{4}$ pounds.
7. $41\frac{1}{3}$ bushels.
8. $4.98.

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9. $1\frac{1}{2}$ minutes.
10. 1440 matches.

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1. $5\frac{5}{8}$.
2. $12\frac{1}{2}$.
3. $15\frac{3}{8}$.
4. 20.
5. $27\frac{1}{3}$.
6. $42\frac{5}{6}$.
7. $38\frac{1}{6}$.
8. $56\frac{1}{4}$.
9. $134\frac{1}{3}$.
10. $134\frac{7}{8}$.
11. $88\frac{1}{2}$.
12. 50.
13. $44\frac{1}{2}$.
14. $26\frac{2}{3}$.
15. $36\frac{3}{4}$.
16. $9\frac{5}{8}$.
17. $46\frac{7}{8}$.
18. $147\frac{1}{3}$.
19. $37\frac{1}{4}$.
20. $73\frac{1}{6}$.

21. $7\frac{1}{4}$.
22. $4\frac{1}{4}$.
23. $86\frac{3}{8}$.
24. $29\frac{1}{8}$.
25. $34\frac{1}{8}$.
26. $59\frac{1}{8}$.
27. $61\frac{1}{6}$.
28. $19\frac{1}{2}$.
29. $17\frac{1}{3}$.
30. $7\frac{1}{6}$.
31. $8\frac{1}{6}$.
32. $18\frac{1}{6}$.
33. $49\frac{1}{2}$.
34. $25\frac{1}{2}$.
35. $74\frac{1}{8}$.
36. $23\frac{1}{2}$.

Page 180.

1. 46,512.
2. 144,536.
3. 253,840.
4. 132,435.
5. 306,130.
6. 354,488.
7. 87,084.
8. 199,014.
9. 784,770.
10. 934,164.
11. 784,770.
12. 934,164.
13. 30,504.
14. 54,756.
15. 37,260.
16. 138,624.
17. 616,302.
18. 104,148.
19. 805,460.
20. 93,912.
21. 151,782.
22. 548,730.
23. 846,300.
24. 969,600.
25. 617,120.
26. 434,420.
27. 47,196.
28. 47,272.
29. 47,082.
30. 137,598.
31. 59,660.
32. 59,508.
33. 137,427.
34. 78,150.
35. 209,664.
36. 844,662.
37. 979,016.
38. 998,016.
39. $17,329\frac{1}{3}$.
40. $58,867\frac{1}{3}$.
41. 760,249.
42. 369,123.
43. $17,367\frac{5}{7}$.
44. 30,250.
45. 850,950.
46. $95,482\frac{4}{5}$.
47. 938,475.
48. 935,712.
49. $781,137\frac{1}{2}$.
50. 954,320.

Page 183.

1. 2,706,230.50.
3. 188,889.
5. 12,635,930.
6. 321.
7. $375.
8. $133.
9. 128,489.
10. $6.

Page 184.

1. $20\frac{7}{10}$.
2. $25\frac{9}{10}$.
3. $39\frac{1}{10}$.
4. $70\frac{3}{10}$.
5. 81.
6. $9\frac{3}{10}$.
7. $21\frac{1}{10}$.
8. $7\frac{1}{10}$.
9. $92\frac{3}{10}$.
10. $57\frac{3}{5}$.

Page 185.

11. $\frac{9}{20}$.
12. $2\frac{9}{20}$.
13. $4\frac{9}{20}$.
14. $4\frac{19}{20}$.
15. $18\frac{19}{20}$.
16. $33\frac{13}{20}$.
17. $53\frac{17}{20}$.
18. 76.
19. 105.
20. $48\frac{1}{20}$.
21. $9\frac{1}{20}$.
22. $61\frac{3}{20}$.
23. $63\frac{7}{20}$.
24. $7\frac{11}{20}$.
25. $15\frac{1}{20}$.
26. $32\frac{7}{10}$.
27. $29\frac{13}{20}$.
28. $29\frac{19}{20}$.
29. 82.
30. $82\frac{7}{8}$.
31. $21\frac{1}{2}$.
32. $31\frac{2}{3}$.
33. $41\frac{3}{4}$.
34. $51\frac{4}{5}$.
35. $61\frac{5}{6}$.
36. $61\frac{7}{8}$.
37. $71\frac{8}{9}$.
38. $84\frac{1}{6}$.
39. $79\frac{1}{12}$.
40. $65\frac{1}{20}$.
41. $59\frac{1}{3}$.
42. $70\frac{1}{6}$.
43. $38\frac{3}{10}$.
44. $7\frac{1}{4}$.
45. $9\frac{3}{8}$.
46. $8\frac{2}{15}$.
47. $78\frac{1}{3}$.
48. $55\frac{1}{12}$.
49. $47\frac{3}{20}$.
50. $39\frac{3}{10}$.

Page 187.

1. 3210.
2. 4321.
3. 765.
4. $3450\frac{4}{184}$.
5. 5403.
6. $4506\frac{4}{186}$.
7. 6063.
8. 7006.
9. 6003.
10. 6005.
11. 7001.
12. 5203.
13. $6715\frac{60}{185}$.
14. 5701.
15. $1020\frac{22}{153}$.
16. 2034.
17. 3240.
18. 4003.
19. $5041\frac{1}{133}$.
20. $4774\frac{52}{194}$.
21. $1789\frac{3}{215}$.
22. $1509\frac{30}{235}$.
23. $1155\frac{187}{256}$.
24. $2631\frac{167}{277}$.
25. $2347\frac{165}{293}$.
26. $2981\frac{314}{318}$.
27. $1435\frac{16}{347}$.
28. $499\frac{340}{375}$.
29. $1545\frac{353}{356}$.
30. $720\frac{60}{424}$.

31. $2117\frac{19}{452}$.
32. $1707\frac{246}{433}$.
33. $1607\frac{251}{331}$.
34. $1615\frac{200}{562}$.
35. $1191\frac{546}{594}$.
36. $1053\frac{447}{644}$.
37. $1008\frac{222}{689}$.
38. $879\frac{506}{724}$.
39. $990\frac{713}{847}$.
40. $600\frac{40}{981}$.
41. $608\frac{196}{1032}$.
42. $461\frac{106}{2165}$.
43. $307\frac{429}{3254}$.
44. $185\frac{1574}{4316}$.
45. $153\frac{4759}{5409}$.
46. $25\frac{2248}{6621}$.
47. $70\frac{2298}{7611}$.
48. $30\frac{4575}{8794}$.
49. $32\frac{3622}{9801}$.
50. $283\frac{1445}{1453}$.
51. $230\frac{608}{2043}$.
52. $251\frac{2512}{3087}$.

Page 189.

1. 240 bushels.
2. 58 cents.
3. 5 cows.
4. 90 cents.
5. $5500.
6. $226\frac{1}{4}$ acres.
7. $5.88.
8. $90.
9. 402.
10. 4 cows.
11. 16 days.
12. 3 yards.
13. 80 quarts.

Page 193.

1. $18\frac{7}{12}$.
2. $24\frac{7}{12}$.
3. $45\frac{11}{12}$.
4. $66\frac{1}{12}$.
5. $79\frac{5}{12}$.
6. $75\frac{1}{12}$.
7. $17\frac{5}{12}$.
8. $16\frac{5}{12}$.
9. $49\frac{5}{12}$.
10. $38\frac{1}{12}$.
11. $18\frac{1}{12}$.
12. $42\frac{5}{12}$.
13. $65\frac{7}{12}$.
14. $29\frac{11}{12}$.
15. $75\frac{11}{12}$.
16. $116\frac{5}{8}$.
17. $50\frac{7}{12}$.
18. $65\frac{1}{3}$.
19. $92\frac{3}{4}$.
20. $97\frac{1}{12}$.
21. $28\frac{7}{8}$.
22. 59.
23. $12\frac{7}{9}$.
24. $99\frac{1}{12}$.
25. $96\frac{3}{4}$.
26. $25\frac{1}{3}$.
27. $71\frac{1}{3}$.
28. $56\frac{1}{6}$.
29. $25\frac{1}{12}$.
30. $3\frac{5}{12}$.
31. $14\frac{1}{6}$.
32. $59\frac{1}{6}$.
33. $38\frac{3}{4}$.
34. $31\frac{3}{8}$.
35. $18\frac{1}{2}$.
36. $31\frac{1}{2}$.
37. $30\frac{2}{3}$.
38. $22\frac{3}{4}$.
39. $41\frac{5}{6}$.
40. $37\frac{7}{8}$.
41. $13\frac{8}{9}$.
42. $3\frac{1}{3}$.
43. $27\frac{1}{4}$.
44. $31\frac{1}{8}$.
45. $13\frac{1}{6}$.
46. $32\frac{2}{9}$.
47. $13\frac{1}{9}$.
48. $14\frac{1}{3}$.
49. $41\frac{1}{8}$.
50. $20\frac{1}{12}$.

Page 194.

1. 117 ounces.
2. 4 lb. 5 oz.
3. 20 gal. 2 qt.
4. 59 quarts.
5. 23 qt. 1 pt.
6. 57 pints.
7. 75 pecks.
8. 143 quarts.
9. 12 pk. 1 qt.
10. 21 bu. 3 pk.
11. 1568 quarts.
12. 180 inches.
13. 44 feet.
14. 159 inches.
15. 9 ft. 11 in.
16. 23 yd. 1 ft.
17. 44 pounds.
18. 88 gallons.
19. 65 quarts.
20. 151 bushels.
21. 43 pecks.
22. 85 ft. 3 in.
23. 60 bu. 3 pk.
24. 13 ft. 1 in.
25. 67 gal. 2 qt.

Page 196.

1. 80.
2. $1200 ; $240.
3. $1.

Page 197.

4. $4.
5. 29 tons.
6. 195 days.
7. 13 cents.
8. 416 yards.
9. $405.
10. 537 pounds.
11. 35 plants.
12. 341 passengers.
13. $3000.
14. Lost $20.
15. 1799.
16. 8 years.

Page 198.

17. $3.
18. $420.
19. $225.
20. 46 boys.
21. $2.
22. $216.
23. 10 cents.
24. 9 months.
25. 200 eggs.

---

1. 835,539.
2. 759,645.
3. 888,732.
4. 869,649.
5. 805,050.
6. 746,108.
7. 902,000.
8. 963,214.
9. 855,922.
10. 957,032.
11. 704,175.
12. 593,164.
13. 986,592.
14. 962,304.
15. 943,114.

16. 831,875.
17. 833,316.
18. 505,134.

Page 199.

19. 190$\frac{2}{3}$.
20. 500.
21. 420.
22. 1845.
23. 987.
24. 1071.
25. 1612.
26. 1645.
27. 2583.
28. 3885.
29. 4100.
30. 780,096.
31. 991,782.
32. 943,260.
33. 984,328.
34. 892,320.
35. 952,408.
36. 933,450.
37. 875,706.
38. 952,714.
39. 970,169.
40. 954,530.
41. 3519.
42. 3616.
43. 6132.
44. 4557.
45. 9568.
46. 10,791.
47. 17,572.
48. 39,333.
49. 76,775.
50. 97,460.
51. 69,000.
52. 2218$\frac{25}{29}$.
53. 786$\frac{16}{39}$.
54. 1618$\frac{11}{49}$.
55. 1046$\frac{38}{59}$.
56. 1033$\frac{18}{59}$.
57. 841$\frac{23}{79}$.
58. 609$\frac{22}{89}$.
59. 215$\frac{56}{99}$.
60. 223$\frac{51}{192}$.
61. 260$\frac{287}{293}$.
62. 89$\frac{375}{394}$.
63. 83$\frac{113}{495}$.
64. 40$\frac{250}{596}$.
65. 78$\frac{48}{697}$.
66. 81$\frac{33}{798}$.
67. 32$\frac{468}{891}$.
68. 99$\frac{784}{999}$.
69. 43$\frac{826}{987}$.
70. 260$\frac{48}{1907}$.
71. 233$\frac{511}{1971}$.
72. 281$\frac{2528}{2908}$.
73. 179$\frac{2807}{3907}$.
74. 166$\frac{576}{4906}$.
75. 51$\frac{298}{5905}$.
76. 107$\frac{1455}{6904}$.
77. 20$\frac{3082}{7903}$.
78. 50$\frac{5889}{8902}$.
79. 90$\frac{6847}{9901}$.
80. 579$\frac{13}{28}$.
81. 2332$\frac{34}{35}$.
82. 767$\frac{10}{43}$.
83. 628$\frac{33}{58}$.
84. 1398$\frac{56}{68}$.
85. 1021$\frac{75}{78}$.
86. 1051$\frac{20}{83}$.
87. 974$\frac{67}{93}$.
88. 108$\frac{171}{187}$.
89. 278$\frac{20}{286}$.
90. 84$\frac{101}{386}$.
91. 184$\frac{14}{489}$.
92. 905$\frac{4}{585}$.
93. 554$\frac{221}{687}$.
94. 951$\frac{651}{786}$.
95. 285$\frac{504}{885}$.
96. 144$\frac{444}{985}$.
97. 821$\frac{871}{974}$.
98. 91$\frac{237}{1863}$.
99. 241$\frac{2110}{2864}$.
100. 237$\frac{2292}{3863}$.
101. 63$\frac{894}{4867}$.
102. 181$\frac{4462}{5432}$.

Page 200.

2. \$289.75.
3. 86,940.
4. 84,099.
5. 971$\frac{61}{83}$.
6. \$927.

Page 201.

7. 200 days.
8. 24 feet; \$1.44.

Page 202.

1. 1$\frac{8}{15}$.
2. 4$\frac{8}{15}$.
3. 7$\frac{8}{15}$.
4. 16$\frac{11}{15}$.
5. 37$\frac{13}{15}$.
6. 13$\frac{2}{15}$.
7. 11$\frac{7}{15}$.
8. 31$\frac{4}{15}$.
9. 8$\frac{1}{15}$.
10. 71$\frac{2}{15}$.
11. 18$\frac{11}{15}$.
12. 26$\frac{13}{15}$.
13. 53$\frac{1}{3}$.
14. 99$\frac{1}{3}$.
15. 91$\frac{1}{3}$.
16. 47$\frac{5}{12}$.
17. 77$\frac{1}{15}$.
18. 86$\frac{13}{20}$.
19. 51$\frac{1}{3}$.
20. 63$\frac{1}{3}$.
21. 88$\frac{7}{15}$.
22. 93$\frac{5}{8}$.
23. 94$\frac{3}{4}$.
24. 93$\frac{7}{8}$.
25. 81$\frac{11}{12}$.
26. 8$\frac{1}{4}$.
27. 76$\frac{11}{12}$.
28. 88$\frac{5}{8}$.
29. 95$\frac{9}{20}$.
30. 93$\frac{1}{12}$.
31. 71$\frac{1}{3}$.
32. 3$\frac{2}{3}$.
33. 24$\frac{2}{3}$.
34. 18$\frac{2}{3}$.
35. 21$\frac{2}{3}$.
36. 43$\frac{1}{12}$.
37. 44$\frac{2}{15}$.
38. 19$\frac{1}{3}$.
39. 14$\frac{7}{12}$.
40. 35$\frac{1}{12}$.

Page 205.

2. 193,535.
3. 5191 men.
4. 14,112 lb.

Page 206.

5. 88 horses.
6. \$9.08.

---

2. 74 loads.
3. \$18.
4. 45 cu. yd.
5. \$9.62$\frac{1}{2}$.

---

2. 172,673.
3. 12,238 men.

Page 207.

4. 20,445 lb.
5. 76 horses.
6. \$23.

2. 11,315.
3. $31.50.
4. 2450 bricks.
5. 43 yards.

Page 208.

2. 134,793.
3. $1138.
4. $147.50.
5. $7546.50.
6. $5.25.

---

2. 65,584.
3. 1242 miles.
4. $823.

Page 209.

5. $175.
6. $3.33.

---

2. 132,224.
3. $1225.
4. $11.27.
5. $18.90.
6. $3.75.

---

1. 260,512.
2. 110,124.
3. 387,024.
4. 237,394.
5. 505,580.
6. 594,672.
7. 531,696.
8. 178,654.
9. 194,508.
10. 177,045.
11. 329,141.
12. 537,966.

Page 210.

13. 503,036.
14. 354,585.
15. 348,087.
16. 781,529.
17. 75,854.
18. $63,616\frac{2}{3}$.
19. $56,818\frac{1}{4}$.
20. $80,647\frac{1}{5}$.
21. $77,371\frac{5}{6}$.
22. $114,608\frac{2}{7}$.
23. 220,676.
24. $313,985\frac{1}{4}$.
25. $434,661\frac{3}{5}$.
26. 447,673.
27. 488,748.
28. $551,536\frac{7}{8}$.
29. $486,029\frac{2}{9}$.
30. $881,001\frac{1}{3}$.
31. 101,376.
32. 162,582.
33. 145,116.
34. 130,245.
35. 84,632.
36. 577,771.
37. 194,142.
38. 806,922.
39. 834,725.
40. 696,822.
41. $92,501\frac{1}{2}$.
42. 205,979.
43. 301,058.
44. $293,336\frac{3}{5}$.
45. 397,087.
46. $564,389\frac{6}{7}$.
47. $378,670\frac{1}{2}$.
48. $489,303\frac{1}{3}$.
49. $571,693\frac{1}{5}$.
50. $352,315\frac{1}{2}$.
51. $3646\frac{19}{27}$.
52. $2376\frac{18}{37}$.
53. $1002\frac{35}{47}$.
54. $1578\frac{44}{57}$.
55. $326\frac{40}{67}$.
56. $711\frac{7}{77}$.
57. $361\frac{10}{87}$.
58. $441\frac{7}{97}$.
59. $3297\frac{21}{26}$.
60. $977\frac{14}{36}$.
61. $2159\frac{14}{46}$.
62. $311\frac{25}{56}$.
63. $1157\frac{8}{66}$.
64. $1073\frac{4}{76}$.
65. $1048\frac{11}{86}$.
66. $314\frac{46}{96}$.
67. $63\frac{164}{273}$.
68. $172\frac{87}{872}$.

Page 211.

69. $59\frac{156}{475}$.
70. $18\frac{137}{574}$.
71. $81\frac{393}{677}$.
72. $123\frac{669}{776}$.
73. $21\frac{715}{879}$.
74. $33\frac{156}{378}$.
75. $3223\frac{155}{269}$.
76. $370\frac{44}{368}$.
77. $410\frac{182}{467}$.
78. $908\frac{342}{566}$.
79. $930\frac{192}{665}$.
80. $460\frac{780}{784}$.
81. $417\frac{255}{863}$.
82. $263\frac{409}{962}$.
83. $255\frac{1322}{2738}$.
84. $197\frac{1899}{3729}$.
85. $194\frac{2518}{4756}$.
86. $116\frac{2448}{5747}$.
87. $54\frac{2529}{6774}$.
88. $68\frac{1472}{7765}$.
89. $29\frac{3897}{8792}$.
90. $76\frac{4629}{9788}$.
91. $135\frac{1150}{2692}$.
92. $228\frac{1186}{3683}$.
93. $29\frac{2425}{4674}$.
94. $119\frac{4822}{5665}$.
95. $47\frac{5025}{6656}$.
96. $97\frac{3241}{7627}$.
97. $50\frac{8055}{8635}$.
98. $63\frac{428}{9629}$.
99. $7\frac{6248}{27387}$.
100. $5\frac{13510}{37298}$.

ANSWERS. — Part II.

Page 216.

16. $44\frac{5}{24}$.
17. $137\frac{3}{8}$.
18. $131\frac{5}{12}$.
19. $72\frac{3}{4}$.
20. $150\frac{1}{8}$.
21. $259\frac{3}{16}$.
22. $201\frac{1}{25}$.
23. $363\frac{1}{4}$.
24. $261\frac{7}{24}$.
25. $121\frac{5}{8}$.

Page 218.

1. $97\frac{9}{20}$.
2. $137\frac{1}{4}$.
3. $176\frac{3}{8}$.
4. $40\frac{1}{7}$.
5. $35\frac{7}{30}$.
6. $51\frac{11}{16}$.
7. $61\frac{5}{9}$.
8. $205\frac{3}{4}$.
9. 85.
10. $234\frac{1}{4}$.
11. $144\frac{1}{2}$.
12. $151\frac{1}{20}$.
13. $223\frac{1}{3}$.
14. $563\frac{7}{24}$.
15. $1096\frac{4}{5}$.
16. $749\frac{1}{5}$.
17. $1256\frac{13}{20}$.
18. $99\frac{4}{5}$.
19. $332\frac{7}{20}$.
20. 193.

Page 220.

11. $27\frac{7}{20}$.
12. $54\frac{3}{7}$.
13. $10\frac{1}{24}$.
14. $30\frac{1}{12}$.
15. $97\frac{1}{20}$.
16. $36\frac{1}{18}$.
17. $1\frac{3}{20}$.
18. $6\frac{5}{24}$.
19. $78\frac{7}{18}$.
20. $8\frac{11}{20}$.
21. $382\frac{1}{2}$.
22. $291\frac{3}{4}$.
23. $109\frac{2}{3}$.
24. $115\frac{3}{5}$.
25. $1\frac{1}{6}$.
26. $24\frac{4}{7}$.
27. $599\frac{3}{8}$.
28. $860\frac{7}{9}$.
29. $85\frac{7}{10}$.
30. $4\frac{10}{11}$.

Page 221.

31. $2\frac{3}{4}$.
32. $17\frac{3}{4}$.
33. $6\frac{2}{3}$.
34. $7\frac{5}{6}$.
35. $11\frac{1}{2}$.
36. $8\frac{7}{9}$.
37. $25\frac{7}{8}$.
38. $17\frac{9}{10}$.
39. $25\frac{7}{10}$.
40. $10\frac{7}{8}$.
41. $23\frac{9}{10}$.
42. $19\frac{4}{5}$.
43. $69\frac{1}{16}$.
44. $3\frac{11}{21}$.
45. $8\frac{7}{24}$.
46. $7\frac{1}{10}$.
47. $5\frac{17}{18}$.
48. $106\frac{5}{16}$.
49. $12\frac{21}{22}$.
50. $7\frac{11}{12}$.
51. $2\frac{13}{24}$.
52. $45\frac{8}{9}$.
53. $25\frac{1}{10}$.
54. $38\frac{5}{18}$.
55. $21\frac{9}{10}$.
56. $3\frac{23}{24}$.
57. $18\frac{17}{18}$.
58. $1\frac{23}{24}$.
59. $7\frac{9}{10}$.
60. $15\frac{11}{12}$.

---

1. $7\frac{1}{12}$ yards.
2. $57\frac{3}{4}$ gallons.
3. 26 yards.
4. 31 cents.
5. $ 2.60.

Page 222.

6. $ 1.40.
7. $ 2.00.
8. $13\frac{7}{8}$ pounds.
9. $ 9.64.
10. $ 20.
11. 360.
12. $ 36.
13. $ 3.40.
14. $ 48.
15. $ 9.30.

Page 223.

16. 16 marbles.
17. 20 stamps.
18. $64\frac{1}{2}$ yards.
19. 35 cents.
20. 2 days.

---

1. 258.
2. 2449.
3. 228.
4. 568.
5. 124.
6. 9920.
7. 339.
8. $199\frac{1}{2}$.
9. $293\frac{1}{4}$.
10. $1231\frac{1}{3}$.
11. $31\frac{1}{4}$.
12. $14\frac{1}{5}$.
13. $14\frac{1}{5}$.

14. $14\frac{1}{3}$.
15. $16\frac{1}{6}$.
16. $21\frac{2}{13}$.
17. $50\frac{1}{10}$.
18. $45\frac{1}{9}$.
19. $75\frac{2}{25}$.
20. $144\frac{1}{19}$.
21. $246\frac{1}{11}$.
22. $302\frac{1}{15}$.

Page 225.

1. 225,506,736.
2. 804,580,398.
3. 561,276,891.

Page 226.

1. $609,340.37.
2. $680,494.41.
3. $840,200.33.

Page 227.

4. 350,879,581.
5. 627,020,401.
6. 589,140,749.
7. 668,386,689.
8. 777,993,982.
9. 713,200,695.
10. 578,616,033.

Page 228.

11. 65,461,219.
12. 615,808,906.
13. 99,090,910.
14. 200,290,240.
15. 249,054.
16. 26,081.
17. 102,900,999.
18. 31,660,868.
19. 82,816,981.
20. 6,543,211.
21. 3,264,973.
22. 53,386,521.

---

1. $ 2,706,230.50.
3. 120,263,455 miles.

Page 229.

4. $608.
5. $18.

Page 230.

3. 12,642,968.
4. 8625.
5. 980,304.
6. $439.11.
7. $314.87.
8. 7225.
10. $55,350.
11. 1,207,053.
13. 1376 yards.
14. 998,392.

Page 231.

25. $12\frac{3}{5}$.
26. $19\frac{1}{4}$.
27. $42\frac{1}{6}$.
28. $79\frac{1}{3}$.
29. $80\frac{3}{8}$.
30. $128\frac{4}{7}$.
31. $101\frac{1}{6}$.
32. $169\frac{1}{10}$.
33. $215\frac{4}{11}$.
34. $177\frac{1}{12}$.
35. $111\frac{5}{6}$.
36. $96\frac{1}{4}$.
37. $163\frac{1}{6}$.
38. $266\frac{1}{3}$.
39. $245\frac{3}{5}$.
40. $348\frac{4}{7}$.
41. $244\frac{1}{3}$.
42. $378\frac{1}{10}$.
43. $468\frac{4}{11}$.
44. 257.
45. $309\frac{1}{12}$.
46. $155,090\frac{1}{4}$.
47. $6,108,538\frac{4}{5}$.
48. $3,761,048\frac{4}{7}$.
49. $25,011\frac{2}{3}$.
50. $28,508\frac{1}{3}$.
51. $69,763\frac{3}{4}$.
52. $598,686\frac{1}{6}$.
53. $3,600,925\frac{6}{7}$.
54. $21,436,213\frac{1}{6}$.

Page 232.

1. $18,016.14.
2. 1,058,213.
3. $3405.78.
4. 8502.
5. 21,263,502.
6. $747\frac{2}{5}$.
7. $22,432\frac{122}{647}$.

Page 233.

3. 19,656.
4. 381.
5. $62.50.

---

1. 674,022,122 pieces.
2. $ 2,466,338.49.
3. 38,788.
4. $9332.86.
5. 75 pounds.
6. 1000.

Page 234.

8. $672.
   $1.20.

Page 235.

1. 6.
2. 12.
3. 12.
4. 24.
5. 4.
11. 5.
12. 15.
13. 13.
14. 31.
15. 5.
16. 11.
17. 17.
18. 25.
19. 5.
20. 8.
31. 16.
32. 8.
33. $20\frac{1}{4}$.
34. $30\frac{1}{4}$.
35. 10.

Page 236.

41. 24.
42. 40.
43. 17.
44. 61.
45. 9.
46. 90.
47. 6.

---

1. $1283.35.
2. $845.95.
3. $2121.75.
4. $857.62.
5. $1247.80.
6. $769.89.
7. $530.25.
8. $853.58.
9. $1250.93.
10. $712.50.

Page 237.

1. $1115.02.
2. $505.44.
3. $1592.64.
4. $9263.05.
5. $1526.25.
6. $967.20.
7. $7133.80.
8. $1072.56.
9. $23.76.
10. $58.24.

Page 238.

11. $92.88.
12. $989.90.
13. $102.52.
14. $133.38.
15. $140.60.
16. $34.98.
17. $53.07.
18. $103.60.
19. $1591.62.
20. $4879.77.

———

1. $4,680,785\frac{3}{7}$.
2. 2,478,208.
3. $258,715,000.
4. 4919.

5. 80,191.
6. 74 acres.
7. $46,200\frac{3}{17}$.
8. 18 patterns.

Page 239.

9. $2582\frac{634}{637}$.
10. $89.60.
13. 5 quarts.
14. 71 quarts.
15. 4 pecks.
16. 9.
17. $2.10.
18. 49,275.

Page 246.

1. 40 days.
2. 1050 yards.
3. 25 sheep.
4. 149 pounds.
5. 300 bushels.
6. 1402 pounds.
7. 36 cents.
8. $66\frac{2}{3}$ cents.

Page 247.

9. 522 rabbits.
10. $2837.50.
11. $11,159.
12. 24 horses.
13. 1000 bars.

Page 249.

5. $915\frac{685}{765}$.
6. 16 days.
7. 19,208 lb.
8. 57 inches.

———

1. $86,362\frac{880}{1819}$.
2. 1288 pieces.
3. $566.

4. $378.75.
5. 41 days.
6. $5578\frac{1}{8}$ bu.
7. $148\frac{1}{16}$ gal.

Page 250.

8. 1,268,459,565 pounds.
9. 1,125,025,619 yards.
10. 30 hours.
12. 2,111,310,206.
13. 10,209,990 pieces.
14. 3684 quarts.
15. $1645.56.

Page 251.

1. 180 hours.
2. 180 hours.
3. 7200 seconds.
4. $12.75.
5. $2.48.
6. 40 pints.
7. 160 hf. pt.
8. 20 packages.
9. 25 cents.

Page 252.

10. 6 gallons.
11. 18,000 sec.
12. 10,080 min.
13. 800 minutes.
14. 512 quarts.
15. 757 ounces.
16. 14 lb. 13 oz.
17. 24 hr. 54 min.
18. 1008 hours.
19. 744 hours.
20. 98 inches.
21. 128,000 oz.
22. 9280 ounces.
23. 750 pounds.
24. 15 cwt.
25. $1.
26. $\frac{3}{4}$ ton.
27. 4 da. 16 hr.
28. 50 yards.
29. $8.25.
30. 32 cents.
31. $\frac{3}{5}$ cwt.
32. 348 pints.

Page 253.

4. 39,312.
   24,568.
5. $152\frac{6}{16}$.
   $69\frac{585}{822}$.

Page 258.

1. $134,083.44.
2. $108,350.78.
3. $27,437.70.
4. $56,284.66.
5. $11,672.66.
6. $96,229.43.

Page 259.

7. 1,000,342.
8. 854,822.
9. 3,649,094.
10. 11,810,804.
11. 77,472,965.
12. 33,845,968.
13. 27,749,898.
14. $28,338,290\frac{7}{8}$.
15. 32,136,750.
16. $6,248,365\frac{1}{2}$.
17. 29,654,230.
18. $63,257,616\frac{3}{8}$.
19. 857,375.
20. 274,170.

21. 72,243.
22. 109,243,616.
23. 115,242$\frac{2}{3}$.
24. 16,610,750.

Page 260.

25. 93$\frac{58}{370}$.
26. 2175.
27. 1025$\frac{112}{624}$.
28. 296$\frac{4138}{72100}$.
29. 530$\frac{1404}{3510}$.
30. 6943$\frac{612}{3004}$.
31. 5565$\frac{3780}{12345}$.
32. 15,168$\frac{44588}{56789}$.
33. 2708.
34. 920$\frac{2840}{4900}$.
35. 870$\frac{28005}{39783}$.
36. 216$\frac{13158}{57095}$.

---

1. 39.
2. 11.
3. 12.
4. 88.
5. 181.
6. 77.
7. 690.
8. 17.
9. 11.
10. 123.

Page 261.

11. 13.
12. 20.

Page 262.

1. $3.
2. $3.
3. $550.
4. $1120.
5. $105.
6. 5 cents.
7. $1536.65.
8. 49 T. 820 lb.
9. 22.
10. $80.
11. $1.25.
12. 25 cents.
13. 60 yards.

Page 263.

14. $22.
15. 10 papers.

---

1. $12.04.
2. $19.63.
3. $107.52.
4. $8.29.

Page 264.

5. $122.75.

Page 266.

1. 34,876$\frac{4}{5}$ average applications. $245,468.65$\frac{1}{5}$ average surplus.
5. 4.45\frac{5}{8}$.
6. 50$\frac{950}{1981}$ cts.

Page 267.

5. 122.995.
6. 293.056.
7. 59.556.
8. 404.529.
9. 390.732.
10. 300.417.
11. 480.507.
12. 939.186.
13. 1180.106.
14. 104.231.
15. 23.495.
16. 18.168.
17. 2359.925.
18. 748.311.
19. 1062.556.
20. 799.511.

Page 269.

41. 1.08.
42. 400.4.
43. 780.8.
44. 780.8.
45. 2.68.
46. 1.536.
47. 1.536.
48. 55.272.
49. .485.
50. 4344.
51. 330.
52. 960.
53. 18.
54. 3801.
55. 59.13.
56. 725.56.
57. 376.68.
58. 334.508.
59. 62.7.
60. 1.8.

Page 271.

31. $241.25.
32. 1968.5 yd.
33. 35.4 pounds.
34. 1212.5 lb.
35. 2.64 tons.
36. $3364.02.
37. 19 pints.
38. .125 peck.
39. $58.50.
40. 1751.56\frac{1}{4}$.

Page 273.

1. 6.375 yards.
2. $95.386.
3. 2060$\frac{5}{37}$ acres.
4. $14,000.
5. 67 yards.

Page 274.

6. 60 gills.
7. 12 bushels.
8. 26$\frac{7}{16}$ gallons.
9. $544.
10. $1960.
13. $4.50 gain.
14. $1.76.
15. 704 pints.
16. $679.
17. $15.66 gain.
18. $2.16.
20. 8 marbles.
21. 90 cents.
22. 88 quarts.

Page 275.

23. 90 cents.
24. 558 pupils.
25. 110 feet.
26. $1806.
27. 20 cents.
28. $2.48.
29. 491 gills.

---

4. 21$\frac{19}{60}$.
5. 7$\frac{53}{56}$.
6. 95 cents.

Page 276.

7. $1.25.
8. 22$\frac{7}{8}$ yards.
9. $80.50.
10. 1050 hours.

11. $33\frac{3}{4}$ hours.
13. $2\frac{18}{25}$ miles.
14. $148.03.
15. $7\frac{1}{5}$ weeks.
16. 1.87\frac{1}{2}$.
17. $2.02.
18. 254 hf. pt.
19. $9717\frac{700}{902}$.

Page 277.

1. 182 sq. in.
2. 153 sq. in.
3. 126 sq. in.
4. 345 sq. in.
13. 180 sq. in.
14. 144 sq. in.
15. 192 sq. in.
16. 360 sq. in.
17. 450 sq. in.
18. 1419 sq. in.
19. 1180 sq. in.
20. 2205 sq. in.

Page 278.

1. 168 sq. ft.
2. 255 sq. ft.
3. 209 sq. ft.
4. 345 sq. ft.
5. 288 sq. ft.
6. 348 sq. ft.
7. 186 sq. ft.
8. 186 sq. ft.
9. 300 sq. ft.
10. 300 sq. ft.
11. 423 sq. ft.
12. 444 sq. ft.
13. 308 sq. ft.
14. 426 sq. ft.
15. 386 sq. ft.
16. 843 sq. ft.
17. $321\frac{3}{4}$ sq. ft.
18. 320 sq. ft.
19. 400 sq. ft.
20. 150 sq. ft.

Page 279.

1. 10,937.5 sq. ft.
2. $2\frac{3}{4}$ sq. yd.
3. $4\frac{1}{2}$ sq. yd.
4. 117 sq. m.
5. $15.
6. $48.75.
7. $2.40.
8. 672 sq. rods.
9. 30 sq. yd.
10. 4 sq. yd.

Page 280.

1. $22\frac{71}{72}$.
2. $12\frac{88}{165}$.
3. $18\frac{109}{120}$.
4. $48\frac{387}{504}$.
5. $47\frac{1}{10}$.
6. $90\frac{17}{30}$.
7. $3\frac{1}{20}$.
8. $48\frac{19}{24}$.
9. $103\frac{4}{5}$.
10. $99\frac{18}{25}$.

Page 281.

11. $8\frac{11}{12}$.
12. $6\frac{13}{24}$.
13. $38\frac{5}{8}$.
14. $17\frac{13}{24}$.
15. $19\frac{4}{5}$.
16. $16\frac{13}{60}$.
17. $24\frac{17}{60}$.
18. $22\frac{29}{30}$.
19. $6262\frac{67}{84}$.
20. $199\frac{47}{48}$.
21. 74.
22. $3\frac{5}{24}$.
23. $16\frac{19}{24}$.
24. 318.
25. $4\frac{1}{10}$.
26. $22\frac{2}{3}$.
27. $46\frac{2}{7}$.
28. 2437.
29. $507\frac{7}{8}$.
30. $\frac{2}{45}$.

Page 282.

1. 2, 43.
2. 3, 29.
3. 2, 2, 2, 11.
4. 2, 3, 3, 5.
5. 7, 13.
6. 2, 2, 23.
7. 3, 31.
8. 2, 47
9. 5, 19.
10. 2, 2, 2, 2, 2, 3.
11. 2, 2, 5, 5.
12. 2, 2, 2, 3, 5.
13. 2, 3, 5, 7.
14. 2, 2, 2, 2, 3, 5.
15. 2, 2, 2, 3, 3, 5.
16. 2, 2, 2, 2, 2, 2, 3, 3.
17. 2, 2, 2, 3, 5, 7.
18. 2, 2, 2, 2, 2, 2, 2, 3, 3.
19. 2, 2, 2, 2, 2, 2, 3, 3, 3.
20. 2, 2, 2, 2, 2, 3, 3, 7.

Page 283.

1. $\frac{6}{7}$.
2. $\frac{11}{12}$.
3. $\frac{2}{3}$.
4. $\frac{3}{4}$.
5. $\frac{11}{14}$.
6. $\frac{1}{4}$.
7. $\frac{27}{32}$.
8. $\frac{17}{23}$.
9. $\frac{4}{5}$.
10. $\frac{3}{4}$.
11. $\frac{24}{35}$.
12. $\frac{2}{3}$.

Page 284.

3. $\frac{4}{7}$.
4. $\frac{14}{33}$.
5. $\frac{1}{2}$.
6. $\frac{7}{15}$.
7. $\frac{4}{5}$.
8. $\frac{7}{8}$.
9. $\frac{6}{17}$.
10. $\frac{4}{9}$.
11. $\frac{7}{12}$.
12. $\frac{3}{4}$.
13. $\frac{7}{11}$.
14. $\frac{5}{7}$.
15. $\frac{9}{10}$.

Page 286.

1. 120.
2. 900.
3. 840.
4. 240.
5. 600.
6. 48.
7. 360.
8. 231.
9. 720.
10. 420.

1. $16\frac{58}{65}$.
2. $84\frac{11}{60}$.
3. $66\frac{33}{100}$.
4. $31\frac{5}{8}$.
5. $61\frac{7}{10}$.
6. $2\frac{47}{300}$.
7. $82\frac{19}{60}$.
8. $101\frac{627}{1000}$.
9. $41\frac{91}{120}$.
10. $23\frac{73}{125}$.

Page 287.

11. $8\frac{7}{48}$.
12. $9\frac{1}{24}$.
13. $85\frac{97}{144}$.
14. $74\frac{27}{80}$.
15. $79\frac{29}{30}$.
16. $223\frac{94}{105}$
17. $471\frac{68}{105}$.
18. $10\frac{267}{1000}$.
19. $43\frac{13}{96}$.
20. $9\frac{59}{1000}$.
21. $\frac{2}{75}$.
22. $\frac{1}{25}$.
23. $17\frac{7}{20}$.
24. $6\frac{31}{40}$.
25. $13\frac{1}{3}$.
26. $674\frac{1}{2}$.
27. $1\frac{1}{15}$.
28. 3.
29. $1036\frac{5}{7}$.
30. $8\frac{86}{105}$.

Page 289.

1. $19\frac{1}{8}$ cents.
2. 75 cents.
3. $416\frac{1}{4}$ miles.
4. 2.
5. $5.
6. $107.48.
7. $\frac{3}{4}$.
8. 532 bags.

Page 290.

9. 52.272 acres.
10. 26,100 sec.
11. $6.35.
12. $7.10.
13. $20\frac{5}{8}$ yards.
14. 50 cts. 15. $\frac{1}{8}$.
16. 68 marbles.
17. 80 cents.
18. $119.46.
19. $58\frac{1}{4}$ miles.
20. $6.90.

Page 293.

1. $581,812,541,-985.20.
2. 3,417,600 acres.

Page 294.

3. $2480.
6. 156 pounds.
7. $422.
8. 14,688 ounces.
10. 3 cents.

---

1. $4,532,088.68.

Page 295.

2. 2551 gal. 1 qt. 1 pt.
3. $25.74 lost.
6. $524,470,971.-05.
7. $$3.33\frac{1}{3}$.
8. $2600.
9. $1017.25.

Page 296.

1. 135.
2. $105\frac{44}{81}$.
3. $117\frac{29}{75}$.
4. $121\frac{31}{84}$.
5. $219\frac{78}{97}$.
6. $344\frac{76}{125}$.
7. $213\frac{299}{345}$.
8. $168\frac{429}{624}$.
9. $420\frac{598}{1008}$.
10. $342\frac{665}{2534}$.

Page 297.

3. 36 days.
4. 68 bags.
5. $2.99.
6. $78.
7. $2844.
8. $1.50.
9. $12.
10. $147.60.
11. $704.
12. 36 days.

Page 298.

13. $2.31.
14. 60 days.
15. $87\frac{1}{2}$ cents.

Page 299.

1. 64.
2. 96.
3. $\frac{12}{35}$.
4. $\frac{35}{48}$.
5. $\frac{1}{2}$.
6. 246.
7. 436.
8. $2221\frac{5}{7}$.
9. $3115\frac{7}{15}$.
10. 15.
11. 9.
12. $1\frac{1}{2}$.
13. 115.
14. $3\frac{1}{3}$.
15. $37\frac{2}{5}$.
16. 3.
17. 42.
18. $3\frac{23}{25}$.
19. $\frac{1}{4}$.
20. $\frac{1}{3}$.
21. $\frac{9}{28}$.
22. $\frac{8}{27}$.
23. $57\frac{3}{8}$.
24. $41\frac{7}{8}$.
25. $47\frac{15}{16}$.
26. $22\frac{1}{10}$.
27. $10\frac{1}{3}$.
28. $9\frac{1}{11}$.
29. $11\frac{3}{8}$.
30. $15\frac{5}{8}$.
31. $3\frac{1}{4}$.
32. $\frac{37}{48}$.
33. $\frac{13}{36}$.
34. $5\frac{1}{16}$.
35. $5\frac{17}{144}$.
36. $178\frac{7}{8}$.
37. $19\frac{11}{12}$.
38. $7\frac{89}{90}$.
39. $2\frac{5}{8}$.
40. $1\frac{3}{40}$.

Page 301.

1. 10.
2. $\frac{1}{6}$.
3. $23\frac{1}{3}$.
4. $3\frac{3}{4}$.
5. $\frac{3}{70}$.
6. $\frac{4}{15}$.
7. $\frac{7}{20}$.
8. $2\frac{6}{7}$.
9. $\frac{74}{99}$.
10. $\frac{49}{187}$.
11. $1\frac{5}{24}$.
12. $\frac{3}{4}$.

13. $1\frac{1}{3}$.
14. $2\frac{3}{4}$.
15. $\frac{3}{4}$.
16. $\frac{4}{11}$.
17. 6.
18. $\frac{1}{6}$.
19. $\frac{1}{17}$.
20. $\frac{11}{197}$.
21. $\frac{4}{9}$.
22. $\frac{8}{15}$.
23. $3\frac{11}{15}$.
24. $2\frac{1}{16}$.
25. $2\frac{188}{375}$.
26. $2\frac{98}{171}$.
27. $1\frac{41}{70}$.
28. $1\frac{97}{748}$.
29. $1\frac{7}{32}$.
30. $3\frac{1}{2}$.
31. $6\frac{1}{4}$.
32. $4\frac{1}{6}$.
33. $17\frac{1}{7}$.
34. $17\frac{1}{7}$.
35. $30\frac{10}{21}$.
36. $30\frac{10}{21}$.
37. $5\frac{3}{8}$.
38. $33\frac{9}{10}$.
39. $37\frac{1}{2}$.
40. $6\frac{2}{3}$.
41. $101\frac{13}{115}$.
42. $2\frac{359}{1008}$.

Page 305.
7. $1\frac{8}{9}$.
8. \$ $1\frac{23}{42}$.
9. $10\frac{1}{2}$.
10. 4.
11. $59\frac{2}{9}$.
12. \$ $1\frac{43}{60}$.
13. $1\frac{10}{39}$.
14. $1\frac{1}{24}$.
15. $4\frac{68}{175}$.

Page 307.
1. \$ $2.43\frac{1}{2}$.
2. 1 ft. 3 in.
3. $10\frac{1}{8}$ pounds.
4. $\frac{7}{72}$.
5. 60 cents.
6. 96 hours.
7. $1.14.
8. 30 days.
9. $ 76.25.

Page 308.
10. 32 days.
11. 6 yards.
12. 1,260,000 cu. ft.
13. $ 7.50.
14. 90 cents.
15. 964 pounds.
16. 60 cents.
17. $31\frac{1}{4}$ cents.
18. $\frac{16}{45}$.
19. $161.
20. Twice as old.
21. $37\frac{2}{5}$.
22. $1430.
23. 2 days.
24. $\frac{35}{102}$.
25. $ 160.

Page 311.
1. $ 161.85.
2. $ 27.36.
3. $ 35.96.
4. $ 54.95.

Page 312.
5. $ 78.27.

---

3. $ 882,258.55.
4. $ 4,211,587.67.

Page 313.
4. 53 hours 20 minutes.
5. $ 4044.63.
$ 2.36.
$ 2.26.
$ 2.15.
6. $ 5,400,000.

---

3. 930,435,246 pieces.
4. 10,209,990 pieces.
7. $ 320.
8. \$ $1.33\frac{82}{117}$.

Page 314.
1. 29 pupils.
2. $\frac{4}{5}$.
4. \$ $255.93\frac{3}{4}$.
5. 8.
6. $2\frac{11}{60}$.
7. $\frac{2}{5}$.
8. $\frac{7}{9}$.
9. $16\frac{8}{15}$.
10. $ 45.

Page 315.
Carriages, 2,698,526.
Equestrians, 132,137.
Pedestrians, 13,730,597.
Total, 16,567,956.

Page 318.
1. 80.
2. 80.
3. 28.8.
4. 27.
5. 72.96.
6. 10.8.
7. 81.666.
8. 47.25.
9. 24.
10. 124.962.

Page 319.
11. 112.
12. 335.
13. 445.9.
14. 120.
15. 26.748.
16. 372.6.
17. 473.184.
18. 222.
19. 10.92.
20. 15,701.57.
21. 32.
22. 3.2.
23. 700.
24. 40.
25. 40.
26. 2.
27. 3.2.
28. 30.6.
29. 200.
30. 144.
31. 32,000.
32. 121.
33. 13,500.
34. 12.5.
35. 42.1.
36. 15,100.
37. 1510.
38. 151.

Page 320.
39. 21.

40. 21.
41. 21.
42. 21.
43. 240.
44. 300.
45. 3.5.
46. 12,360.
47. 122.5.
48. .016.
49. 400.
50. .007.
51. 47.
52. .064.
53. 13.5.
54. 43647.89+.
55. 2.384+.
56. .264+.
57. 20.001+.
58. 4.405+.
59. 24.8.
60. 2.634+.

Page 321.

4. .812.
5. .105.
6. 1.06.
7. .143.
8. .025.
9. .044.
10. .05625.
11. .105.
12. .288.
13. 36.4.
14. .088.
15. .605.

Page 323.

1. $562.68.
2. 105.45 sq. rods.
3. 14 rods.
4. 420.168+ marks.
5. 103.771.
6. 2699.73.
7. 77.7.
8. .3.
9. 1864 lb.
10. 62.832 in.

---

3. 61.
4. 540.
6. $7\frac{17}{42}$.
7. $4\frac{13}{16}$ pounds.

Page 324.

8. $ $\frac{11}{12}$.
9. 12 days.
10. 14 yr. 1 mo.
11. 6 miles.
12. $ 1.92.
13. $87\frac{1}{2}$.
$3\frac{9}{10}$.
14. $ 5.86.

Page 325.

2. 5,026,101.
3. 1,359,908.
4. $9319\frac{412}{867}$.
5. $2\frac{5}{8}$ yards.
6. $ 12.40.
7. 1 mile.
8. $ 2.58.

Page 327.

1. 17 days.
2. $\frac{1}{4}$ week.
3. $ 216.
4. 5400 min.
5. 5 da. 15 hr.
6. 18 hours.
7. 9 hr. 36 min.
8. $ 28.
9. 82 cents.
10. 4 lb. 10 oz.
11. 6 bu. 1 pk. 5 qt.
12. 4 gallons. $1\frac{1}{2}$ pints.
13. $ 3.05.
14. 9 bu. 1 qt.

Page 328.

15. 63,360 in.
16. 924 feet.
17. 14,080 rails.
18. 3840 steps.
19. 55 minutes.
20. 15 hr. 17 min. 14 hr. 18 min.
21. 276 ounces.
22. 169,560 lb.
23. 151 quarts.
24. 360 pints.
25. 127 pints.
26. 111 pecks.
27. 391 quarts.
28. 1344 pints.
29. 47,520 yd.
30. 91 yards.
31. 10 da. 10 hr.
32. 12 T. 1124 lb.
33. 100 rods.

Page 329.

34. 109 gal. 2 qt.
35. 6 yd. 1 ft.
36. 5 mi. 50 rd.
37. 5 wk. 1 da.
38. 9 bu. 1 pk.
39. 19 lb. 11 oz.
40. 61 yr. 11 mo.
41. 25 ft. 2 in.
42. 105 min. 57 sec.
43. 50 years.
44. 35 wk. 2 da.
45. 13 miles.
46. 60 yards.
47. 50 gal. 2 qt.
48. 13 pk. 2 qt.
49. 74 bu. 1 pk.
50. 50 quarts.
51. 50 quarts.
52. 74 bu. 1 pk.
53. 48 pk. 3 qt.
54. 146 gallons.
55. 200 yards.
56. 63 mi. 175 rd.
57. 77 wk. 1 da.
58. 72 yr. 6 mo.
59. 81 min. 10 sec.
60. 113 feet.
61. 6 ft. 5 in.
62. 22 min. 43 sec.
63. 6 yr. 2 mo.
64. 31 wk. 5 da.
65. 6 mi. 177 rd.

Page 330.

66. 14 yd. 2 ft.
67. 145 gal. 3 qt.
68. 24 pk. 2 qt.
69. 84 bu. 3 pk.
70. 68 quarts.
71. 43 qt. 1 pt.

72. 10 min. 7 sec.
73. 17 yr. 5 mo.
74. 24 wk. 3 da.
75. 7 mi. 65 rd.
76. 25 yd. 1 ft.
77. 64 gal. 2 qt.
78. 24 bu. 3 pk.
79. 37 qt. 1 pt.
80. 9 bu. 1 pk. 4 qt.
81. 2.
82. 5.
83. 8.
84. 9.
85. 10.

---

1. 378 sq. yd.
2. 378 sq. yd.
3. 6 sq. yd.
4. 893 sq. yd.
5. 5963 sq. yd.
6. 396 sq. yd.
7. 288 sq. yd.
8. 36 sq. yd.
9. 240 sq. yd.
10. 36 sq. yd.

Page 331.

13. 420 sq. in.
14. 32 sq. yd.
15. 256 sq. yd.
16. 1500 sq. ft.
17. 270 sq. ft.

Page 332.

18. 96 sq. in.
19. 72 cents.
20. $40.

Page 334.

1. 10,061,280 minutes.
2. $6336.12.
3. 7609 cords.
4. $6934.89+.
5. $1833.72.
6. $70.20.
7. $608.
8. $101,790.
9. 3192; 45.
10. $1200.
11. 1.955 yards.
12. 7960.
13. 2.35\frac{3}{8}$.

Page 335.

14. $2.56.
15. 18 pounds.
16. 327 feet.
17. $17.88.
19. 62 days.
20. $2.81.
21. $2.
22. 48 geographies.
23. 96.

Page 336.

24. $340.30.
25. 21 bushels.
26. 11,608 bot.
27. 192 pounds.
28. $72.06.
29. 6.
30. 60.

Page 337.

1. .00125.
2. .025.
3. .08.
4. .78125.
5. .265625.
6. .1875.
7. .006.
8. .03125.
9. .00525.
10. .0035.
11. .24.
12. .056.
13. 2.875.
14. 2.9375.
15. .044.
16. .0016.
17. 5.859375.
18. .1015625.
19. .00390625.
20. .013671875.
21. .0009765625

Page 338.

22. $\frac{3}{400}$.
23. $\frac{9}{25}$.
24. $\frac{11}{400}$.
25. $\frac{11}{25}$.
26. $\frac{1}{32}$.
27. $\frac{243}{500}$.
28. $\frac{3}{8}$.
29. $\frac{3}{8}$.
30. $\frac{18}{125}$.
31. $\frac{3}{5000}$.
32. $\frac{27}{100}$.
33. $\frac{27}{1000}$.
34. $\frac{73}{20000}$.
35. $\frac{24}{25}$.
36. $\frac{9}{10000}$.
37. $\frac{102}{125}$.
38. $\frac{5}{32}$.
39. $\frac{3}{80}$.
40. $\frac{1}{160}$.
41. $\frac{12}{125}$.
42. 304.134.
43. 1167.3946.
44. 883.2429.
45. 2759.5755.
46. 2283.9171.
47. 314.7032.
48. 1013.0418.
49. 639.7105.
50. 49.21619.
51. 563.7625.

Page 339.

52. 188.26.
53. 288.3623.
54. 999.999.
55. 13.615.
56. 184.7569.
57. 15.0885.
58. 1999.96875.
59. 113.1991.
60. 17.84375.
61. 1503.5975.
62. 79.2.
63. .045264.
64. 1850.3125.
65. 4.566.
66. .13875.
67. .009438.
68. 6.3784.
69. 16.93542.
70. .45953125.
71. 45.78644.
72. 25.327+.
73. 81519.856+.
74. .222+.
75. .321+.
76. 88.4507+.
77. 23.328+.
78. 2626.595+.
79. .2025+.
80. .0655+.
81. 16.841+.

82. 544.382+.
83. 9.245+.
84. 5.343+.
85. .0438+.
86. 60.331+.
87. 304.977+.
88. 15.472+.
89. 17.426+.
90. 74.3802+.
91. 88.537+.

Page 340.

1. .034375.
2. 1200.
3. $407,294\frac{1080}{1754}$.
4. 70,234,730,841.

Page 341.

5. $444.75.
6. 6.
7. .2955.

---

1. $152.50.
2. $7.22.
3. $136.08.
4. 836.02\frac{1}{2}$.
5. $12.75.
6. $2392.39.
7. $111.45.
8. 26.23\frac{1}{2}$.
9. $157.50.
10. $31.50.
11. $579.72.
12. $47.41.
13. $546.48.
14. $1129.11.
15. $644.62.

Page 342.

16. $433.07.
17. $1787.95.
18. $125.66.
19. $1580.25.
20. $35.40.
21. 9.96.
22. .0002075.
23. 2.292.
24. 26.
25. 9.2.
26. 900.
27. 5.67.
28. .01008.
29. .33375.
30. .04375.
31. $\frac{1}{3}$.
32. $\frac{1}{5}$.
33. $\frac{1}{6}$.
34. $\frac{1}{7}$.
35. $\frac{1}{160}$.
36. $\frac{1}{90}$.
37. $\frac{1}{15}$.
38. $\frac{5}{6}$.
39. $\frac{1}{110}$.
40. $\frac{1}{24}$.
41. $\frac{10}{13}$.
42. $\frac{3}{80}$.

Page 343.

4. 5229 miles.
5. $\frac{1}{2}$ cent.
6. $1,303,095.17.
7. 38 clerks.

Page 344.

8. $1914.65.
9. 34,888 packages.
10. 21,781.53696.

Page 345.

1. 1512 sq. in.
2. 1278 sq. in.
3. 1850 sq. in.
4. 4788 sq. in.
5. 31,104 sq. in.
6. 14,256 sq. in.
7. 810 sq. in.
8. 5980 sq. in.
9. 6264 sq. in.
10. 8424 sq. in.
11. 432 sq. ft.
12. 12 sq. ft.
13. 432 sq. ft.
14. 12 sq. ft.
15. 14 sq. ft.
16. 12 sq. ft.
17. 14 sq. ft.
18. 437$\frac{1}{2}$ sq. ft.
19. 14 sq. ft.
20. 1755 sq. ft.
21. 450 sq. yd.
22. 20 sq. yd.
23. 108 sq. yd.
24. 15 sq. yd.
25. 18 sq. yd.
26. 24 sq. yd.
27. $5\frac{5}{27}$ sq. yd.
28. $3\frac{19}{24}$ sq. yd.
29. $7\frac{1}{2}$ sq. yd.
30. 90 sq. yd.

Page 348.

1. 270 sq. ft.
2. 5 lb. 14 oz.
3. $127.32.
4. 20 cents.
5. $\frac{23}{40}$.
6. $\frac{7}{45}$.
7. $\frac{13}{28}$.
8. $4581\frac{9}{11}$ sec.

Page 349.

9. 216.
10. 29.
11. $6000.
12. 6 hr. 49 min. 45 sec. A.M.
13. A, $3600; B, $2400.
14. $1.35.
15. 45 sq. yd.
16. Increased $\frac{1}{12}$.
17. $12.
18. 360 oranges.
19. $68.40.
20. $750.

Page 350.

1. $15,373.84.
2. $15,697.16.
3. $40,525.88.

349,129 pupils.

Page 351.

1. $4\frac{91}{96}$ bushels.
2. $13,691.16.
3. Tea, $3555\frac{5}{9}$ lb.; coffee, 6000 lb.; sugar, $37,012\frac{4}{83}$ lb.; $4,080 remaining.
4. 68.81495 lb.
5. Lost 45.97\frac{1}{2}$.
6. $94.51.
7. $70.20.
8. 24.75 tons.
9. 204.0278267.

Page 352.

1. 233,675.
2. 64,725.
3. $101,537\frac{1}{2}$.
4. 216,100.

5. 1,015,375.
6. 2,336,750.
7. 23,367$\frac{1}{2}$.
8. 701,025.
9. 432,200.
10. 243,300.
11. 428,400.
12. 80,250.
13. 185,100.
14. 129,000.
15. 230,400.
16. 21,100.
17. 525,500.
18. 145,312$\frac{1}{2}$.
19. 24,062$\frac{1}{2}$.
20. 1,828,500.

Page 354.

1. 107,136.
2. 604,665.
3. 96,145.
4. 494,312.
5. 473,484.
6. 191,597.
7. 410,896.
8. 1,297,479.
9. 347,332.
10. 1,301,234.
11. 113,542.
12. 73,350.
13. 132,790.
14. 110,808.
15. 101,085.
16. 852,120.
17. 73,072.
18. 325,815.
19. 167,892.
20. 304,856.
21. 169,344.
22. 212,175.
23. 710,046.
24. 301,392.
25. 474,300.
26. 385,600.
27. 1,497,300.
28. 2,300,400.
29. 324,000.
30. 2,984,800.

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1. \$54,659,-886.61.

Page 355.

2. \$145,543,-810.71.
5. \$69.75.
6. 12 pages.
7. \$13,614.07.
8. $\frac{1}{6}$.

Page 358.

2. 42 gal. 2 qt.
3. 637 gal. 2 qt.
4. 25,000 times.
5. 9$\frac{1}{2}$ yards.
6. 1501 min.
7. 66 days.
8. 41 hr. 15 min.
9. 1440 steps.

Page 359.

10. 80 rods.
11. 1$\frac{2}{15}$ min.
12. 4 gal. 1 qt. 1 pt.
13. 8 hr. 43 min.
14. 2 bu. 3 pk. 5 qt.
15. 75 rods.

---

1. 109$\frac{3}{10}$.
2. 25$\frac{7}{24}$.
3. 146$\frac{17}{48}$.
4. 97$\frac{19}{60}$.
5. 206$\frac{143}{480}$.
6. 240$\frac{17}{40}$.
7. 152$\frac{9}{20}$.
8. 211$\frac{4}{15}$.
9. 829$\frac{11}{24}$.
10. 224$\frac{41}{60}$.

Page 360.

11. 18$\frac{1}{8}$.
12. 75$\frac{29}{35}$.
13. 36$\frac{9}{50}$.
14. 30$\frac{47}{60}$.
15. 49$\frac{59}{72}$.
16. 42$\frac{31}{35}$.
17. 37$\frac{67}{80}$.
18. 68$\frac{101}{108}$.
19. 16$\frac{29}{30}$.
20. 228$\frac{1}{90}$.
21. 17$\frac{2}{5}$.
22. 3$\frac{1}{3}$.
23. 210$\frac{4}{7}$.
24. 52$\frac{5}{8}$.
25. 15$\frac{5}{8}$.
26. 3$\frac{7}{20}$.
27. 248.
28. 84$\frac{4}{5}$.
29. 4$\frac{2}{7}$.
30. 83$\frac{1}{3}$.
31. 1$\frac{1}{6}$.
32. 3$\frac{3}{4}$.
33. 1384$\frac{3}{8}$.
34. 19$\frac{29}{77}$.
35. 8$\frac{1}{28}$.
36. 2$\frac{31}{132}$.
37. 2$\frac{38}{75}$.
38. 5$\frac{11}{35}$.
39. $\frac{6}{11}$.
40. $\frac{5}{9}$.
41. 31$\frac{1}{15}$.
42. 5$\frac{15}{22}$.
43. 43$\frac{5}{8}$.
44. 5$\frac{5}{284}$.
45. 49$\frac{7}{32}$.
46. $\frac{23}{120}$.
47. $\frac{2}{5}$.
48. 3$\frac{2}{15}$.

Page 361.

2. $\frac{3}{320}$. .046875.
3. .000000140028.
4. \$872.87.
5. 6$\frac{1}{14}$.
6. .09375 bu.
7. 11$\frac{1}{4}$ pounds.
8. \$22,612.50.
9. \$4.05.
10. \$296.25.
11. \$968.88.
12. \$20.16.
13. \$7335.
14. $\frac{201}{400000}$.
15. $\frac{3}{13}$.
16. 560.22 yards.

Page 362.

17. \$7.96.
18. .04.
19. \$676.
20. \$6.
21. Gained \$8.
22. 21 clerks.
23. 1280 sheep.
24. 4 boxes.
25. $\frac{16}{45}$.
26. \$45.
27. \$1033.05.
29. 31$\frac{1}{4}$ cents.

Page 363.

30. .0002009877.

31. 82.
32. 31 years.
33. $21\frac{127}{144}$.
34. $ 108.
35. 399 yr. 2 mo. 17 da.
36. 219 hats.
37. 7 years.
38. $ 999.
39. .00012.
40. $ 3.
41. $ 110.
42. 63 miles.

Page 364.

1. 777 ounces.
2. 190 yards.
3. 3520 yards.
4. 89 hours.
5. 1455 seconds.
6. 17,675 lb.
7. 180 quarts.
8. 600 pints.
9. 79 pecks.
10. 632 quarts.
11. 62 lb. 8 oz.
12. 62 lb. 8 oz.
13. 2 ft. 4 in.
14. 2 ft. 3 in.
15. 3 qt. 1 pt.
16. 2 qt. 1 pt.
17. 1 pk. 7 qt.

Page 365.

1. $47,789\frac{5}{7}$.
2. (*a*) 14.75605; (*b*) 5999.25.
3. 598 bu. 3 pk.
4. $15\frac{5}{8}$.
5. 4 weeks.
6. 34 cords.
7. $ 357.50.

Page 366.

1. 60 lb. 15 oz.
2. 11 yards.
3. 21 da. 13 hr.
4. 28 min. 14 sec.
5. 4 T. 1314 lb.
6. 123 gal. 1 qt. 1 pt.
7. 185 pk. 5 qt.
8. 46 bu. 1 pk.
9. 5 weeks.
10. 990 inches.

Page 367.

1. 44 lb. 9 oz.
2. 23 yd. 1 ft.
3. 14 hr. 14 min.
4. 26 min. 13 sec.
5. 4 yd. 2 ft. 10 in.
6. 28 gal. 2 qt.
7. 18 bu. 3 pk.
8. 4 pk. 2 qt.
9. 12 weeks.
10. 16 T. 904 lb.

---

1. 3 lb. 9 oz.
2. 5 yd. 2 ft.
3. 7 hr. 10 min.
4. 33 min. 45 sec.
5. 1 ft. 4 in.
6. 18 gal. 2 qt.
7. 21 bu. 3 pk.
8. 3 quarts.
9. 1 wk. 3 da.
10. 4 T. 912 lb.

Page 368.

1. 42 days.
2. 12 days.
3. 28 days.
4. 56 men.
5. 33 horses.
6. 18 lines.
7. 900 steps.
8. 3072 bricks.
9. 11 hours.
10. 77 cents.
11. 12 days.

Page 369.

1. 168,932.
5. 15 hr. 16 min. $21\frac{9}{11}$ sec.
6. $ 40.
7. 6.0625.

Page 370.

8. $ 130.

---

1. 37 lb. 5 oz.
2. 22 hr. 10 min.
3. 49 T. 835 lb.
4. 69 bu. 3 pk.
5. 14 wk. 2 d.
6. 21 yd. 2 ft.
7. 73 minutes.
8. 19 gal. 2 qt.
9. 22 feet.
10. 9 yards.
11. 4 lb. 9 oz.
12. 3 gal. 2 qt.
13. 6 hr. 27 min.
14. 5 bu. 1 pk.
15. 5 min. 13 sec.
16. 2 yd. 2 ft.
17. 1 ft. 11 in.
18. 8 T. 1234 lb.
19. 2 wk. 6 da.
20. 4 yd. 2 ft. 3 in.
21. 4.
22. 6.
23. 8.
24. 9.
25. 7.
26. 9.
27. 8.

Page 371.

28. 12.
29. 15.
30. 13.
31. 16.
32. 11.
33. 18.

---

1. 1125 sq. ft.
2. 432 sq. ft.
3. 48 sq. yd.
4. 12 sq. yd.
5. 13 sq. yd.
6. 44 sq. yd.
7. 7975 sq. ft.

Page 373.

1. $ 548.80.
2. $ 37.45.
3. $ 72.
4. $ 187.60.
5. $ 137.10.
6. 11 pupils.
7. $ 480.

Page 374.

8. $333.
9. 3 words.
10. 60 cents.

Page 375.

2. 1.33\frac{1}{3}$.
3. 1550.
4. .000007.
5. 4.975.
6. 2633.0045.
7. $6.75.
8. 66 cents.
9. 4 quarts.
10. 160 acres.
11. 5$\frac{1}{2}$ hours.
12. $\frac{2}{3}$.
13. $\frac{16}{100}$; $\frac{8}{50}$; $\frac{4}{25}$; 16%.

Page 376.

1. $27.56.
2. $31.40.
3. $18.99.
4. $45.52.

Page 378.

1. $11.46.
2. $29.10.
3. $18.77.
4. $11.37.
5. $21.87.
6. $7.47.
7. $3.99.
8. $22.26.
9. $5.08.
10. $7.46.
11. $87.99.
12. $6.30.
13. $13.42.
14. $64.97.
15. $2.55.
16. $56.25.
17. $49.02.
18. $3.99.
19. $95.02.
20. $96.58.
21. $23.20.
22. $189.
23. $568.
24. $225.
25. $589.60.
26. $51.30.
27. $62.40.
28. $320.
29. $133.25.
30. $52.92.
31. $13.50.
32. $2.75.
33. $55.

Page 379.

4. 37$\frac{1}{2}$ sq. yd.
5. 900 sq. ft.
6. 2 sq. ft.
7. 3$\frac{3}{4}$ sq. ft.
8. 8$\frac{3}{4}$ sq. ft.
9. 13$\frac{1}{8}$ sq. ft.
10. 8100 sq. ft.

Page 380.

11. 5062$\frac{1}{2}$ sq. ft.
12. 7$\frac{1}{2}$ sq. ft.
13. 750 sq. ft.
14. 61$\frac{7}{8}$ sq. ft.
15. 308$\frac{1}{3}$ sq. ft.

---

1. 38$\frac{2}{15}$ cents.
2. $10.04.
3. 18 yd. 2 ft.
4. 3520 rails.
5. 497$\frac{48}{105}$ min.
6. $247.50.
7. 135 pounds.
9. 23 tons.
10. 231 pints.

Page 381.

1. 121$\frac{1}{3}$.
2. 210$\frac{1}{3}$.
3. 88$\frac{1}{5}$.
4. 331$\frac{1}{54}$.
5. 139$\frac{5}{66}$.
6. 118$\frac{2}{7}$.
7. 591$\frac{3}{32}$.
8. 382$\frac{4}{15}$.
9. 247$\frac{23}{27}$.
10. 263$\frac{5}{8}$.
11. 5$\frac{2}{45}$.
12. 13$\frac{3}{180}$.
13. 21$\frac{1}{9}$.
14. 81$\frac{3}{8}$.
15. 12$\frac{15}{64}$.
16. 11$\frac{62}{85}$.
17. 12$\frac{2}{7}$.
18. 72$\frac{5}{36}$.
19. 30$\frac{5}{26}$.
20. 40$\frac{31}{56}$.

Page 383.

1. 117$\frac{113}{176}$ yd.
2. $3000.
3. $27.56.
4. 244\frac{1}{6}$.
5. 5.83\frac{1}{3}$.

Page 384.

6. $87.
7. $\frac{3}{400}$; $\frac{5}{8}$.
8. $4.42.
9. 300.04; 6.568$\frac{1}{3}$.
10. 1$\frac{1}{12}$ oranges.
11. 502$\frac{1}{3}$ days.
12. 5833.33\frac{1}{3}$.

Page 385.

13. 1360 pounds.
14. 2178 feet.
15. $52.50.
16. $1.12.
17. $4.50.
18. 52\frac{1}{5}$.
19. $379.50.
20. 2880 tiles.
21. $15,000; $350.
22. $\frac{3}{7}$.
23. $877.22.
24. 724 bushels.
25. $1828.50.

Page 386.

26. 31 pounds.
27. 5$\frac{1}{4}$ miles.
28. $\frac{9}{35}$.
29. 3240 bushels.
30. $360.

Page 387.

1. 2$\frac{23}{54}$.
2. 1$\frac{33}{100}$.
3. $\frac{7}{11}$.
4. 6$\frac{1}{6}$.
5. 1$\frac{1}{3}$.
6. $\frac{32}{63}$.
7. 34$\frac{24}{31}$.
8. $\frac{23}{35}$.
9. 1.
10. 15.
11. 3.679+.

12. .005.
13. .004375.
14. 3.78.
15. 102.390561.
16. 19,700.

Page 388.

17. .125; 8.
18. 90.
19. 1.36.
20. .26285.

---

1. $\frac{3}{11}$.
2. $\frac{5}{16}$.
3. $\frac{148}{254}$.
4. $\frac{3}{5}$.
5. $\frac{18}{41}$.
6. $\frac{11}{39}$.
7. $\frac{3}{11}$.
8. $\frac{300}{401}$.
9. $\frac{3}{8}$.
10. $\frac{4}{7}$.
11. $\frac{7}{11}$.
12. $\frac{2172}{2265}$.

Page 389.

1. 131 pints.
2. 220 pints.
3. 128 pints.
4. 129 pints.
5. 279 pints.
6. 252 pints.
7. 77 pints.
8. 85 pints.
9. 217 pints.
10. $39\frac{1}{2}$ pints.
11. 39 gal.
12. 19 gal. 3 qt.
13. 51 gal.
14. 162 gal. 3 qt.
15. 7 gal. 3 qt. 1 pt.
16. 34 gal. 2 qt. 1 pt.
17. 17 gal. 1 qt. 1 pt.
18. 42 gal. 3 qt.
19. 15 gal. 3 qt.
20. 88 gal. 3 qt. 1 pt.

Page 390.

21. 633 inches.
22. 7594 yards.
23. 2391 quarts.
24. 2507 ounces.
25. 2271 inches.
26. 611 quarts.
27. 192 feet.
28. 510 pints.
29. 102 quarts.
30. 34,369 lb.
31. 54,960 sec.
32. 827 hours.
33. 120 hours.
34. 165 yards.
35. 40 ounces.
36. 52 yd. 4 in.
37. 29 lb. 11 oz.
38. 22 bu. 3 pk. 1 qt.
39. 6 da. 35 min.
40. 2 T. 972 lb.
41. 3 mi. 12 rd.
42. 14 gal. 2 qt. 1 pt.
43. 2 hr. 38 min. 3 sec.
44. 27 bu. 1 pk. 5 qt.
45. 93 lb. 7 oz.
46. 10 yd. 1 ft. 1 in.
47. 54 gallons.
48. 2 mi. 236 rd.
49. 39 gal. 3 qt. 1 pt.
50. 2 miles.

Page 391.

1. .0015 T.
2. $\frac{5}{576}$ day.
3. 45 minutes.
4. 45 minutes.

Page 392.

5. .00625 day.
6. \$ $76.87\frac{1}{2}$.
7. 3 T. 1504 lb.
8. \$ 46.87.
9. 14 T. 1244 lb.
10. $\frac{31}{36}$ yard.
11. .890625 bu.
12. 3 pk. 6 qt.
13. 75 cents.
14. \$ 6.86.
15. 2 qt. $1\frac{1}{2}$ pt.
16. $\frac{49}{150}$.
17. $\frac{174}{203}$.
18. 1 bu. 1 pk. 1 qt.
19. .885 day.
20. 5280 feet.

---

1. 46 lb. 7 oz.
2. 43 bu. 2 pk. 1 qt.

Page 393.

3. 34 yd. 2 ft. 6 in.
4. 40 da. 19 hr. 55 min.
5. 186 gal. 3 qt.
6. 22 hr. 30 min. 28 sec.
7. 18 T. 862 lb.
8. 9 wk. 1 da.
9. 53 mi. 294 rd.
10. 76 yr. 8 mo.
11. 866 T. 899 lb.
12. 140 lb. 3 oz.
13. 38 hr. 40 min. 2 sec.
14. 180 gal. 3 qt. 1 pt.
15. 137 yd. 7 in.
16. 194 mi. 183 rd.
17. 128 yr. 4 mo. 21 da.
18. 36 wk. 5 hr.
19. 22 hr. 24 min. 22 sec.
20. 17 bu. 4 qt.

Page 394.

21. 17 lb. 7 oz.
22. 3 bu. 2 pk. 5 qt.
23. 7 yd. 2 ft. 7 in.
24. 10 da. 9 hr. 20 min.
25. 3 gal. 2 qt. 1 pt.
26. 13 hr. 44 min. 30 sec.

27. 246 T. 1676 lb.
28. 11 wk. 16 hr.
29. 15 mi. 311 rd.
30. 11 yr. 3 mo.
31. 16 lb. 12 oz.
32. 8 bu. 3 pk. 6 qt. 1 pt.
33. 8 yd. 1 ft. 9 in.
34. 12 da. 23 hr. 45 min.
35. 56 gal. 1 pt.
36. 67 yr. 6 mo.
37. 42 mi. 245 rd.
38. 38 T. 546 lb.
39. 16 lb. 12 oz.

Page 395.

40. 38 wk. 3 da. 17 hr.
41. 10 gal. 1 qt. 1 pt.
42. 17 hr. 24 min. 35 sec.
43. 8 yd. 1 ft. 10 in.
44. 57 bu. 1 qt.
45. 38 da. 18 hr. 55 min.
46. 13 bu. 1 pk. 6 qt.
47. 16 gal. 2 qt. 1 pt.
48. 6 hr. 29 min. 40 sec.
49. 3 lb. 10 oz.
50. 25 bu. 1 pk. 7 qt.
51. 27 bu. 3 pk. 4 qt.
52. 76 gal. 3 qt. 1 pt.
53. 30 lb. 8 oz.
54. 16 hr. 16 min. 15 sec.
55. 138 bu. 2 pk. 2 qt.
56. 224 gal. 3 qt. 1 pt.
57. 202 pounds.
58. 5 hr. 1 min. 57 sec.
59. 7 bu. 6 qt.
60. 9 gal. 2 qt. 1 pt.
61. 54 years.
62. 94 wk. 6 da.
63. 74 T. 500 lb.
64. 77 yards.
65. 65 mi. 160 rd.
66. 1 da. 14 hr. 18 min.

Page 396.

67. 1323 gal.
68. 4 bu. 2 pk. 4 qt.
69. 2 hr. 34 min. 5 sec.
70. 94 wk. 3 da. 20 hr.
71. 20 bu. 2 pk. 7 qt.
72. 73 yd. 2 ft. 3 in.
73. 21 days.
74. 54 yr. 10 mo.
75. 41 gal. 1 qt.
76. 5 lb. 3 oz.
77. 7 ounces.
78. 3 quarts.
79. 1 bu. 1 pk.
80. 1 hr. 10 min.
81. 5 lb. 13 oz.
82. 18 bu. 3 pk. 7 qt.
83. 16 yd. 2 ft. 9 in.
84. 11 da. 5 hr. 19 min.
85. 93 gal. $3\frac{1}{3}$ qt.
86. 5 hr. 35 min. 5 sec.
87. 22 T. 825 lb.
88. 2 wk. 4 da. 4 hr. 48 min.
89. 18 mi. 180 rd.
90. 5 yr. 9 mo.
91. 13 bu. 3 pk. 6 qt.
92. 25 gal. 2 qt. 1 pt.
93. 33 min. 33 sec.
94. 2 wk. 5 da. 12 hr.
95. 5 yd. 6 in.
96. 7 bu. 2 pk. 1 qt.
97. 1 da. 6 hr. 49 min.
98. 3 qt. 1 pt.
99. 2 yd. 2 ft. 2 in.
100. 10 wk. 2 da. 18 hr. 15 min.
101. 50 hr. 50 min. 50 sec.
102. 11 gal. 1 qt. 1 pt.
103. 22 bu. 2 pk. 2 qt.
104. 17 yd. 1 ft. 9 in.
105. 31 mi. 108 rd. 4 yd.
106. 25 da. 23 hr. 48 min.

Page 397.

108. 13 gal. 1 pt.
110. 17 lb. 3 oz.
111. 4 T. 960 lb.
112. 1 mi. 110 rd.
113. 3 yr. 6 mo.
114. 12 bu. 3 pk. $2\frac{2}{3}$ qt.
115. 17 yd. 4 in.
116. 5 hr. 20 min. 10 sec.
117. 18 gal. 1 qt. 1 pt.
118. 3 da. 5 hr. 20 min.
119. 14 T. 110 lb.
120. 16 yd. 2 ft. 11 in.
121. 14 gal. 3 qt. $1\frac{1}{2}$ pt.

Page 399.

1. 1 hr. 18 min. 17 sec.
2. 25 bu. 3 pk. 4 qt.
3. $3\frac{1}{2}$ inches.
4. 2 pk. 6 qt.
5. 14 mi. 17 rd.

6. 10 hr. 28 min.
7. 1 ft. $10\frac{1}{4}$ in.
8. 4 hr. 43 min. 30 sec.
9. 27 min. 10 sec.
10. $37.50.
11. $91\frac{2}{3}$ cents.
12. $202\frac{1}{2}$ miles.

Page 403.

1. $1129\frac{1}{7}$.
2. 10,665.
3. $8077\frac{1}{11}$.
4. $28,813\frac{2}{5}$.
5. $31,523\frac{1}{5}$.
6. $61,903\frac{7}{11}$.
7. $206,783\frac{1}{5}$.
8. 403,270.
9. $834,085\frac{5}{7}$.
10. 15,940,572.
11. 775,665.
12. 933,273.
13. 601,227.
14. 542,817.
15. 2,758,239.
16. 8,296,695.
17. 1,232,766.
18. 3,855,141.
19. 9,733,680.
20. 7,467,570.
21. 67,100.
22. 310,700.
23. $108,662\frac{1}{2}$.
24. $324,133\frac{1}{3}$.
25. $113,437\frac{1}{2}$.
26. 216,500.
27. 426,300.
28. 2150.
29. 105,300.
30. 690,300.

Page 404.

31. $4187\frac{8552}{9999}$.
32. $62,132\frac{505}{1987}$.
33. $9555\frac{10220}{31416}$.
34. $9593\frac{10810}{26543}$.
35. $52,073\frac{8054}{16074}$.

Page 405.

10. $2\frac{5}{24}$.
11. $\frac{2}{35}$.
12. $14\frac{7}{8}$.
13. $7\frac{1}{7}$.
14. 71.01.
15. .89575.
16. 148.28125.
17. .2.
18. $31,370.38.
19. 1 cwt. 3 qr. 10 lb. 10 oz.

---

1. $216,671,399,071.35.

Page 406.

6. $24.90.
7. 21,945 cu. in.
8. $175.
9. $10 gain.
10. $2.47.

Page 407.

4. 20 pounds.
5. 81.
6. $127,581,911,264.12.
7. $1.03 loss.
8. 4800 steps.
9. $120.
10. $912.92.

Page 408.

2. 100 envelopes.
3. 24 rugs.
4. 72 boards.

Page 409.

7. 1944 bricks.
8. 72 tiles.
9. 240 boards.
10. 264,000 stones.
11. 4816 sq. yd.
12. 4840 sq. yd.
13. 80 by 121, 40 × 242, etc.
14. 16 times.
15. 9000 sq. ft.

Page 410.

16. 41,400 sq. ft.; 8600 sq. ft.
17. 9400 sq. ft.
18. $2800; $330.

Page 411.

21. 160 sq. yd.
22. 160 rods.
23. 64 yards.
24. 18 sq. yd.; $20\frac{1}{4}$ yd.
25. $59\frac{1}{16}$ sq. yd.

---

1. 18 hours.
2. 31 days.
3. 7 o'clock; 3 o'clock; 5 o'clock.
4. $18.

Page 412.

5. 25 cases.
6. 21 posts; 2 posts; 3 posts.
7. 31 days; 29 days.
8. 43 days.
9. 23 chapters.
10. 27 problems.

Page 413.

1. 238 days.
2. 140 days.
3. 109 days.
4. 76 days.
5. 151 days.
6. 284 days.
7. 179 days.
8. 139 days.
9. 91 days.
10. 151 days.
11. $196.
12. 235 days.
13. Tuesday.
14. 66 days.
15. August 15.
16. 170 days.

Page 414.

17. 44 yr. 4 mo. 12 da.
18. 4 yr. 1 mo. 11 da.
19. 8 yr. 4 mo. 14 da.

20. 128 yr. 2 mo. 9 da.
21. Mar. 4, 1841.
22. 33 yr. 1 mo. 8 da.
23. 3 yr. 9 mo. 15 da.
24. 49 yr. 3 mo. 15 da.
25. July 21, '61.
26. 117 yr. 5 mo. 27 da.

Page 415.

1. $34.56.
2. $15.30.
3. 469 bushels.
4. 108 cows.
5. 216 yards.
6. 192 soldiers.
7. 116 gallons.
8. 31 cents.
9. $39.
10. $1.56.
11. 39 cents.
12. 240 hours.
13. 2 hr. 24 min.
14. $1.77½.
15. $2.20.
16. 27 days.

Page 416.

1. $34.40.
2. 48 cents.
3. $15.60.
4. $3.90.
5. 35 cents.

Page 417.

6. $10.40.
7. $22.40.
8. $1.
9. $108.
10. $9.60.

Page 418.

1. $\frac{469}{905}$.
2. 11 miles.
3. 3 bu. 7 qt.
4. A, $750; B, $500; C, $250.
5. $39.38.
6. $2.15.
7. $1407.
8. $7.05.
9. 48 tiles.
10. $75.60.
11. $216.66⅔.
12. 12 bu. 3 pk. 4 qt.

Page 419.

1. 18,500 sq. ft.
2. 28 sq. yd.

Page 420.

4. 864 bricks.
5. 1728 bricks.
6. 112 sq. in.
7. 36 sq. ft.
8. 45 rolls.
9. 24 lots.
11. 9000 sq. ft.
12. $1200.

Page 421.

13. 16 fields.
14. 275 yards.
15. 120 sq. rd.
16. 5 acres.
17. 405 sq. yd.
18. 18,755 sq. yd.

Page 422.

19. ½ acre.
20. 160 sq. in.

---

1. $7\frac{9}{11}$ rods.
2. 7 rd. 4½ yd.
3. 7 rd. 4 yd. 1½ ft.
4. 7 rd. 4 yd. 1 ft. 6 in.
5. 13 rd. 1 ft. 6 in.
6. 12 rods.

Page 423.

7. 8 rd. 5 yd.
8. 8 rd. 5 yd.
9. 8 rd. 5 yd.
10. 8 rd. 5 yd.
11. 9 rd. 1 ft. 6 in.
12. 9 rd. 1 yd. 1 ft. 6 in.
13. 9 rd. 1 yd. 1 ft. 6 in.
14. 9 rd. 1 yd. 1 ft. 6 in.
15. 9 rd. 1 yd. 1 ft. 7 in.
16. 9 rd. 2 yd. 1 ft. 6 in.
17. 9 rods.
18. 9 rods.
19. 9 rods.
20. 9 rods.
21. 7 rd. 2 yd. 2 ft. 1 in.
22. 4 rd. 5 yd. 1 ft.
23. 6 rd. 4 yd. 1 ft. 1 in.
24. 14 rd. 2 ft.
25. 5 rd. 3 yd. 1 ft. 6 in.
26. 7 rd. 2 yd. 1 ft.
27. 17 rd. 2 yd. 1 ft. 3 in.
28. 6 rd. 2 yd. 1 ft. 6 in.
29. 7 rd. 5 yd. 10 in.
30. 990 inches.
31. 5 rods.
32. 1422 inches.
33. 7 rd. 1 yd.

Page 424.

34. 17 rd. 3 yd.
35. 15 rd. 2 yd. 2 ft. 6 in.
36. 13 rods.
37. 22 rd. 3 yd. 2 ft. 6 in.
38. 5 rd. 5 yd. 6 in.
39. 12 rd. 4 yd. 6 in.
40. 23 rd. 2 yd. 6 in.
41. 111 rd. 1 yd. 6 in.
42. 3 rd. 4 yd. 2 ft. 6 in.
43. 3 rd. 4 yd. 1 ft. 6 in.

Page 425.

5. 70 cu. in.
8. 46,656 cu. in.
9. $\frac{1}{3}$ cu. yd.
10. 8 ft. × 4 ft.; 16 ft. × 2 ft.; etc.
11. 5184 cu. ft.
12. 3 × 7 × 11; 6 × 7 × 5$\frac{1}{2}$; etc.
13. 1 cu. ft. smaller.
14. $132.

Page 426.

15. 3 feet.
16. About 7$\frac{1}{2}$ gal.
17. About 1$\frac{1}{4}$ cu. ft.
18. 30 gallons.
19. 30 bushels.
20. 9 cords.
21. 162,000 bricks. 10,368,000 cu. in.
22. 31,104 bricks.
23. 27 bricks.
24. 40,000 bricks.
25. $2048.

Page 428.

4. 1562.5.
5. $39.81.
8. 88 cents; $\frac{5}{2376}$ rod.
9. $\frac{5}{18}$ bbl.; 127$\frac{7}{9}$ cu. yd.

---

1. $3.62.

Page 429.

2. $46.55.
3. $7.32.
4. 795 minutes.
5. 1 mi. 85 rd. 2 ft. 6 in.
7. 40 sq. in.
8. .625 year. 643 lb. 9.6 oz.
10. 31.416. 2 inches.

---

1. 3\frac{5}{12}$.

Page 430.

3. $28,800.
5. $\frac{4}{5}$ ton.
7. $75.47.
9. 40 bushels.
10. $\frac{2}{3}$.

---

1. $693.
2. 45 cents.
3. $\frac{126}{135}$ day.
4. $2.39.
5. 24 miles.

Page 431.

1. $136.57.
2. 60$\frac{20}{21}$ acres.
3. $\frac{375}{3608}$.
4. 5$\frac{33}{40}$.
5. Increased $\frac{1}{18}$.
6. 8.384964.
9. 47 min. 12$\frac{2}{35}$ sec.
10. $\frac{160}{389}$.
11. 7.41\frac{2}{3}$.
13. 453$\frac{1}{3}$ miles.
14. 99$\frac{15}{16}$ cents.

Page 432.

15. $16.50.
16. $25.
17. 149$\frac{47}{77}$ gal.
18. $2.51.
19. 2500.
21. $6.66.
24. 23$\frac{52}{103}$.
25. $2.80.

Page 433.

1. $\frac{19}{20}$; $\frac{263}{300}$.
2. .571$\frac{3}{7}$; .625.
3. 199.925.
4. .012.
5. $27.
6. 36 spoons.

Page 434.

7. $20.
8. $71.28.
9. 4 feet.
10. $120.
11. 76 sq. yd.
12. $24.75.
13. $23.52.
14. 12,960 lb.
15. $14.28.

Page 435.

3. 10$\frac{17}{90}$; 84$\frac{2}{7}$.
4. 41.00679.
5. 2750 sq. yd.
6. 926$\frac{1}{4}$.

Page 437.

1. 447.77\frac{3}{5}$.
2. 493.76\frac{1}{4}$.
3. $24.75.
4. $141.95.
5. 100.
6. $10.55. 320 feet.
7. 9.37\frac{1}{2}$.
8. 141.
10. $4.50. 5$\frac{55}{141}$ tons.
11. $\frac{8}{11}$ acre.

Page 438.

12. $6; $18; $24; $36.
13. 672 hens.
14. 51$\frac{29}{40}$ miles.
15. $114.60.
16. $1250.
17. 106,294.4. 3757.2.
18. 54 sq. yd. 160 sq. in.
19. $400.
20. 1232 mi.; 1730 yd.; 1,020,304 lb.
21. 24.37\frac{1}{2}$.
22. 173,218.35; 814.43.

Page 439.

23. 3300 ft.; $\frac{37}{48}$ day; 110,672 oz.
24. 31$\frac{31}{54}$ sq. yd.; 42 feet.
25. $\frac{2}{5}$.
26. $714.
27. $9120; $14,820.
28. $\frac{4}{105}$; 278$\frac{3}{40}$.
29. $166.72.

30. \$60.75.

---

1. $38\frac{11}{12}$.
2. $37\frac{1}{40}$.
3. $58\frac{11}{30}$.
4. $31\frac{29}{30}$.
5. $61\frac{1}{24}$.
6. $68\frac{3}{25}$.
7. $66\frac{7}{15}$.
8. $95\frac{17}{30}$.
9. $38\frac{19}{24}$.
10. $89\frac{7}{30}$.

Page 440.

11. $6\frac{2}{3}$.
12. $13\frac{11}{15}$.
13. $54\frac{19}{22}$.
14. $67\frac{23}{24}$.
15. $17\frac{19}{30}$.
16. $61\frac{5}{6}$.
17. $37\frac{31}{36}$.
18. $18\frac{29}{36}$.
19. $39\frac{16}{21}$.
20. $25\frac{11}{24}$.
21. $131\frac{1}{4}$.
22. $211\frac{1}{4}$.
23. $663\frac{2}{5}$.
24. $185\frac{5}{12}$.
25. $103\frac{13}{16}$.
26. $95\frac{2}{5}$.
27. $81\frac{1}{4}$.
28. $98\frac{2}{3}$.
29. $513\frac{10}{21}$.
30. $431\frac{3}{32}$.
31. $15\frac{21}{26}$.
32. $13\frac{13}{22}$.
33. $16\frac{41}{60}$.
34. $12\frac{31}{60}$.
35. $21\frac{55}{126}$.
36. $31\frac{64}{217}$.
37. $23\frac{11}{192}$.
38. $22\frac{37}{50}$.
39. $30\frac{41}{256}$.
40. $20\frac{1}{15}$.

Page 441.

1. 460.12.
21,355.74.
2. .4551.
3. $\frac{523}{2520}$.
4. $\frac{269}{660}$.
5. $11\frac{701}{2310}$.
6. $725\frac{131}{140}$.
7. $\frac{27}{128}$.
8. .675.
9. \$227.60$\frac{5}{12}$.
10. \$3800.
11. 23 hr. 2 min. $8\frac{4}{7}$ sec.
12. \$11.08$\frac{1}{3}$.
13. $1\frac{1}{4}$ yards.
14. 1.
15. \$187.50.
16. 7 days.

Page 442.

17. 216 sq. in.; $1\frac{1}{2}$ sq. ft.; 216 cu. in.; $\frac{1}{8}$ cu. ft.
18. $28\frac{1}{8}$ feet.
19. $10\frac{29}{35}$ years.
20. \$21.67$\frac{1}{2}$.
21. \$199.50.
22. 7.92 inches.
23. 40 cents.
24. \$1800; \$6300.
25. \$874.80.
26. $4142\frac{1}{4}$ lb.
27. $62\frac{1}{2}$ %.
28. \$518.40.
29. \$183.

Page 445.

1. 150 sq. in.
2. 1536 sq. yd.
3. 117 sq. yd.
4. 1225 sq. ft.
5. 1554 sq. yd.
6. 6111 sq. ft.
7. 924 sq. m.
8. 81 sq. ft.

Page 446.

9. 81 sq. ft.
10. 735 sq. yd.

Page 448.

2. 20 and 80.
3. \$2000; \$4000; \$12,000.
4. 18 girls; 36 boys.
5. 13 and 65.
6. 13.

Page 449.

7. 11.
8. \$3000; \$6000; \$18,000.
9. 12 and 60.
10. 9 marbles; 18 marbles; 27 marbles.
11. 36 years; 6 years.
12. 8.
13. 1; 4; 12; 24.
14. 30; 15; 135.
15. 9 pounds.
16. 19 rods.
17. 85 feet.

Page 450.

18. Son, \$40; daughter, \$80.
19. 25 days.
20. Girl, \$80; boy, \$40.
21. Father, 30 da.; son, 15 da.
22. 3 dimes, 6 nickels, 18 cents.
23. 25 yards.
24. 25 rods; 100 rods.
25. Speller, 15 ¢.; reader, 45 ¢.
26. 60 and 12.
27. 18 nuts; 9 nuts; 27 nuts.

Page 452.

1. 24.
2. 24.
3. 42.
4. 84.
5. 24.
6. 70.
7. 72.
8. 40.
9. 360.
10. 160.
11. 18.
12. 18.
13. 8.
14. 16.

15. 12.
16. 20.
17. 900.
18. 60.
19. 60.
20. 32.

Page 453.

21. 36.
22. 222.
23. 180.
24. 72.
25. 320.
26. 7.

---

1. 15 and 75.
2. $28\frac{4}{7}$; $71\frac{3}{7}$.
3. $816.
4. $180.
5. 89.
6. 100.
7. 40; 15.
8. $\frac{28}{32}$.
9. $\frac{60}{84}$.

Page 454.

10. 60; 420.
11. 540; 18.
12. 9.
13. 20 peaches; 5 plums.
14. $200; $600; $700.
15. $60; $140.
16. $300.
17. 64 marbles.
18. $2; $3; $10.
19. $4; $2.
20. 3 horses; 12 cows.

Page 455.

1. 19.
2. 22.
3. 47.
4. 14.
5. 9.
6. 10.
7. 6.
8. 33.

Page 456.

9. 27.
10. 3.
11. 28.
12. 96.
13. 144.
14. 18.
15. 24.
16. 6.
17. 32.
18. 18.
19. 12.
20. 20.

---

2. 15.
3. 9.
4. 15 marbles; 33 marbles.

Page 457.

5. 25 ft.; 100 ft.
6. 39 acres; 47 acres.
7. 1059 votes; 1377 votes.
8. 62 years.
9. 84; 12.
10. $108.
11. 17; 28.
12. $16; $11.
13. Cows, $45; horses, $125.
14. 3 dimes; 14 half dimes.
15. 74 and 26.
16. 21 boys; 33 girls.

Page 458.

17. $3600; $6000; $8400.
18. 44; 11.
19. 5 five-cent stamps; 20 two-cent stamps; 35 postal cards.
20. 8 horses; 25 cows; 55 sheep.

ANSWERS. — Part III.

Page 448.

2. 20 and 80.
3. $2000; $4000; $12,000.
4. 18 girls; 36 boys.
5. 13 and 65.
6. 13.

Page 449.

7. 11.
8. $3000; $6000; $18,000.
9. 12 and 60.
10. 9 marbles; 18 marbles; 27 marbles.
11. 36 years; 6 years.
12. 8.
13. 1; 4; 12; 24.
14. 30; 15; 135.
15. 9 pounds.
16. 19 rods.
17. 85 feet.

Page 450.

18. Son, $40; daughter, $80.
19. 25 days.
20. Girl, $80; boy, $40.
21. Father, 30 da.; son, 15 da.
22. 3 dimes, 6 nickels, 18 cents.
23. 25 yards.
24. 25 rods; 100 rods.
25. Speller, 15 ¢.; reader, 45 ¢.
26. 60 and 12.
27. 18 nuts; 9 nuts; 27 nuts.

Page 452.

1. 24.
2. 24.
3. 42.
4. 84.
5. 24.
6. 70.
7. 72.
8. 40.
9. 360.
10. 160.
11. 18.
12. 18.
13. 8.
14. 16.
15. 12.
16. 20.
17. 900.
18. 60.
19. 60.
20. 32.

Page 453.

21. 36.
22. 222.
23. 180.
24. 72.
25. 320.
26. 7.

---

1. 15 and 75.
2. $28\frac{4}{7}$; $71\frac{3}{7}$.
3. $816.
4. $180.
5. 89.
6. 100.
7. 40; 15.
8. $\frac{28}{32}$.
9. $\frac{60}{84}$.

Page 454.

10. 60; 420.
11. 540; 18.
12. 9.
13. 20 peaches; 5 plums.
14. $200; $600; $700.
15. $60; $140.
16. $300.
17. 64 marbles.
18. $2; $3; $10.
19. $4; $2.
20. 3 horses; 12 cows.

Page 455.

1. 19.
2. 22.
3. 47.
4. 14.
5. 9.
6. 10.
7. 6.
8. 33.

Page 456.

9. 27.
10. 3.
11. 28.
12. 96.
13. 144.
14. 18.
15. 24.
16. 6.
17. 32.
18. 18.
19. 12.
20. 20.

---

2. 15.
3. 9.

4. 15 marbles; 33 marbles.

Page 457.

5. 25 ft.; 100 ft.
6. 39 acres; 47 acres.
7. 1059 votes; 1377 votes.
8. 62 years.
9. 84; 12.
10. $108.
11. 17; 28.
12. $16; $11.
13. Cows, $45; horses, $125.
14. 3 dimes; 14 half dimes.
15. 74 and 26.
16. 21 boys; 33 girls.

Page 458.

17. $3600; $6000; $8400.
18. 44; 11.
19. 5 five-cent stamps; 20 two-cent stamps; 35 postal cards.
20. 8 horses; 25 cows; 55 sheep.

Page 460.

1. $6.34.
2. $4.50.
3. $9.60.
4. $1.55.
5. $4.158.
6. $415.80,
7. $10.32.
8. $1255.80.
9. $9.16.
10. $3.68.
11. $1.60.
12. $1.58.
13. $1.62¼.
14. $326.80.
15. 3 cents.
16. $13.09.
17. $19.98.
18. $1.17.
19. $3.16.
20. $7.50.
21. $3.21¼.
22. $11.32.
23. $63.04.
24. $19.95.
25. $550.
26. $44.40.
27. $323.40.
28. $4.65.

Page 461.

30. $\frac{13x}{20}$.
31. 40%.
32. $\frac{x}{4}$.
33. 168.
34. $\frac{6x}{5}$.
35. 110.
36. $\frac{2x}{3}$.
37. 117.
38. $\frac{x}{150}$.
39. 500%.
40. $\frac{x}{800}$.
41. 18,400.
42. 40%.
43. $133\frac{1}{3}$.
44. 72.
45. 68.
46. 75%.

Page 462.

47. $83\frac{3}{4}$%.
48. $\frac{11x}{200}$.
49. $800.
50. $\frac{2}{3}$.
51. $37\frac{1}{2}$%.
52. 99.
53. $218.75.
54. $500.
55. 1.
56. 500%.
57. 32%.
58. $183.20.
59. 1100.
60. 738.
61. 10,800.
62. $1547.
63. $764.80.

Page 463.

64. $4120.
65. $170.
66. 3500 bu.
67. $1440.
68. $592.

Page 464.

1. Selling price, $2157.40.
2. Selling price, $29.40.
3. Selling price, $1181.25.
4. Selling price, $831.25.
5. Selling price, $1051.38.

Page 465.

6. 3%.
7. $33\frac{1}{3}$%.
8. 15%.
9. 20%.
10. $6\frac{1}{4}$%.
11. $6\frac{1}{4}$%.
12. 20%.
13. 5%.
14. $12\frac{1}{2}$%.
15. $16\frac{2}{3}$%.
16. Cost, $375.
17. Cost, $92.30.
18. Cost, $1234.56.
19. Cost, $240.
20. Cost, $63.75.
21. $55.
22. 25%.
23. $800.

Page 466.

24. 50%.
25. $14\frac{2}{7}$%.
26. $12\frac{1}{2}$%.
27. $37\frac{1}{2}$ cents.
28. $40.
29. $219.
30. $200.
31. $37\frac{1}{2}$%.
32. 25%.
33. $1646.
34. $4053.
35. $113.75+.

Page 467.

1. 3 feet.

2. 18 feet.
3. $\frac{1}{4}$ acre.
4. 1430 yards.
5. 8 strips.

Page 468.

6. 3600 sheets.
7. 250 ft. 1500 sq. ft.
8. 250 boards.
9. $710.
10. 6400 cakes.
11. 76,800 cu. ft.; 2208 tons.
12. 160 feet.
13. 281,600 sq. ft.; 240,000 sq. ft.
14. 2773\frac{1}{3}$.
15. 96 lots.
16. 164 sq. yd.

Page 469.

1. 11$\frac{1}{9}$ %.
2. 10 %.
3. 11$\frac{1}{9}$ %.
4. $26.40.
5. $2.80.
6. 1.22\frac{1}{2}$.
7. 45 %.
8. 1350 %.
9. 4$\frac{1}{2}$ %.
10. $\frac{2}{25}$ %.
11. 25 %.
12. 33$\frac{1}{3}$ %.
13. 16$\frac{2}{3}$ %.
14. $75.
15. 21$\frac{87}{73}$ %.
16. $35.
17. 71.34\frac{3}{4}$.
18. $19.20.
19. 33$\frac{1}{3}$ %.
20. 12 %.
21. $1.92.
22. $24.24.
23. $39.20.
24. $1.57.

Page 470.

25. $18.33.
26. 119$\frac{1}{21}$ %.
27. $200.
28. $893.20.
29. $1.
30. 2 %.
31. $18.
32. $626.05.
33. $1.57.
34. $68.18.
35. 9$\frac{32}{225}$ %.
36. 14$\frac{2}{7}$ %.
37. 18$\frac{26}{33}$ %.
38. $27.20.
39. 15 cents.
40. $6300.
41. $871.83.
42. $309.14.
43. 8 %.
44. 7$\frac{9}{13}$ %.
45. $21.
46. $677.25.
47. $7692.
48. $2.36.
49. 7$\frac{1}{2}$ %.
50. $21.55.

Page 471.

1. $112.50.
2. 90$\frac{5}{8}$ cents.
3. 85$\frac{5}{8}$ cents.
4. $19.52–.
5. $1.65.
6. $1.16–.
7. $19.55.
8. $2.94.
9. 1.11\frac{1}{9}$.
10. 67.72\frac{1}{2}$.
11. $49.77+.
12. 235.75\frac{1}{2}$.

Page 472.

13. $18.88.
14. $7.80.
15. $18.74–.
16. 1050.80\frac{1}{2}$.
17. 1035.73\frac{1}{3}$.
18. $33.73.–.
19. $49.04–.
20. 154.87\frac{1}{2}$.
21. $884.53+.
22. $6191.20.
23. $2841.81–.
24. $2344.50+.
25. $161.00+.
26. $835.31–.
27. $886.17–.
28. $411.65+.

Page 473.

29. $1550.21–.
30. $118.14–.

---

1. $\frac{7}{8}$.
2. 7$\frac{1}{2}$ feet.
3. 10 mi. 249$\frac{3}{4}$ rd.

Page 474.

18. 63 cents.
19. $5.70.
20. $4.20.
21. $1.50.
22. $2.75.
23. $7.20.
4. $\frac{45}{112}$.
5. 16 %.
6. $7.35.
7. $\frac{19}{27}$.
8. 33$\frac{1}{3}$ times.
9. $\frac{47}{400}$ cent.

Page 475.

10. 1.15; .000625; .0040625; 1$\frac{3}{20}$; $\frac{1}{1600}$; $\frac{13}{3200}$.
11. 9$\frac{3}{8}$ %.
13. 15 times.
14. $530.
15. .0223125 mile.
16. 2.432013984.
17. 3$\frac{3}{4}$ %.
18. $2.64–.
19. $3999.24.
20. 162 days.
21. 16$\frac{2}{3}$ %.
22. $5.69+.
23. $753.
24. $11.28–.
25. $\frac{3}{16}$; $\frac{45}{104}$.

Page 476.

1. 3613 flags; 1$\frac{4}{45}$ yards remaining.
2. 200 men.
3. 317 acres; $14,325.59$\frac{1}{4}$; 45.19\frac{143}{1000}$.
4. $288.75.
5. 7,002,079.003129.
6. 482.96388599.
7. 2.0635+.
8. $10.38–.

Page 477.

9. $56\frac{32}{47}$ %; $13\frac{14}{47}$ %; $30\frac{5}{47}$ %.
10. \$.06703+. \$.04648–; \$.0055; \$.00675; \$.00475; \$.0225; \$.005359+.
12. 8272.08512.
13. 8.522.
14. 3.7857+.
15. 30,000 men.
16. $\frac{1}{4}$.
17. 15 miles.

Page 478.

18. 170 bushels.
19. 1500 letters; 750 letters.
20. \$ 2630.20.

---

1. 1,163,117,-683.002129.
2. 69,092.-80236843.
3. 176.303–; 2.0247–.
4. \$ 66.45–.

Page 479.

5. \$ 42.96–.
6. 2, 3, etc. 18 G. C. D.
7. 25 %.
8. 400 %.
9. $6\frac{13}{20}$.
10. $88\frac{56}{113}$ cents.
11. 7.625.
12. $1\frac{1}{23}$.
13. $\frac{371}{500}$.
14. $7\frac{7}{8}$.

Page 480.

1. $2\frac{1}{11}\frac{1}{3}\frac{0}{11}$.
2. 82.
3. \$ $1.26\frac{1}{2}$.
4. 113.
5. $53\frac{1}{3}$ %; $46\frac{2}{3}$ %.
6. \$ 1.60.
7. 192 planks.
8. $12\frac{1}{2}$ %; $11\frac{1}{9}$ %.
9. \$ 17.76+.
10. \$ 6.
11. $266\frac{2}{3}$ %.

Page 484.

1. 165.
2. 252.
3. 4048.
4. 910.
5. $2594\frac{2}{5}$.
6. $5646\frac{2}{3}$.
7. $1977\frac{4}{7}$.
8. $6373\frac{1}{2}$.
9. $5067\frac{9}{10}$.
10. $8852\frac{1}{4}$.
11. 46,018.
12. 79,520.
13. $25,554\frac{3}{8}$.
14. 106,908.
15. 65,471.
16. $65,635\frac{1}{2}$.
17. 86,855.
18. $27,702\frac{1}{6}$.
19. 37,411.
20. 26,969.
21. 41,382.
22. 58,975.
23. 899,100.
24. 426,000.
25. 16,800.
26. 1,172,880.
27. 4290.
28. 71,400.
29. 67,716.
30. $293,249\frac{3}{8}$.

Page 485.

"All other articles," \$ 73,327,274; \$ 72,122,469; Increase, \$ 26,-976,455.

Page 486.

1. $6975\frac{4}{5}\frac{2}{7}\frac{3}{8}\frac{4}{5}\frac{2}{9}$.
2. $6126\frac{4}{4}\frac{1}{5}\frac{1}{6}\frac{6}{7}\frac{2}{8}$.
3. $11,202\frac{2}{3}\frac{0}{4}\frac{4}{5}\frac{7}{8}\frac{1}{7}$.
4. $11,699\frac{1}{3}\frac{1}{3}\frac{0}{4}\frac{3}{5}\frac{2}{9}$.
5. $786\frac{2}{3}\frac{7}{7}\frac{2}{8}\frac{3}{2}\frac{2}{5}\frac{2}{8}$.
6. $9809\frac{4}{7}\frac{6}{1}\frac{2}{8}\frac{3}{8}\frac{2}{5}$.
7. $49,167\frac{1}{1}\frac{3}{8}\frac{2}{7}\frac{3}{8}\frac{3}{9}$.
8. $9631\frac{5}{5}\frac{2}{9}\frac{7}{7}\frac{4}{8}\frac{4}{4}$.
9. $7288\frac{1}{1}\frac{3}{6}\frac{3}{9}\frac{3}{8}\frac{2}{7}\frac{0}{5}$.
10. $6462\frac{1}{4}\frac{3}{5}\frac{3}{8}\frac{2}{7}\frac{0}{2}\frac{2}{9}$.

---

1. 630 boards; 140 posts.
2. 10 feet.
3. \$ 1.50.

Page 487.

4. 4840 sq. yd.; about 70 yards.
5. 400 rods.
6. 18 sq. ft.; 270 cu. ft.
7. $106\frac{2}{3}$ yards.
8. $86\frac{2}{3}$ bundles.
9. 810 sq. yd.
10. 114 rods.

Page 488.

11. 300 panes.
12. 20,736 gal.
13. 72 cords.
14. 13,440 bu.
15. 7000 cu. yd.
16. 2250 cu. yd.

---

1. \$ 313.31–.
2. \$ 136.50.
3. \$ 500.
4. 8 %.
6. \$ 410.16–.

Page 489.

7. 29.623– feet.
8. \$ $30.87\frac{1}{2}$.
9. $12\frac{1}{2}$ tons.
10. 3.

---

1. \$ .41+; (\$ $.37\frac{1}{2}$).
2. \$ .55+; (\$ .46–).
3. \$ 2.89+; (\$ 2.75+).
4. \$ .37–; (\$ .32).
5. \$ 6.89–; (\$ 6.72).

Page 490.

6. \$ $173.59\frac{1}{2}$ (\$ 173.70).
7. \$ 36.23+ (\$ 36.25+).

8. \$791.42+; (\$791.89–).
9. \$176.74$\frac{1}{2}$; (\$176.85).
10. \$985.41$\frac{2}{3}$; (\$986).
11. \$3.10; (\$2.80).
12. \$2.60; (\$2.48).
13. \$5.64; (\$5.46).
14. \$1.74–; (\$1.69+).
15. \$5.38–; (\$5.29+).
16. \$2.52; (\$2.40).
17. \$5.58; (\$5.40).

Page 491.

18. Proceeds, \$87.34–; (\$87.38+).
19. Proceeds, \$122.66+; (\$122.75–).
20. Proceeds, \$502.05–; (\$502.34+).
21. Proceeds, \$71.65–; (\$71.68+).
22. Proceeds, \$232.99–; (\$233.10+).
23. Proceeds, \$95.58+; (\$95.63–).
24. Proceeds, \$162.65+; (\$162.76+).
25. Proceeds, \$81.91+; (\$81.95+).

---

1. \$24.28.
2. \$10.15+.
3. \$2.52+.
4. \$367.42–;
5. \$10.99+.
6. \$2.13.
7. \$11.23+.
8. \$12.78+.

Page 492.

674.7+ yards. \$613.98+.

Page 493.

1. 371$\frac{7}{8}$.
2. 530$\frac{1}{3}$.
3. 129$\frac{1}{15}$.
4. 127$\frac{25}{48}$.
5. 517$\frac{1}{3}$.
6. 289$\frac{9}{16}$.
7. 233$\frac{38}{49}$.
8. 260$\frac{35}{64}$.
9. 734$\frac{13}{24}$.
10. 1078$\frac{1}{8}$.
11. 85$\frac{1}{4}$.
12. 89$\frac{5}{6}$.
13. 264$\frac{16}{35}$.
14. 361$\frac{13}{27}$.
15. 337$\frac{17}{40}$.
16. 674$\frac{11}{16}$.
17. 81$\frac{7}{12}$.
18. 401$\frac{1}{24}$.
19. 421$\frac{37}{50}$.
20. 434$\frac{1}{3}$.

Page 495.

1. $\frac{1}{32}$; .01020201; 10.01.

Page 496.

2. \$4.90.
3. \$115.54–.
4. \$5.37$\frac{1}{2}$; \$224.07–.
5. \$728; \$950$\frac{6}{7}$.
6. \$25,000.
7. 20 %.
8. The first; 5 % more.
9. 1$\frac{2}{5}$ acres.
10. .0006216.
11. \$1200; \$4608; 25 %.
12. \$6$\frac{2}{7}$.

Page 497.

13. \$55.91–.
14. 4$\frac{7}{80}$; 6$\frac{22}{63}$ days; \$85,333$\frac{1}{3}$.
15. \$173,668; \$201.880618.
17. \$1453.76.
18. 4.23+ times; 64.68+ inhabitants; 6.07– inhabitants; \$15,124,032.
19. 300 sheep.
20. \$8572.20.

Page 498.

21. 257 sq. yd.
22. 6$\frac{1}{2}$ %; \$287.50.
23. 52 weeks.
24. 33,750 qt.; 2250 qt. average.
25. \$153.75.
26. \$33,519.20.
27. \$85.
28. 70 feet.
29. \$187.36.
30. 9$\frac{1}{11}$ %; 8$\frac{1}{3}$ %; 100 %.

Page 499.

1. \$46.80.
2. \$18.40.
3. 17 lb. 11 oz. 5 pwt. 19 gr.

Page 500.

4. 23,220 gr. gold; 2322 gr. silver; 258 gr. copper.
5. 24 spoons.
6. \$2800.
7. $\frac{144}{175}$; 109$\frac{5}{7}$ %.
8. \$105.
9. 27+ cents.
10. 23$\frac{3}{11}$ gal.
11. 704 sq. ft.
12. 1$\frac{5}{6}$ cu. ft.
13. 1316$\frac{17}{24}$ lb.

14. $13,405\frac{1}{11}$ gal.; 1440 bu.
15. 180 %.
16. $82\frac{6}{7}$ %.
17. $ 309.42.

Page 501.

18. 300 pounds.
19. A, $43\frac{1}{2}$ %; B, 36 %; C, $20\frac{1}{2}$ %.
20. 672 yards.
21. $ 315.

Page 502.

1. 101.5901+.
2. $71\frac{1}{3}$.
3. 72.
4. 390.
5. 3 yards.
6. $ 4.
7. $20\frac{5}{8}$ %.
8. $ 81.25.

---

1. $11\frac{4}{9}$ rolls.
2. 1.5548–.
3. $5333\frac{1}{3}$ bu.
4. $ $3.02\frac{14}{31}$.

Page 503.

5. $ $60.62\frac{1}{2}$.
6. 70 cents.
7. $11\frac{593}{2120}$ cts.
8. 17 spoons.
9. $48\frac{3}{34}$ bu.

---

1. $ 49.01–; ($ 48.99–); $ 48.88–; ($ 48.88–).
2. $ 175.94–; ($ 175.94–).
3. $ 350.55+; ($ 350.55+).

Page 504.

4. $ 846.26–; ($ 845.77+).
5. $ 724.85+; ($ 724.69–).

---

1. 38 rd. 3 yd. 11 in.
2. 113 rd. 3 yd. 1 ft. 6 in.
3. 175 rd. 1 yd. 1 ft. 6 in.
4. £ 16 13 s. 4 d.
5. 21,090 d.
6. £ 26 5 s.
7. $ 27.37+.
8. £ 108 4 s. 6 d.
9. £ 45 18 s. 8 d.
10. $773\frac{7}{16}$ oz.
11. $ 175.

Page 505.

12. $ 123.
13. $ 72; $ 119.25; $ 92.25.
14. $ 450; $ 750; $ 600.
15. 24 days.
16. $2\frac{2}{3}$ hours; 48 miles from A.
17. $ 45; $ 135; $ 270; $ 450.
18. $ 237; $ 189; $ 114.
19. 7 days.
20. 240 eggs.
21. 5 months.
22. 24 weeks.

Page 506.

23. $ $37,033.13\frac{1}{3}$.
24. 10 inches.
25. $ 360.

---

1. 125,422,928,-368.01.
6. 12 T. 6 cwt. 2 qr. 13 lb.

Page 507.

7. 300.
8. $11\frac{2}{3}$ days.
9. $3416\frac{14}{25}$ lb.
10. £ 3068 15 s. 10 d.

---

4. 302.
6. $11\frac{1}{20}$.
7. $9\frac{3}{5}$.
9. 1250.

Page 508.

10. $10\frac{212}{253}$.
11. $37\frac{1}{3}$ pieces.
12. 43,200 min.
13. $24\frac{8}{9}$ inches.
14. 362.16 mi.
15. $ 6.72–.
17. $ 4761.90+.
18. $ $8.28\frac{4}{7}$.
19. $ 870.48.
20. A, $ 50; B, $ 90; C, $ 110.
21. $ 380.
22. $ 158.40.

Page 509.

1. $ 554.23.
2. $ 171.20.
3. $ $1782.67\frac{1}{2}$.
4. $ 158.40.
5. $ 28.50.
6. $ $60.56\frac{1}{4}$.
7. $ 50.85.
8. $ 13.
9. $ 7.45.

Page 510.

10. $ 27.84–.
11. 40 and 10 %; $ 2 difference.
12. $ 60; 40 % discount; 60 % net.
13. 52 %.
14. $ 100.
15. 72 %.
16. 72 %.

---

1. $ 81.
2. 75 %.
3. $\frac{1}{10}$ %.
4. 627.5.
5. .0075; $\frac{3}{400}$.
6. $77\frac{7}{9}$ %.
7. $ $2.63\frac{1}{8}$.
8. $ 24.
9. 600.

Page 511.

10. \$3575.
11. $6\frac{3}{4}$ %.
12. \$65.
13. \$101.50.
14. \$375.
15. \$10.96–.
16. \$23.14–.

Page 512.

1. \$19.
2. 35.
3. $2624\frac{2}{3}$ yards.
4. \$27.20 gain.
5. \$20.
6. 1767.5.
7. \$5.49–.
8. $35\frac{5}{8}$ %.
9. \$1431.27.

Page 513.

10. \$30.
11. $4\frac{5}{31}$.
12. .00007865.
13. 108.86– bu.
14. 2 bu. 1 pk. 4 qt.
15. \$7425.
16. 25 %.
17. \$49.31+.
18. \$594.50; (\$594.80).
19. 25 %; $32\frac{1}{7}$ %; $42\frac{6}{7}$ %.
20. \$$1.56\frac{1}{4}$.
21. Loss, \$177.
22. \$3.60.
23. $.01\frac{2}{3}$.

Page 514.

24. 15,203.
25. \$$4.16\frac{2}{3}$.
26. 80 hours.
27. 250 cu. ft.
28. \$$7.12\frac{1}{2}$.
29. \$18.
30. \$77.95; (\$79).

---

1. $\frac{9x}{80}$.
2. \$1600.
3. $255x$.
4. 3 %.
5. \$2000.

Page 515.

6. 5 years.
7. $150x$.
8. 6 %.
9. $1875 + \frac{375x}{32}$.
10. 682.90+ $\frac{6829x}{3200}$.
11. $\frac{21x}{2000}$; $\left(\frac{x}{100}\right)$.
12. $\frac{11x}{30}$; $\left(\frac{x}{3}\right)$.
13. $\frac{x+3}{20}$; $\left(\frac{x}{20}\right)$.
14. $\frac{41x}{60}$; $\left(\frac{2x}{3}\right)$.
15. $\frac{5997-x}{10}$; $\left(600-\frac{x}{10}\right)$.
16. $\frac{477x}{400}$.
17. $\frac{1969x}{2000}$; $\left(\frac{197x}{200}\right)$.

Page 517.

1. $121\frac{1}{2}$ cu. ft.
2. \$162.

Page 518.

3. \$126.
4. \$85.50.
5. 16.8 tons.
6. 2 pieces, 12×12; 2 pieces, 12×14; 2 pieces, 14×14.
7. $44\frac{48}{54}$ lb.
8. 5280 lb.
9. Outside dimensions, $14 \times 14 \times 14$; 2744 cu. in. wood and marble; 1728 cu. in. marble; 1016 cu. in. wood.
10. 8 times; $\frac{1}{4}$ ton; $6\frac{3}{4}$ tons.

Page 520.

1. 5 %.
2. 2 years.
3. \$96.
4. 3 %.
5. \$1.43–.
6. \$36.
7. 1 yr. 6 mo.
8. \$42.17+.
9. \$5000.
10. 6 %.
11. 1 yr. 9 mo. 2 da.
12. \$144.
13. \$$83.26\frac{1}{2}$.
14. 3 %.
15. 2 yr. 1 mo. 7 da.
16. \$80.
17. \$2181.99–.
18. \$72.

Page 521.

19. 5 mo. 23 da.
20. 6 %.
21. \$16.92.
22. \$402.22.
23. 37 days.
24. 5 %.
25. 11 mo. 29 da.

Page 522.

1. 570,073,438,-098.53.

Page 523.

6. .37875.
7. \$70.20.
8. \$281.25.
9. \$15,000.
10. \$67.71–.

Page 524.

2. \$41.99+.
3. \$951.13+.
4. \$112.74–.

5. $119.43–.
6. $13.91–.

Page 525.

7. $147.19–.
8. $8.10–.
9. $52.33–.
10. $1005.50.
12. $967.78.
13. $21.79–.
14. $16.53–.
15. $1274.21–.

Page 526.

16. ($860.50–).
17. $24.74–; ($23.94).
18. $761.06–; ($761.45).
19. $43.99–.
20. $786.39–.
21. $65.40.
22. $625.03+.
23. $98.49+.
24. $993.27+.
25. $61.68–.
26. $252.37+; ($252.52–).
27. $13.09–; ($12.87½).
28. $486.10–; ($486.43–).
29. $2.33+; ($1.94+).
30. $989.67–. ($990).
31. $1938.43–.
32. $8.06+.

Page 527.

33. $1473.52–.
34. $6.55+.
35. $278.16–.
36. $1.23–.
37. $196.64+.
38. $1.10.
39. $389.60.
40. $6.11+.
41. $4.09+.
42. $56.32–.
43. $16.72–.
44. $95.43–.
45. $15.71+.
46. $594.30; ($594.60).
47. $10.73+; ($10.38+).
48. $793.73+; ($794.13+).

Page 528.

49. 45 cents; (nothing).
50. $968.83+; ($970).

---

1. 470,952.

Page 529.

5. 8614.20375.
6. 2 4/7.
7. $110.85+ gain.
8. $16.25.
9. 45 T. 5 cwt. 2 qr.
10. $43.33⅓.

Page 530.

4. $196.17.
5. $600.01+.
6. $1506.12.
7. 4269.22+ francs.
8. $1563.55–.
9. $1563.55–.
10. $1547.37.
11. 18x/25

Page 531.

12. 400 – 4x.
13. $500.
14. 10%.
15. Same.

---

1. 688,965,549,-176.65.

Page 532.

6. .0019.
7. 32 cents.
8. $46 gain.
9. 60 cents.
10. $16.98.

Page 535.

1. $106.33+; ($106.38).
2. 7%; (7 7/30%).
3. 24 days.
4. $1200.
5. $4.68; ($3.90).
6. 5%.
7. 72 days.
8. $1200. ($1260).
9. $304.26–. ($304.05+).
10. Apr. 8, 1894.
11. $853.27+; ($853.71–).
12. $56.62½; ($57.50).

Page 536.

3. 2 4/15.
4. 375.
5. $288.
6. $922.20.
7. $573.47½.
8. $1700.
9. $1400; $1200.
10. $1337.

Page 539.

1. $165.37½.
2. $1453.42–.
3. $50,625.50.

Page 540.

4. $893,615,929.
7. 284,106,409,-352.02.
8. 45.29–%.
9. $223,852.8835.
10. 107 2/5 cents.

Page 541.

11. $20,000.
12. $728.17½.
15. Profits, $4414.10.
16. ¾; ⅛; 3/16; ⅖.

Page 542.

17. $4752.
18. 37½%; 25%; 12½%; 25%.
19. $81.90.

20. $299.88–.

---

1. 1,287,400.
2. 3,370,185.
3. 598,969.
4. 2,883,736.
5. 816,669.
6. 5,127,460.
7. 2,455,038.
8. 42,327,198.
9. 2,513,420.
10. 22,944,747.

Page 543.

11. 857,712.
12. 6,482,112.
13. 1,230,828.
14. 921,776.
15. 3,460,704.
16. 5,888,304.
17. 1,460,025.
18. 10,563,960.
19. 3,911,322.
20. 2,982,840.
21. 714,186.
22. 3,277,719.
23. 456,375.
24. 174,600.
25. 362,250.
26. 104,787½.
27. 128,550.
28. 625,975.
29. 213,966⅔.
30. 413,866⅔.
31. 8650½.
32. 10,757⅔.
33. 21,873⅓.
34. 24,292½.
35. 485,072.
36. 167,276.
37. 24,418½.
38. 24,130⅔.
39. 651,329 $\frac{5}{11}$.
40. 1,932,560.
41. $277,133.11.
42. $60,887.10.
43. $48,554.08.

Page 544.

44. £7 5 s. 9 d.
45. 11 yd. 1 ft. 11 in.
46. 12 bu. 2 pk. 5 qt.
47. 11.76+.
48. 13.72+.
49. 14.41+.
50. 13.34+.
51. 19.05–.
52. 22.30–.

---

1. 79.98 %.
2. 5.02 %.
3. 4.28 %.
4. 3.83 %.
5. 3.81 %.
6. 2.44 %.
7. .54 %.
8. .10 %.
Total, $872,-270,283.

Page 545.

1. 237.49 %.
2. 234.60 %.
3. 563.36 %.

Page 546.

4. 0.04 %.
5. 25 %.
6. 20 %.
7. $24; $30.
8. $3.
9. £73 3 s.
10. 6⅔ days.

---

2. $58.93–.
3. 6 %.
4. $3400; $3570.

Page 547.

5. $1.30–.
6. $10,000.
8. $264.25.
9. $135.40–.

Page 548.

1. $107.65+.
2. 6 yr. 6 mo.
3. 7½ %.
4. $750.
5. $882.
6. $97.
7. $6; $144.
8. 33⅓ %.
9. $8960.
10. $360.

Page 549.

11. $50.
12. 20.
13. $61.87½.
14. $1150.
15. $\frac{5}{49}$.

---

1. 4 $\frac{22}{80}$.
3. $\frac{5}{11}$.
4. 16 $\frac{23}{36}$.
5. $\frac{22}{155}$.
6. 8 $\frac{12}{35}$.
7. 73; 240.
8. 4 $\frac{35}{48}$.
9. $\frac{2}{11}$; $\frac{8}{13}$.
10. Largest, $\frac{4}{9}$; smallest, $\frac{2}{3}$ of $\frac{5}{8}$.

Page 550.

11. 2 $\frac{8}{99}$.
12. 2 da. 15 hr. 50 min. 35 sec.
14. 11 $\frac{79}{147}$.
15. 1 $\frac{1}{12}$.
16. $\frac{87}{80}$.
17. 180.
18. 3¾.
19. $\frac{26}{119}$.
20. 36⅔.
21. 1 $\frac{2}{13}$.
22. 3 hr. 22 min.
23. 85⅓ rods.
24. 1 $\frac{31}{36}$.
25. 15⅜.

---

1. 4½ feet.
2. 120 yards.

Page 551.

3. 8750 sq. yd.
4. $3000 + 30x$; 80 yards.
5. $100x$; 40 yards.
6. $60x + 1200$; 80 yd. and 120 yd.
7. 10,000 sq. yd.
8. 1920 flag-stones.
9. $9.26–.
10. 46⅔ yards.

11. $90\frac{11}{64}$ lb.
12. $35\frac{5}{12}$ sq. ft.

Page 552.

13. 10 ft. 8 in.
14. 950 bushels.
15. 810 gallons.
16. $416.
17. 10 miles.
18. 615 cords.
19. $8\frac{409}{1152}$ lb.
20. $27\frac{709}{4608}$ lb.
21. $706\frac{2}{3}$ sq. yd.
22. $722\frac{2}{3}$ sq. yd.

Page 553.

23. $690\frac{2}{3}$ sq. yd.

---

1. $24\frac{11}{36}$ yards.
2. 28 cents.

Page 554.

3. 84 days.
4. 9 days.
5. $49.
6. 180 bushels.

Page 555.

8. 60 bushels.
9. 720 pounds.
10. 4600 sheets.
11. $312.
12. 8 da. 4 hr.
13. $28.80.
14. 96 rods.
15. 50 days.
16. $45.
17. 120 men.

Page 556.

18. 360 men.
19. 10 hours.
20. $1.20.

---

2. 329.
3. $$7\frac{1}{7}$.
4. 10 bushels.
5. $4\frac{2}{45}$.
6. 12 feet.
7. 90 cents.

Page 557.

8. 1680.
9. 85,800.
10. Loss, $93.75.
11. $63\frac{7}{11}$ %.
12. $2.35+.
13. 99.99.
14. 14 feet.
15. $69.05–.
16. 1050 acres.
17. $9792\frac{4}{5}$ lb.
18. $275.69–.

Page 558.

20. $873; $184;
21. $143.
22. Estate, $45,000; son's share, $10,000.
23. $10.86+.
24. $6\frac{4}{9}$ days.

---

1. $78; $104.
2. $36; $17; $48.
3. $25; $20.
4. $16.20; $13.80.
5. 45 lb. gold; $4\frac{1}{2}$ lb. silver; $\frac{1}{2}$ lb. copper.

Page 559.

6. Saltpeter, 54 lb.; sulphur, $7\frac{1}{5}$ lb; charcoal, $10\frac{4}{5}$ lb.
7. $20.25; $18.63; $15.12.
8. 864 bales 540 bales; 396 bales.
9. $12; $20;
10. $60; $72.

---

2. 326.9843.
3. 6408.
5. 4.2633; 1.405712.

Page 560.

6. 16.5393.
9. 28.165; 305.36721642.
10. £.3375.
12. Sum, 8.08690625.
14. .019104141.
15. .05; .03965.
16. 1093.3524.
17. 2.8.
18. .215625 m.
19. 12 da. 20 hr. 31 min. 12 sec.
20. 5934.47– meters.
21. 6 quarts.
22. .8125 pound.
23. .25 rod.
24. 100 links.
25. .1 acre.

Page 561.

£131 3s. $8\frac{1}{4}$– d. 368.90 marks; $30.73–.

Page 562.

1. $40.
2. 8 %.
3. $150; 150 %.

Page 563.

4. $30.
5. $5\frac{1}{4}$ %.
6. $435.
7. Same.
8. 5 %; $2\frac{1}{2}$ %.
9. $2267.26+.
10. $2.85.
11. $4\frac{4}{19}$ %.
12. $3\frac{101}{233}$ %.

Page 564.

17. $$459.37\frac{1}{2}$.
18. $24,000; $66\frac{2}{3}$ %.
19. $1\frac{1}{2}$ %.
20. $18,000.
21. 101 rd. 2 yd. 1 ft. 6 in.
22. 35 shares; $87.50.

23. 37 rd. 4 yd. 11 in.
24. $106.12.

Page 565.

25. 229 rd. 4 yd. 2 ft. 6 in.
27. $428.40–; $53.40–.
28. $119.10+.
29. $149.14+.

Page 566.

30. 51 cents.

---

1. 15 %.
2. $400.
3. $102.50.
4. $2.70.
5. $40.
6. $1.60.
7. 1 hour.
8. $20.
9. $12.

Page 567.

10. $20 loss.

---

4. 1.
5. 962 feet.
6. $6.50 gain.
7. $3.54.

Page 568.

8. $509.25.
9. 19 T. 62 lb. 8 oz.
10. 24 cents.

Page 570.

1. $45.
2. 10$\frac{7}{8}$ acres.
3. 452.32\frac{1}{2}$.
4. $4.50.
5. $1.50; $5.40; 20 %; 20 %.
6. $6000.
7. 3333\frac{1}{3}$.

Page 571.

8. $212.51–; $439.79–.
9. $5250; 18 %.
10. $402.50; 35 %.
11. $3330; $4.25.
12. $9.30.
13. $201.60; $125.93–; $722.40+.
14. 2$\frac{7}{8}$ pounds.
15. 98$\frac{26}{163}$ lb.
16. $71.25.

Page 572.

17. $85.
18. $594.20; ($594.50).
19. 20 %.
20. $11,356.01$\frac{1}{2}$
21. 700.
22. $38.59+.
23. 12.
24. 1006.13\frac{1}{2}$.
25. 63 sq. ft.; 10$\frac{16}{25}$ rods.

26. $57,600.
27. 2.37\frac{1}{2}$.
28. $387.36.

Page 573.

29. $1000; 100 %.
30. $5487.

---

2. 688,450.
3. 3$\frac{28}{63}$; 18$\frac{26}{27}$.
4. .140625; 88.088.
5. 5.820068; 1000.

Page 574.

6. 400 yards.
7. 2\frac{3}{8}$.
8. $10,500.
9. 7400 inhabitants.
10. $10,500.

Page 575.

1. (*a*) 1575.355671; (*b*) .028376604.
2. 49,999.74999.
3. (*a*) 73; (*b*) 2016.
4. $72.
5. $53.95+.
6. $160; $140; $240.
7. $262.50.

Page 576.

1. $3481.07+.
2. 1874.77\frac{1}{2}$.
3. $2461.76+.
4. $1000.50+.
5. $1632.

Page 577.

6. $1845.90+.
7. $946.04–.
8. $632.65–.
9. $985.22+.
10. $326.34.

---

1. 9 hr. 45 min.
2. 14$\frac{1}{2}$ minutes past 5 P.M.
3. 4 ft. 6 in.

Page 578.

4. 748 plants; 754 plants.
5. 20 da. 6 hr. 40 min.
6. 5544 revolutions; 1320 revolutions.
7. 28 mi. 130$\frac{10}{11}$ rd.
8. 1388$\frac{8}{9}$ mi.
9. .15708 foot.
10. 694$\frac{4}{9}$ miles.
11. 3211$\frac{15}{32}$ mi.

Page 579.

12. 1104$\frac{27}{32}$ mi.

---

1. $828.45. ($828.87).
2. $397.30; ($397.50).

3. \$554.40; (\$554.68).

Page 580.

4. \$625.33+.
5. $\frac{3959x}{4000}$; $\left(\frac{3961x}{4000}\right)$.
6. $\frac{11979-2x}{10}$; $\left(\frac{11985-2x}{10}\right)$.
7. $\frac{7956+8x}{5}$; $\left(1592+\frac{8x}{5}\right)$.
8. \$1200; (\$1199.39+).
9. 30 days; (33 days).
10. \$1.50 discount per \$1000. (\$2 discount per \$1000).

Page 581.

1. 56°.
2. 2 hr. 29 min. 12 sec.
3. 5 hr. 50 min. 20 sec.; 7 A.M.
4. 37° 30′.
5. 1 hr. 14 min. 52 sec.

Page 582.

6. 33° 30′ east longitude.
7. $39\frac{1587}{5057}$ miles per hour.
8. 1800 lb.
9. \$1000.80.
10. 8 yd. 6 in.
11. 15 ft. 7 in.
12. 257 bu. 2 pk. 2 qt.
13. 20 spoons.
14. 16° 40′.

Page 583.

1. \$67.46; \$168.65; \$236.11.
2. 1 lb. 3 oz.
3. .93.
4. 1 yr. 4 mo. 26 da. nearly.
5. 16 days.
7. $\frac{12}{253}$.

Page 584.

8. \$630.76–.
9. $46\frac{2}{3}$ inches.
10. .0005207.
11. $\frac{50}{121}$ acre.
12. 30 min. 50 sec.
13. \$225.67–.
14. Dec. (10) 13, 1888.
15. Dec. 29, 1892.
16. A, $107\frac{2}{3}$; B, $92\frac{2}{3}$; C, $119\frac{2}{3}$.
17. 3 bu. 3 pk. 3 qt.
18. \$$7.87\frac{1}{2}$.
19. $19\frac{898}{1225}$.
20. $\frac{1}{30}$; $\frac{1}{16}$; $\frac{61}{900}$; $\frac{281}{40000}$.
21. 1.299609375.
23. $109\frac{11}{18}$ feet.

Page 585.

24. A, 4340 votes; B, 5551 votes.
25. \$133.32–.

---

1. \$880.86+.
2. \$1229.01–.

Page 586.

3. \$193.70+.
4. \$446.33–.
5. 4188.48+ marks.
6. 8490.44– francs.
7. £307 7 s. $6\frac{1}{2}$ d. nearly.
8. \$2050.72+.
9. \$4138.97+.

Page 587.

10. \$908.87–.

Page 589.

1. \$224.46+.
2. \$261.19–.

Page 590.

4. \$772.37–.
5. \$899.91+.

---

3. 12.
4. $2\frac{1}{16}$.
5. \$1089.19.

Page 591.

6. \$150; \$7500.
7. 61%.
8. 15,544,041.45 francs.

---

1. \$3640.
2. \$4; $6\frac{2}{3}$%.

Page 592.

5. \$2 loss.
7. \$701.53–.
8. \$13.65.
9. $8\frac{1}{3}$%.
10. \$$200.02\frac{1}{2}$.

---

1. \$137.61+.

Page 593.

2. \$12.39–.

---

1. \$8500.
2. \$282.25.
3. 23 min. $53\frac{1}{3}$ sec.
4. \$17.82.
5. 11 feet.
6. 12 rods.
7. \$56.
8. $1\frac{1}{4}$ cents.
9. \$300.
10. 3600 yards.

Page 594.

11. \$497; (\$497.25).
12. 5%. 13. $\frac{3}{10}$ ct.
14. \$700.
15. \$375.10–.

---

1. \$1,181,021.50.
2. \$1,037,124.44.
3. \$214,854.74.
4. £14 7 s. 2 d.
5. 25 bu. 1 pk. 4 qt.
6. 23 rd. 2 yd. 1 ft. 6 in.

Page 595.

7. \$249,981.53.
8. \$318,808.78.
9. \$202,722.44.

Page 597.

1. \$11.
2. \$85.25.
3. \$95.48–.
4. 24 cents.
5. \$18,228.
6. 150 sq. ft.
7. \$39,700.
8. $6\frac{1}{2}$%.
9. 138 feet.

Page 598.

10. \$$48.43\frac{3}{4}$.
11. 44%.
12. \$$33.12\frac{1}{2}$.
13. 114 sq. yd.
14. \$594.
15. \$2090.25.
16. $\frac{25}{28}$.
17. $26\frac{2}{3}$%.
18. \$363.
19. 6400 lb.; \$11.40.
20. \$870.62–; (\$870.14–).
21. \$$4.31\frac{1}{4}$.
22. $45\frac{315}{764}$ ¢.
23. \$150; \$225.

Page 599.

24. \$33.60; $15\frac{3}{16}$ cords.
25. $2\frac{1}{2}$; $1\frac{7}{10}$.
26. 10%.
27. \$653.48–; (\$654.36).
28. \$$2688\frac{8}{9}$.
29. \$3682.19–; (\$$3684.37\frac{1}{2}$).
30. \$4940.28.

Page 600.

3. 135.62.
4. $12\frac{80}{181}$.
5. 13 feet.
6. 16 T. 19 cwt. 3 qr. $10\frac{1}{2}$ lb.
7. \$1235.21–.
8. $88\frac{1}{2}$.
9. 25 cents.
10. \$16.48–.

Page 601.

1. $1\frac{1}{4}$ yards.
2. 18.72 feet.
3. \$78.75.
4. 8 ft. 4 in.
5. \$14,910.75.
6. $108\frac{17}{36}$ sq. ft.
7. 44.17875 sq. ft.
8. 2970 bu.
9. 252 gallons.
10. 146 sq. yd.
11. 3 hr. 12 min.; 4 hr. $6\frac{6}{7}$ min.; 1 hr. 48 min.;

Page 602.

13. $40\frac{1}{2}$ yards; \$$44.02\frac{1}{2}$.
14. $1\frac{1}{2}$ ft.; 108 sq. ft.
15. 984 sq. ft.; 71 ft.
16. \$11,200.
17. \$56.

Page 603.

1. .00007722.
2. 1.485135.
3. .450522.
4. 7.70904.
5. .0712111.
6. .0048393.
7. .63672.
8. .0374715.
9. .8220672.
10. .00004768.

---

1. .68515625.
2. 589.84.
3. 153.6.
4. 3265.
5. 50.
6. .064.
7. .00002375.
8. .0115.
9. 79,000.
10. 219.32.

---

3. 4.120275.

Page 604.

4. $1\frac{1}{15}$.
5. 7 fur. 16 rd. 3 yd. 1 ft. 9.888 in.
6. \$4.13.
7. \$1900.
8. 3 T. 7 cwt. 2 qr. 17 lb. $4\frac{1}{9}$ oz.
9. \$15,000.
10. \$6.98–.

Page 605.

2. 1,345,595.
3. $3\frac{3}{20}$.

Page 606.

4. 80 miles.
5. .49184.
6. \$9.27; \$12.36.
7. 468 bricks.
8. \$46.08.
9. 55 feet.
10. \$5700.
11. \$1700.
12. \$77.14.
13. \$44.48.
14. \$215.36+.
15. \$747.44–.

Page 609.

1. 14.
2. 16.
3. 18.
4. 24.
5. 26.
6. 36.
7. 35.
8. 42.
9. 44.
10. 51.

11. 53.
12. 54.
13. 61.
14. 63.
15. 72.
16. 75.
17. 83.
18. 84.
19. 91.
20. 95.

---

1. 7568.
2. 5107.
3. 6008.
4. $6285\frac{122064}{607482}$.
5. 6098.
6. 3007.
7. 98,640.
8. 75,064.
9. 70,921.
10. 78,905.

Page 610.

1. \$183.27–.
2. £150 8s. 4d.

Page 612.

1. $\frac{128}{167}$.
2. $\frac{4}{5}$.
3. $\frac{88}{414}$.
4. $\frac{128}{107}$.
5. $\frac{525}{556}$.
6. $\frac{7}{9}$.

Page 613.

7. $\frac{1250}{5927}$.
8. Equal.
9. $\frac{7}{20}$.
10. $\frac{22}{3}$.

---

1. \$389.61–.
2. \$4330.80+.
3. 4 lb. tea, 27 lb. coffee.
4. \$458.16$\frac{2}{3}$.
5. \$8.25.

Page 614.

6. Dec. (12) 15; (27) 30 da.; \$1275.88+; (\$1276.00+).
7. \$77.61–.
8. \$332.50; \$525.
9. 18 men.
10. 256 barrels.

---

1. \$2167.09–.
2. \$543.16–.
3. \$1335.23+.
4. \$911.66–.

Page 615.

5. \$784.54–.
6. \$724.03+.
7. \$1586.88–.
8. \$1008.10+.
9. \$2607.48–.
10. \$718.00–.
11. $4\frac{1}{2}$%.
12. \$600.
13. 4 yr. 6 mo. 9 da.
14. 2 yr. 8 mo. 1 da.
15. \$600.
16. 4%.
17. 63 days.
18. \$400.
19. \$295.35. (\$295.50).
20. 7%.
21. \$4.10; (\$4).
22. \$2750.
23. 90 da.
24. 18 da.
25. \$475. (\$474.76–).

Page 616.

1. 567.
2. 915.
3. 144,200.
4. 25,758.
5. 2,114,000.
6. 86,526.
7. 106,000.
8. 374,625.
9. 19,800.
10. 7312.
11. 466,000.
12. 50,133.
13. 98,500.
14. 4180.
15. 423,000.
16. 40,096.
17. 419,904.
18. 310,050.
19. $230\frac{5}{17}$.
20. $227\frac{1}{2}$.

---

1. 12 : 6 P.M.

Page 617.

2. 9 : 36 A.M.
3. 142° 55′ 30″ West.
4. 52° 36′ East.
5. 5 : 16 A.M.

Page 618.

6. 8 : 24 A.M.
7. 12 : 1 : 20 P.M.
8. 143° 3′ West.
9. 82° 40′ West.
10. 11 : 33 : 12 A.M.
11. 3 : 37 : 20 P.M.
12. 88° 4′ East.
13. 32° 34′ East.
14. 2 : 45 : 42 A.M.
15. 23° West.

---

3. $\frac{13}{21}$.
6. $\frac{15}{32}$.
7. $\frac{18}{47}$.
8. $\frac{23}{62}$.
9. $\frac{77}{95}$.
10. $\frac{89}{93}$.
11. $\frac{48}{65}$.
12. $\frac{55}{71}$.
13. $3\frac{1}{2}$.
14. $3\frac{1}{3}$.
15. $1\frac{5}{7}$.
16. $2\frac{2}{15}$.
17. $2\frac{11}{18}$.
18. $3\frac{6}{25}$.
19. $12\frac{1}{4}$.
20. $16\frac{1}{4}$.
21. $10\frac{1}{8}$.

Page 619.

1. 666,862,394,588.21.
5. $7\frac{11}{18}$.
6. \$1144.73–.
7. \$5166.69+; \$283.31–.

Page 622.

Table.
1. 63.30%.
2. 13.14%.

3. 8.58 %.
4. 3.83 %.
5. 3.41 %.
6. 2.09 %.
7. 1.11 %.
8. .72 %.
9. 3.82 %.
All others, $ 26,632,801.

Page 623.

1. 63,805,573.34.
2. $ 869,109.89.
3. 524,658,551,-760.
4. $986\frac{3585}{7428}$.
7. (*a*) $1104.02+.
(*b*) $1101.56+; $1116.44+.

Page 624.

8. $9862.33+.
9. $\frac{1}{7}$.
10. $ 72.06.

---

1. 80.
2. 4.
3. $17\frac{1}{2}$.
4. 28.
5. 5.
6. 7.
7. 36 cents.
8. 70 cents.
9. 10 bottles.
10. 4 men.

Page 625.

3. 12.
4. 8.
5. 70.
6. $15\frac{147}{182}$.
7. $\frac{35}{72}$.
8. $\frac{1}{7}$.
9. $1\frac{3}{5}$.
10. $\frac{9}{32}$.
11. 1.
12. $1\frac{43}{45}$.
13. 6.

Page 626.

2. $ 625.92.
3. 3968 revolutions.
4. 27 kilometers.

Page 627.

5. 770 meters; 42 kilos.
6. $ 504.
7. $8\frac{1}{2}$ days.
8. $ 82.50.
9. 14 yards.
10. $ 2.45.

Page 628.

21. 40 and 10 %.
22. 50 and 10 %.
23. 30 and 10 %.
24. 30 and 5 %.
25. 30 and 15 %.
26. 50 and 10 %.
27. 20 and 50 %.
28. 40 and 5 %.
29. 60 and 10 %.
30. 40 and 15 %.

---

1. $ 135.
2. 10 %.
3. $ 150.
4. 10 %.
5. $ 8.96.

Page 629.

6. 60 cents.
7. 20 %.
8. $ 150.
9. $ 120.
10. 20 %.

---

1. 254.
2. 27.1.
3. 1.37.
4. 26.8.
5. 3.76.
6. .838.
7. 16.27.
8. .4876.
9. .306.
10. .069.

Page 630.

1. $ 6.24.
2. 4 hr. 24 min.
3. $ 22.50.
4. 150 barrels.
5. $ 153.60; $ 192; $ 230.40.
6. 1740 tiles.
7. $ 600,000.
8. $4\frac{1}{4}$ %.
9. 5.0004.
10. 2880 gal.
11. .013972.
12. $ 2351.25.

Page 631.

13. $\frac{47}{375}$.
14. 1 mi. 95 rd. 1 yd. 6 in.
16. £ 23.
17. $ 6000; $ 14,000.
18. 50 %.
19. 4 %.
20. $ 372.
21. $1\frac{23}{37}$ days.

---

1. $7\frac{811}{952}$.
2. $\frac{163}{315}$.

Page 632.

3. $22\frac{7}{144}$.
4. $\frac{4}{9}$; 4 s. 6 d.
5. $13\frac{88}{144}$.
6. $\frac{3}{800}$ acre; 218 lb. 12 oz.
7. $\frac{8}{15}$; $\frac{17}{35}$.
8. $\frac{1}{8}$.
9. $42\frac{65}{99}$.
10. 1.
11. 10.
12. £ $\frac{3}{8}$; $\frac{3}{10}$ da.
13. 8.
14. 13 s. $3\frac{11}{21}$ d.
15. $\frac{881}{1247}$.
16. $3\frac{3}{28}$.
17. 370.
18. $ 520.
19. $ 918.75.
20. £ 1060 18 s. 9 d.

Page 633.

1. $49\frac{181}{256}$ cents; $65\frac{163}{256}$ cents.
2. $ 13,227.50.
3. 104 days.
4. $ 591.09+.
5. $\frac{225}{1024}$.
6. $ 782,300.
7. $ 100.
8. $ 1005.50.
9. 60 %.

Page 634.

1. $ 100.02.
2. $ 500.
3. $ 1.75 discount per $ 1000.
4. $ 700.
5. $ 198.80.
6. 33 days.
7. Sight.
8. 7 %.
9. Par.
10. 5 %.

---

1. 23 inches.
2. 1 ft. 11 in.
3. 120 rods; 2640 yards.

Page 635.

4. 66 feet; 7.92 inches.
5. 15 inches.
8. $12\frac{1}{2}$ inches.
9. 24 inches.
10. 5 inches.

Page 636.

1. $ 8.40; ($ 8).
2. $ 629.20.
3. $ 650.39–.
4. $ 225.16+.
5. $ 343.61–. ($ 343.43+).
6. $ 484.75; ($ 485).
7. $ 60.63+.
8. $ 2.

Page 637.

10. 5 acres; $2\frac{8}{81}$ days.

---

1. $ 327.05+.
2. $ 154.44+.
3. $ 291.08–.
4. $ 1276.28+.
5. $ 874.75+.

Page 638.

1. $ 2109.
2. $ 95.25.
3. $ 40.
4. $ $34.37\frac{1}{2}$.
5. $ 5106.25.
6. $ 1080; 4 %.
7. 10 %; $5\frac{5}{7}$ %.
8. $3\frac{3}{4}$ %.
9. $\frac{1}{8}$ %.

Page 639.

10. $ 62.50; $\frac{1}{4}$ %.

---

2. Increased, $ 68.75.
3. $ 357.42.
4. $ 175.
5. 6's.
6. East, 15°.
7. 7 A.M.
8. $ 2437.60+.
9. $ 4800; ($4797.58–).

Page 640.

1. 2.646–.
2. 3.742–.
3. 6.164+.
4. 8.602+.
5. 18.708+.
6. 27.532–.
7. 28.408–.
8. 37.202+.
9. 43.290–.
10. 63.245–.

---

1. .316+.
2. .632+.
3. .949–.
4. .316+.
5. .632+.
6. .949–.
7. 1.265–.
8. 1.586+.
9. 1.897+.
13. 2.214–.
14. 2.530–.
15. 2.846–.
16. 3.162+.
17. .348–.
18. .379+.
19. .411+.
20. .443–.

---

3. 16,203.03.

Page 641.

4. 172,030.
5. 9 %.
6. $ 1160.32+.
7. $ 592.48–.
8. $ 120.75; $ 162.61; $ 62.79; $ 83.72; $ 93.38.

Page 642.

9. $ 126.80.
10. $ 171.98; $ 194.01; $ 174.08; $ 167.87.

Page 643.

1. 17.
2. 12.
3. 36.
4. 29.
5. 28.
6. 33.
7. 73.
8. 48.
9. 16.
10. 113.
11. 180 yards.
12. 10 rods.
13. 12 rd.; 18 rd.
14. $\frac{21}{40}$ acre.

Page 644.

15. $ 420.
16. 192 rods.
17. 50 rods; 1430 yd; $21\frac{1}{4}$ acres.
18. 150 yards.
19. 396 yards.
20. 113.14– rd.

---

1. $16\frac{2}{3}$ %.
2. 3 minutes.
3. 9 ¢; 3 ¢.

Page 645.

4. 300 %.
5. 7 × 10, etc.
6. $ 1.50.
7. 100 %.
8. 20,

9. 12,000.
10. 48 cases.

---

1. £45 16 s.
2. 1 lb. 14 oz.
3. 10 men.
4. 44 yards.
5. 3 hr. 16 min.
6. 319 rd. 4 yd. 1 ft. 6 in.

Page 646.

7. 10 feet.
8. 4 acres; 16 acres.
9. $2\frac{1}{4}$ pounds.
10. 432 pounds.
11. 6 days.
12. 4 %.
13. $4\frac{3}{10}$ %.
14. $4\frac{1}{3}$ %.
15. $8000.
16. $1000; $1400.

Page 647.

17. $70,000.
18. 68 rd. 3 yd. 4 in.
19. $165; $210; $225.
20. $315.
21. $6\frac{126}{1979}$ %; 6 %.
22. 6 hours.

---

1. $337.68+.
2. 1.5625.
3. (*a*) $2\frac{19}{105}$.
(*b*) $2\frac{7}{45}$.
(*c*) 180.
(*d*) $37\frac{1}{2}$.
(*e*) Impossible.

Page 648.

4. $24.
5. 1 mi. 7 fur. 27 rd. 3 yd. 2 ft. $11\frac{1}{2}$ in.
6. L.C.M. 360.
7. 194.45+ ft.
8. $6500.
9. $22\frac{2}{9}$ %.
10. 1200 peaches.

---

1. $4.45–.
2. $3.60+.

Page 649.

3. $5.76.
4. $4.38.
5. $1.17+.
6. $16.72–.
7. $11.94.
8. $51.40.
9. $76.80.
10. $92.38+.

---

1. $1000.
2. $189.92–
3. $31,000.
4. $12\frac{1}{2}$ %.
5. .7525 miles.
6. 10 mo. 17 da. nearly.
7. (10¢ per bu.); $4\frac{1}{6}$ %.

Page 650.

9. $1.72–.
10. $66\frac{2}{3}$ %.
11. 11.
12. 10,000.
13. $\frac{18}{21}$; $\frac{17}{25}$.
14. $15,000.
15. 896 pounds.
16. $472\frac{11}{112}$.
17. 28.84\frac{1}{2}$.
19. $637\frac{1}{5}$.
20. 166.58\frac{1}{3}$.
21. 6000 copies.
22. $18\frac{2}{11}$ years.
23. $28\frac{268}{539}$ bbl.
24. 80 rods.

Page 651.

2. $1091.66+.
3. $128\frac{4}{7}$ %.
4. $34,312.50.
5. $86.44–.
6. $1551.27+; ($1552.06–).
7. $6000.

Page 652.

8. (*a*) $33,000; (*b*) $13\frac{1}{2}$ %.
9. N. Y. & N. E. $50.50.
10. 40 acres.

---

2. 1470 sq. ft.
3. 294 sq. rd.
4. 1764 sq. rd.
5. 300 sq. yd.
6. 42.332+ ft.; 2031.94– sq. ft.

Page 653.

7. 15.588+ sq. ft.
8. 1176 sq. ft.
9. 672 sq. ft.
10. 3.464+ sq. in. Diagonals, 2 in. and 3.464+ in.
11. 21.008+ feet.
12. 50.2656 sq. in.
13. $3.1416\,x^2$.
14. 10 inches.
15. 19.635 sq. ft.

Page 654.

1. $215.75.
2. $414.33+.
3. $125.90.
4. $104.55.

Page 655.

5. $157.68+.
6. $453.61+.

---

1. 84 sq. ft.
2. 234 sq. yd.
3. 264 sq. rd.
4. 84 sq. in.
5. 990 sq. ft.
6. 900 sq. ft.
7. 420 sq. yd.

Page 656.

8. 330 sq. rd.
9. 744 sq. rd.
10. 240 sq. ch.

---

1. $215\frac{422}{715}$.

2. 1.975.
3. $100.
4. $52.02.
5. $489.75.
6. 93 lb. $9\frac{3}{5}$ oz.
7. $4\frac{461}{732}$.
8. $653.08–.
10. 2, 5, 7, 13, 23.

Page 657.

1. 3 months.

Page 658.

2. 9 months.
3. 1 yr. 5 mo.
4. $3\frac{3}{5}$ months.

Page 659.

5. $28\frac{1}{3}$ days.
6. $2\frac{1}{2}$ months.
7. $4\frac{1}{12}$ months.
8. 7 mo. 26 da.
9. $4\frac{1}{2}$ months.
10. August 10.
11. $94\frac{3}{4}$ cents.
12. $31.50; $21; $31.50.
13. A, $3500; B, $3600.

Page 660.

14. 60 bu.; 40 bu.
15. 20 bushels.
16. A, $36; B, $24.
17. A, $875; B, $1458.33+; C, $1166.67–.
18. A, 54 tons; B, $31\frac{1}{2}$ tons; C, $94\frac{1}{2}$ tons.
19. $6\frac{2}{3}$ quarts.

---

1. 186,441.

Page 661.

2. $34{,}538{,}549\frac{4}{7}$.
3. $82{,}739\frac{101}{785}$.

---

1. $540.
2. 220 %.
3. $440.
4. $2\frac{1}{4}$ hours.
5. $1440.
6. 1350 sq. in.

Page 662.

7. $1.29.
8. Feb. 12, 1809.
9. $5 gain.
10. 2025.

---

1. $9.52.
2. 1.464375.
3. 56.65– feet.
4. $21.
6. A, $2; B, $3; C, $3.50; D, $4.50.

Page 663.

8. $\frac{84}{96}$; ($\frac{7}{8}$).
9. 4725 lb.

---

1. 9600 men.
2. $37.75–.
3. $9.84+.
4. $4000.
5. 2.16603.
6. 7.95– %.
7. 5 yr. 5 mo. 20 da.
8. $3800.47+.
9. 58 feet.

Page 664.

10. $8\frac{5}{11}$ feet.
11. 7 %.
12. 1 yr. 10 mo.
13. $9956.86–.
14. $4\frac{5}{6}$.
15. $759.76.

Page 665.

1. $336\frac{48}{157}$.
2. $17\frac{5}{11}$ rods.
3. $1533.75.
4. $116.36–.
5. 7 bu; 5 bu. 1 pk. 3 qt.
8. 10 acres.
9. .66+.
10. 21 lb. 5 oz. 18 pwt. $20\frac{1}{2}$ gr.
11. 7 %.
12. $16,000.
13. 113.14– rd.

Page 666.

14. $\frac{17}{31}$.
15. 4196 cu. ft.
17. $331.86–.
18. $223\frac{3}{5}$ sq. ft.
19. $28.01+.
20. $5\frac{1}{75}$ tons.
21. $40.50.
22. 56.8+ %.
23. $27.84+.
24. 16– cents.
25. $\frac{112}{545}$; $\frac{176}{981}$; $\frac{1001}{4905}$; $\frac{224}{545}$.
26. 240 rods.

Page 667.

1. $6.2832x$.
2. $3.1416x^2$.
3. $.7854x^2$.
4. $.07958x^2$.
5. 1017.8784 sq. ft.
6. 7 yards.
7. 50 rods.
8. $\frac{x^2}{2}$
9. 11,250 sq. rd.
10. 2688 sq. m.
11. 62.35+ sq. ft.
12. 1469.69– sq. yd.
13. 7.958 sq. ft.
14. 960 sq. rd.

Page 668.

15. 4800 sq. yd.
16. 541.27– sq. rd.
17. 3800 sq. rd.
18. 234 sq. rd.
19. 779.42+ sq. yd.
20. 36 feet.
21. 40 rods.
22. 93.53+ sq. in.
23. 113.10– sq. in.
24. 50 sq. ft.

Page 669.
25. 62.832 sq. in.

———

1. 18,990.59+.
3. $\frac{1}{4}$%.
4. $49.02.
5. $21.

Page 670.
6. $220.54–; ($220.67–).
7. $310.85+.
8. £1,411,734 6 s. 1 d.
9. $21,050.
10. $1200.

———

1. $1400.
2. $8.63–.
3. $1917.
4. 284 days.
5. 357.
6. 143.
7. .8.
8. 14 feet.

Page 671.
9. 3 mi. 207 rd. 1 yd. 1 ft. 6 in.
10. 33 mi. 225$\frac{3}{11}$ rd.
11. 1 hr. 2 min. 52 sec. P.M.
12. $72.
13. .6 week.
14. $34.26+.
15. 42$\frac{2}{3}$ yards.
16. 175 sheep.
18. $425.51.
19. 11$\frac{2}{3}$ rolls.
1. $6224.35.
2. 2 yr. 4 mo. 3$\frac{3}{4}$ da.

Page 672.
3. $1441.94+.
4. $150; $270.
5. 10%.
6. 288 boards.
7. 49 rods.
8. $1540.
9. 14 rods.
10. $50,000.

———

1. $69.85–.

Page 673.
2. $12,500.
3. 3%.
7. 20.
10. 14 weeks.
11. 21 men.
12. 240 miles.
13. $96.
14. $15,000.
15. $128.

Page 677.
$432.63–.

Page 678.
1. 306 sq. ft.; 12 ft.
2. 126 sq. yd.; 12 yd.
3. 1110 sq. rd.; 15 rd.
4. 210 sq. ch.; 15 ch.
5. 600 sq. in.; 16 in.
6. 210.
7. 1260.
8. 594.
9. 3366.
10. 3060.

———

1. $88.73+; $86.27–.
2. 309.37\frac{1}{2}$; 464.06\frac{1}{4}$; 513.56\frac{1}{4}$.
3. 27\frac{1}{12}$; 32\frac{1}{2}$; 29\frac{1}{6}$; 41\frac{1}{4}$.

Page 679.
4. $4945.05+; $5934.07–; $4120.88–.
5. $42.96–; $34.80–; $28.35+; $46.39+.
6. 6 men.
7. 31$\frac{3}{7}$ days.
8. 208 acres.
9. 240 men.
10. 20 days; 24 days.

Page 680.
11. $13.
12. $226.80.
13. 1363$\frac{1}{2}$ lb.
14. 12$\frac{1}{2}$ days.

Table.
1883, $25.923.
1884, $26.254.
1885, $28.972.
1886, $26.306.
1887, $27.543.
1888, $27.307.
1889, $29.499.
1890, $25.772.
1891, $25.680.
1892, $27.801.

Page 681.
1. 96 sq. in.
2. 72 sq. in.
3. 144 sq. in.
5. 75.3984 sq. in.
7. 294 sq. in.; $6x^2$.
8. 6 inches.
9. 216 sq. in.

Page 682.
10. 128 sq. in.
11. 10 feet.
12. 1200 sq. in.; 20 in.
13. 702 sq. in.
14. 4.7124 sq. ft.
15. 48 sq. in.
17. 64 sq. in.
18. 576 sq. in.
19. 6.93+ sq. in.
20. 37.6992 sq. in.
21. 122.5224 sq. in.

Page 683.
22. 13 inches.
23. 122.5224 sq. in.

———

1. 130,548 sessions.
2. 11 poems.

3. $237.40; ($237.52).
4. $10,000.

Page 684.
5. $40.
6. 303 feet.

Page 686.
1. $10.34+; $505.38–.
2. $78,133$\frac{1}{3}$.
3. 68$\frac{1}{2}$%.
4. 57 cents.
5. 4 lots.
6. $55.15–.
7. $7600.
8. 285 acres; $213.75 commission.
9. $425.52+; $443; ($446.28+). $900; ($899.54+).
10. $1394.05.

Page 687.
11. 4$\frac{31}{36}$ tons.
12. 21$\frac{1}{3}$ acres.
13. .56; .0012.
14. 144 yards.
15. 26$\frac{11}{14}$ cents.
16. 1$\frac{79}{121}$ cents.
17. $441.
18. $12,204.
19. 27 sq. yd.
20. 7$\frac{1}{7}$%.

Page 688.
21. 395,999.-922186; $$\frac{21}{40}$; $\frac{1}{2}$.
22. 24$\frac{4}{9}$ yards.
23. $2022.22+; $371.44–; ($371.62$\frac{1}{2}$).
24. 20%.
25. $1280.42+; ($1279.54+).
26. $8,575,875.
27. $54,368.52$\frac{1}{2}$.
28. 3960 inches.
29. 5$\frac{5}{9}$% gain; 5$\frac{5}{19}$% loss.
30. 67,750 acres.

Page 689.
1. 18 cu. ft.
2. 400 cu. in.
3. 60 cu. in.
4. 82$\frac{1}{2}$ cu. yd.
5. 83.14– cu. ft.
6. 30.56– cu. ft.
7. 169.65– cu. meters.
8. 1163.29– gal.
9. 9719.325 lb.
10. 57$\frac{3}{5}$ cu. in.

Page 690.
12. 3$\frac{1}{2}$ inches.
13. 301.59+ cu. yd.
14. 13 inches.
15. 700 sq. in.; 896 sq. in.; 1568 cu. in.

---

1. $1,468,380,-830.
2. 1,001,101.
3. 202,100,001.-00006.
6. 1983.38\frac{1}{3}$.

Page 691.
7. $2.382+.
8. 10 T. 17 cwt. 3 qr. 8 oz.
9. $51,000; $1260; $49,740.
10. $353,369,-654.14; 94.84– %.
11. $47,891,-785.50; 3.9083+ %.
12. 5$\frac{111}{343}$%.

Page 692.
13. $8000.
14. Due, $11,-646.19.

Page 693.
1. 16 (board) ft.
2. 7 (board) ft.
3. 8 (board) ft.
4. 14 (board) ft.
5. 4 (board) ft.
6. 18 (board) ft.
7. 7 (board) ft.
8. 24 (board) ft.
9. 14 (board) ft.
10. 16 (board) ft.
11. 40$\frac{1}{2}$ (board) ft.
12. 27 (board) ft.
13. 30 (board) ft.
14. 15 (board) ft.
15. 9 (board) ft.
16. $16.20.
17. 1575 (board) ft.
18. $3.56.

Page 694.
19. $60.48.
20. $52.

---

2. 3$\frac{4}{7}$.
3. 6 lots.
4. 5889\frac{1}{3}$.
5. 36 days.
6. 108.
7. 146.86+ mi.
8. B, $1000; C, $1500.

Page 695.
9. $5.94–.

---

1. 682.32 sq. ft.
2. 117.81 sq. in.
3. 50 sq. in.
4. 28.54 sq. in.
5. 7.854 in; 7.071+ in.
6. 78.54 sq. in.; 28.54 sq. in.
7. 3.1416 sq. in.; 12.5664 sq. in.

8. 1 to 9; area $= R^2 \times 3.1416$.

Page 696.

9. $43.6\frac{1}{3}$ sq. yd.
10. 392.7 sq. ft.
11. 84.8232 sq. in.
12. 3 : 1.
13. 500 sq. ft.
14. 500 sq. ft.
15. 1 : 4.

Page 698.

16. 12.5664 sq. in.
17. 31 cents.
18. Equal.
19. 2 : 3.
20. 127.328 sq. in.

---

1. $1\frac{1}{2}$%; $90.
2. $88,922.4231.

Page 699.

3. $13,173.60.
4. $$112\frac{1}{2}$.
5. $6050.

Page 700.

1. 13.
2. 21.
3. 32.
4. 41.
5. 53.
6. 62.
7. 75.
8. 82.
9. $\frac{8}{9}$.
10. $\frac{11}{12}$.
11. $\frac{20}{23}$.
12. 1.5.
13. $1\frac{1}{8}$.
14. $1\frac{1}{11}$.
15. $1\frac{7}{9}$.

Page 701.

1. 113.0976 cu. in.
2. 14.1372 cu. in.
3. 1 : 8.
4. .5236 : 1.

Page 702.

5. 113.0976 in.; 4071.5136 sq. in.; 24,429.0816 cu. in.
6. 245 lb. 7 oz.
7. 2 : 3.
8. .4764; about $\frac{1}{2}$.
9. .2146; about $\frac{1}{5}$.

---

1. 44 men.
2. A, $100; B, $120; C, $120.
3. 7.62+ ft.; (16.89+ ft.).
4. $821.76+; ($825.50).

Page 703.

5. $4160.30–; ($4157.60–).
6. 452.3904 sq. in.; 904.7808 cu. in.
7. 3 hr. 13 min. 36 sec. fast.
8. 185.61– sq. ft.
9. 192 shares.
10. $3200.
11. 3450 copies.
12. A, $375; B, $150; C, $100.
13. $1059.35+; ($1059.39–).

Page 704.

14. $1434.29+.
15. 2 : 58 : 48 P.M.
16. $19\frac{13}{98}$ days.
17. $7\frac{1}{2}$ days.
18. $13,000.
19. 2013.7824 sq. in.
20. 452.3904 cu. in.

---

2. (*a*) $1\frac{79}{112}$; (*b*) $\frac{5}{16}$.

Page 705.

6. $17,728.53–.
7. 15 hours.
8. $73.80.
9. 4 mi. per hr.
10. 10 hours.

Page 706.

12. (*a*) 8; (*b*) $1\frac{7}{8}$; (*c*) .625.
13. (*a*) 2 rd. 1 yd.
14. (*a*) .001, .0001024, 32.004; (*b*) .003672; (*c*) 1600.
16. (*a*) $\frac{3}{7}$; (*c*) .125 acre.
17. 25%.
18. A, $50; B, $13,600; C, $4350.

Page 707.

19. 100 miles.
20. $2000.
22. (*a*) $\frac{1}{3}$; (*c*) $\frac{11}{32}$.
23. (*a*) $\frac{16}{21}$; (*b*) .00375; (*c*) .16.

Page 708.

25. (*b*) 4000; (*c*) 13.163; (*d*) 1.706.
27. $$33\frac{1}{3}$.
28. $200,000.
29. 2 men.
30. $75.

Page 709.

32. (*a*) 7; (*c*) $1\frac{11}{30}$.
33. (*a*) $\frac{7}{10}$; (*b*) 22 rd. 4 yd. 2 ft. $1\frac{5}{7}$ in.; (*c*) .0625.
35. (*b*) 20,007.253; (*c*) .00003.
36. (*a*) .00091; (*b*) 00006.
37. 8 men.

38. 1333\frac{1}{3}$;
$2000;
2666\frac{2}{3}$.

Page 710.

39. $1514.
40. $60.

---

1. 113.
2. 124.
3. 155.
4. 341.
5. 2.35.
6. 4.06.

Page 711.

1. $109;
$15.95–.
2. 8$\frac{1}{3}$%;
15 years.
3. $600;
90 (87) days.
4. January 12.
5. $950;
$5937.50.
6. 2$\frac{7}{34}$%;
2$\frac{11}{31}$%.
7. 60 feet;
231.
8. 2$\frac{1}{4}$ feet;
263.8944.
9. 171.65+ sq. in.

Page 712.

10. 351.8592 sq. in.;
502.656 cu. in.

---

2. 3$\frac{1}{3}$.

3. 122° 26′ 15″ West.
4. .00005.
5. A, $2870;
B, $7175;
C, $1435.
6. $545.82+.
7. 8%.
8. 117 feet.
9. 2.16+.
10. 16$\frac{2}{3}$%.

Page 714.

1. 44.83\frac{1}{3}$.
2. $\frac{19}{40}$.
3. 25.62\frac{1}{2}$.
4. $9.80.
5. 18 days.
6. 45 men.
7. $44,092.
8. $65.
9. $246.36+;
($246.15+).

Page 715.

10. £6 3 s. 11$\frac{1}{4}$ d.

---

1. 17,000;
.0002938.
1st, greatest;
2d, least.
.000003125.
2. $\frac{37}{850}$;
5$\frac{73}{135}$;
$\frac{2}{14}$, $\frac{16}{63}$.
3. £.4545–;
28,475 m.;
35.17\frac{1}{2}$.

4. A, $105;
B, $87.50.
5 miles.
5. $5600;
$97,920.

Page 716.

6. 30,013;
.1716–.
7. 26 days.
8. $40.80.
9. $189; $147.
10. 17.43–%.
11. 7 mo. 6 da.
12. The latter.
13. $\frac{1}{30}$ less.

Page 717.

14. $5145.
15. 5 miles.
16. 131$\frac{1}{4}$ miles.

---

1. .023825+;
18$\frac{3}{4}$%;
$\frac{3863}{5000}$.
2. $715.
3. 18.000002+;
7.745967–.

Page 718.

4. 2945 kilos nearly.
5. $1521.30–,
($1520.27+);
$1837.99–,
($1836.73+).

---

1. Latter, $\frac{1}{75}$ greater.
2. .909.
3. $101.86+.

4. 23,048,771 sq. in.
5. Thos., $2.25;
Henry, $1.35;
Richard, $0.54.
6. 233$\frac{3}{4}$ qt. dry measure.
7. [illegible] minute.

Page 719.

8. A, 56 times;
B, 35 times;
C, 22 times.

---

1. $756.96.
2. $1530.
3. $821.57+.
4. $1178.46.

Page 721.

1. $150.
2. $360.
3. 5 miles.
4. 162 miles.
5. 15,840 feet.
6. 760 acres.

Page 722.

1. 989.95 francs.
2. 22.525 sq. m.
3. 176.715 sq. m.
4. 93.15 ares.
5. 1029 ares.

Page 723.

6. 276.25 francs.
7. 546,000 liters.
8. 296 bottles.
9. 306 marks.
10. 55,200 kilos.
11. .62137– mi.

12. 15,748 feet.
13. 64 cu. in.; $1\frac{25}{231}$ qt.
14. $2\frac{41}{42}$ bushels; $27\frac{164}{231}$ gal.

Page 724.

15. $2\frac{17}{54}$ pounds.
16. 25,2525.25+ miles.
17. $1\frac{19}{81}$ sq. yd.
18. $2\frac{5398}{9801}$ acres.
19. $20\frac{20}{99}$ rods.
20. $37\frac{1}{27}$ cu. ft.
21. $16\frac{11}{54}$ grains.
22. 1.584 Km.

Page 725.

23. $19.82–.
24. 289.56– mi.
25. $24.80–.

Page 726.

1. 101 lb. 7 oz. 12 pwt. 12 gr.
3. 33 mi. 172 rd. 2 ft. $1\frac{5}{7}$ in.

Page 727.

4. 1 hr. 13 min. $36\frac{4}{5}$ sec.; 26° 15′ east.
5. $96 gain.
6. 18.8496 feet; 28.2744 sq. ft.; 12.5664 cu.in.
7. 48 ¢; $1.20.
8. 40 acres.
9. $192\frac{6}{7}$ bushels; 231 cu. in.
10. 99 yards.

Page 729.

3. $-6\,xy$.
4. $11\,abc$.
5. $-3\,xyz$.
6. $x+6$.
7. $-10\,a-8\,x$.
8. $-a+5\,b-4\,c$.
9. $1\frac{1}{4}\,x-1$.
10. $3\frac{1}{3}\,x+38\frac{1}{2}$.

Page 731.

9. $2\,x+4$.
10. $8\,x+1$.
11. $-4\,x+9$.
12. $-2\,x+20$.
13. $4\,x+5$.
14. $-x-3$.
15. $2\,x+3\,a+4$.
16. $15\,y-z-5\,b$.
17. $-2\,d+e+f$.

Page 732.

1. 6.
2. 9.
3. 6.
4. 4.
5. 9.

Page 733.

7. 9.
8. 6.
9. 6.
10. $3\frac{5}{7}$.

Page 734.

11. 11.
12. 11.
13. 35.
14. 9.
15. $-5\frac{1}{2}$.
16. 2.
17. 2.
18. 4.
19. $6\frac{2}{3}$.
20. 120.
21. 120.
22. $186\frac{2}{3}$.
23. 12.
24. $\frac{1}{2}$.
25. 12.

Page 735.

1. 15.
2. 96.
3. 20 years.
4. $1600.
5. 96 marbles.
6. Father, 88 years; son, 50 years.
7. 20 cents.
8. 3 years.

Page 736.

9. 48 gallons.
10. 432.
11. 118; 548.
12. 14 two-dollar bills; 15 five-dollar bills.
13. 6 years; 36 years.
14. 12; 15.
15. $3, son; $4, father.
16. 32; 15.
17. 13 years.

---

1. 10 cents.

Page 737.

2. 8.
3. 50 cents.
4. 2 pigs.
5. 40 years.
6. 4 cents.
7. 12 yards.
8. 24 cents.
9. 6.

---

1. 4.
2. 5.
3. 12.
4. 7.
5. 24.

Page 738.

6. 4.
7. 6.
8. 9.
9. $8\frac{1}{3}$.
10. 50.
11. Coat, $12; vest, $3.
12. 3.
13. 8.
14. $x=7, y=8$.
15. $x=9, y=6$.
16. $x=3, y=4$.
17. $x=5, y=10$.

Page 739.

18. $x=6, y=8$.
19. $x=1, y=1$.
20. $x=1\frac{1}{2}, y=2\frac{1}{4}$.
21. $x=11, y=7$.

22. $x=2,\ y=3$.
23. $x=12,\ y=3$.
24. $x=1,\ y=2$.
25. $x=3,\ y=4$.
26. $x=\frac{1}{3},\ y=\frac{1}{2}$.
27. $x=7,\ y=5$.
28. $x=4,\ y=-4$.
29. $x=2,\ y=19$.

Page 740.

30. $x=4,\ y=24$.
31. $x=42,\ y=63$.
32. $x=5,\ y=7$.
33. $x=64{,}000,\ y=36{,}000$.
34. $x=6,\ y=3$.
35. $x=8,\ y=9$.
36. $x=11,\ y=15$.
37. $x=23,\ y=18$.
38. $x=13,\ y=9$.
39. $x=10,\ y=5$.

---

1. 15 and 22.
2. 47 and 19.
3. 5 and 4.
4. 23 and 42.

Page 741.

5. 19 two-dollar bills; 13 five-dollar bills.
6. 10 pigs; 15 sheep.
7. Oranges, 3 ¢; peaches, 2 ¢.
8. Tea, 60 ¢; coffee, 25 ¢.
9. 4 horses, 16 cows, 32 sheep, 2 pigs.
10. Raisins, 12 ¢; cheese, 17 ¢.
11. 12 and 7.
12. $\frac{35}{91}$.
13. $\frac{166}{315}$.
14. 40 pounds.

Page 742.

15. 40 pounds green tea; 60 pounds black tea.

Page 743.

2. $x=2,\ y=3,\ z=5$.
3. $x=7,\ y=13,\ z=1$.
4. $x=12,\ y=31,\ z=19$.
5. $x=10,\ y=13,\ z=16$.

Page 744.

6. $x=12,\ y=18$.
7. $x=12,\ y=6$.
8. $x=6,\ y=5$.
9. $x=19\frac{1}{2},\ y=-17$.

---

1. $30,000.
2. A, 20 chestnuts; B, 2 chestnuts.
3. $5; $3.
4. 17, 38, and 45.

Page 745.

5. 21 and 32.
6. $18; $32; $16.
7. $600.
8. $150.
9. $66\frac{1}{3}$ acres; $38\frac{5}{9}$ acres; $25\frac{1}{12}$ acres.
10. 70 yards.
11. $20.

Page 746.

1. $x^2+7x+10$.
2. $x^2+17x+72$.
3. $2x^2+9x+10$.
4. $2x^2+26x+72$.
5. $3x^2+22x+7$.
6. $4x^2+4x+1$.
7. $x^2+6x-27$.
8. x^2+x-42.
9. x^2-25.

Page 748.

1. ± 7.
2. ± 5.
3. ± 5.
4. ± 5.
5. ± 5.
6. 24.
7. 24.
8. ± 5.
9. 13.
10. 5.
11. ± 7.
12. ± 1.
13. ± 8.
14. ± 4.
15. ± 6.
16. 7.
17. 13.
18. ± 6.
19. 13.
20. 13.

---

1. 60 rods; 30 rods.
2. 4 inches.
3. 10 and 8
4. 45.
5. 50.

Page 749.

6. 2500.
7. 12 rods; 20 rods.
8. 12 yards.
9. 8 feet.
10. 24 and 25.

Page 751.

1. $x+3=\pm 7$.
2. $x-6=\pm 8$.
3. $x-4=\pm 6$.
4. $x-8=\pm 5$.
5. $x+9=\pm 10$.
6. $x+1=\pm 5$.
7. $x-7=\pm 8$.
8. $x-11=\pm 12$.
9. $x+7=\pm 10$.
10. $x-11=\pm 13$.

---

1. 7 or -1.
2. 18 or -6.
3. 6 or -8.
4. 5 or -23.
5. 13 or 1.
6. 10.
7. 5 or -25.
8. 2 or -28.
9. 24.
10. 24 or -16.
11. 3 or 1.
12. 5 or -35.

13. 1 or -29.
14. 4 or -26.
15. 8.
16. 2 or -38.

Page 752.

1. 3 or -4.
2. 5 or -2.
3. -1 or -4.
4. 8 or -1.
5. -4 or -5.
6. 7 or 4.
7. -6 or -7.
8. 19 or -4.
9. 18 or -1.
10. -18 or -1.

---

1. 3, -2.
2. 4, -5.
3. 3, -7.
4. 8, 2.
5. 9, -7.
6. 3, -6.
7. 5, 4.
8. 1, -8.
9. 2, -9.
10. 4, 1.

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1. 4 and 8.
2. 80 feet.
3. 60 yards.
4. 25 yards.
5. 5 feet.
6. 4 feet.
7. 68 rods.

Page 754.

8. 34 feet.
9. 65 feet.
10. $3\frac{1}{3}$ feet.

11. $AC = 42$ ft. $ED = 24$ ft.

Page 784.

9. 1 : 4.
10. 90°, 5 in.; $1\frac{1}{2}$ in., 2 in., $2\frac{1}{2}$ in.; 37°, 53°, 90°.

Page 785.

11. $\frac{23}{72}$ in.

---

1. 60 feet.
2. 48 feet.

Page 786.

3. $109\frac{1}{11}$ feet.
4. 2160 feet; 2060 feet.
5. 124 feet.

Page 787.

6. 91 chains.
7. 162 feet.
8. 84 feet.

Page 788.

9. 108 feet.
10. 119 yards.

---

1. 2.0944 in.; 4.1888 in.; 6.2832 in.; 8.3776 in.; 10.4720 in.
2. 2 inches; 3.464+ in.; 4 inches; 3.464+ in.; 2 inches.

3. 35°; 40°; 105°; 105°.

Page 793.

10. 14.1372 sq. in.
11. $1\frac{1}{2}$ in., 3 in.; 1 in., 3 in.

Page 794.

12. 108°.
14. Two, 15 in.; two, 13 in.; 384 sq. in.
15. 240 sq. in.
17. 40 sq. ft.

Page 795.

20. 122.5224 sq. in.
21. 175 sq. in.
22. 257.6112 sq. in.
23. 27 inches.

Page 796.

24. 37.6992 sq. ft.; 21.2058 sq. ft.
25. $4166\frac{2}{3}$ mi.
26. 2000 miles.
27. 12,500 miles.
28. 1 : 2.
29. 2828.4+ mi.

Page 797.

30. 113.0976 sq. in.
31. 56.5488 sq. in.; 28.2744 sq. in.; 84.8232 sq. in.

---

1. 72 cu. in.
2. 36 cu. in.
3. 36 cu. in.

Page 798.

4. 36 cu. in.; 124.71$-$ cu. in.
5. 690 cu. in.
6. 62.882 cu. in.

Page 799.

7. 144 cu. in.; 18 cu. in.
8. 126 cu. in.
9. 1350 cu. in.; 50 cu. in.
10. 1300 cu. in.; 520 sq. in.; 770 sq. in.

Page 800.

11. 1300 cu. in.
12. $65\frac{1}{3}$ cu. ft.
13. 2232 cu. in.

Page 801.

14. 1927.3716 cu. in.
15. 929.9136 cu. in.; 4.0256$-$ gal.

Page 803.

16. 3053.6352 cu. in.
17. 381.7044 cu. in.; 190.8522 cu. in.